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APPROXIMATION ON FIXED POINT FOR F-ITERATIVE SCHEME

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Abstract. In this paper, we study the approximation of fixed points of mappings satisfying the (η, θ)-condition in uniformly convex Banach spaces. Using the F-iterative scheme, we establish weak and strong convergence results under suitable assumptions. Our results extend and generalize several existing fixed point theorems, including those for Suzuki-type mappings. Finally, the applicability of the proposed (η, θ)-condition and the associated iterative scheme is demonstrated through an image restoration problem, supported by numerical experiments.

Keywords. Banach space, $(\eta, \theta)-$ condition, convergence results, $F-$ iteration.
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1. Introduction and Preliminaries

Throughout this article, $\mathbb{I}^+$ denotes the set of all positive integers, $\mathcal{B}$ is a Banach space. $\mathcal{Q}$ is a nonempty closed convex subset of a Banach space $\mathcal{B}$, and $\Phi : \mathcal{Q} \to \mathcal{Q}$ a mapping with $Fix(\Phi)$ the set of all fixed points of Φ.

A mapping $\Phi : \mathcal{Q} \to \mathcal{Q}$ is said to be

(1) A contraction if, for all $\varkappa, \nu \in \mathcal{Q}$, there exists $\varrho \in (0, 1)$ such that

$$\|\Phi(\varkappa) - \Phi(\nu)\| \le \varrho \|\varkappa - \nu\|.$$

(2) [12] A nonexpansive mapping if

$$\|\Phi(\varkappa) - \Phi(\nu)\| \le \|\varkappa - \nu\|,$$

holds for all $\varkappa, \nu \in \mathcal{Q}$.

(3) [11] A mapping $\Phi : \mathcal{Q} \to \mathcal{Q}$ is said to be quasi-nonexpansive if

$$\|\Phi(\varkappa) - p\| \le \|\varkappa - p\|, \quad \forall \varkappa \in \mathcal{Q},\ p \in Fix(\Phi).$$

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Browder [7] showed that, if $\mathcal{B}$ is a uniformly convex Banach space and $\mathcal{Q}$ is a nonempty, closed, bounded, and convex subset of $\mathcal{B}$, then every nonexpansive mapping on $\mathcal{Q}$ has a fixed point.

Definition [29] Let $\mathcal{Q} \neq \emptyset$ be a subset of a Banach space $\mathcal{B}$. A mapping $\Phi : \mathcal{Q} \to \mathcal{Q}$ is said to satisfy the (C) condition if

$$\frac{1}{2}\|\varkappa - \Phi(\varkappa)\| \leq \|\varkappa - \nu\| \Rightarrow \|\Phi(\varkappa) - \Phi(\nu)\| \leq \|\varkappa - \nu\|,$$

for each $\varkappa, \nu \in \mathcal{Q}$. The mapping satisfying condition (C) do not need to be continuous; hence, condition (C) is weaker than the one depicting nonexpansive mappings. However, mappings satisfying condition (C) were stronger than one defining quasi-nonexpansive mappings.

In this paper, we consider a mapping introduced by Sahu et al. [21], which satisfies the $(\eta, \theta)-$ condition. This mapping plays a key role in the development of our main results and serves as a basis for the convergence analysis of the F-iterative scheme.

Let $\mathcal{Q}$ be a nonempty subset of a Banach space $\mathcal{B}$. Let η, θ be real numbers such that

$0 \leq \theta < \frac{1}{4}$, $4\theta \leq \eta \leq 1$. A mapping $\Phi : \mathcal{Q} \to \mathcal{Q}$ is said to satisfy the (η, θ)-condition if for all $\varkappa, \nu \in \mathcal{Q}$,

$$\eta\|\varkappa - \Phi(\varkappa)\| \leq \|\varkappa - \nu\| + \theta\|\nu - \Phi\nu\|,$$

implies

$$\|\Phi(\varkappa) - \Phi\nu\| \leq \frac{1 - \eta + \theta}{1 - \theta}\|\varkappa - \nu\| + \frac{\theta}{1 - \theta}\|\varkappa - \Phi(\varkappa)\|.$$

Remark 1.1. The introduced (η, θ)-condition provides a flexible framework that extends several known classes of nonlinear mappings, including nonexpansive and condition (C) mappings. This generalization allows for a broader applicability of fixed point iterative methods.

Sahu et al. [28] demonstrated that the $(\eta, \theta)-$ condition is a generalized form of (C) using the following example.

Example 1.1. Let $\Phi : [0, 3] \to [0, 3]$ be defined by

$$\Phi(\varkappa) = \begin{cases} 0, & \text{if } \varkappa \neq 3 \\ 2, & \text{if } \varkappa = 3. \end{cases}$$

Here, Φ satisfies $(\eta, \theta)-$ condition, but not (C).

In 1922, Banach [6] established that a fixed-point contraction map can be defined within a complete metric space. Furthermore, Banach's findings lead to the development of the Picard [26] iterative scheme, represented as $\varkappa_{n+1} = \Phi\varkappa_n$, which is used to approximate the unique fixed point of the contraction map. The Picard iterative scheme is recognized as the simplest method for estimating solutions to fixed-point equations. However, the Picard iterative scheme does not converge to a fixed point of a large class of mappings. Therefore, it is natural to investigate new iterative schemes to overcome such problems and get better convergence speed.

We recall some known iterative schemes here:

In 2007, Agarwal et al. [3] introduced S- iterative scheme as follows:

$$\begin{cases} \varkappa_1 \in \mathcal{Q} \\ \varkappa_{n+1} = (1-\vartheta_n)\varkappa_n + \vartheta_n \Phi \nu_n, \\ \nu_n = (1-\varrho_n)\varkappa_n + \varrho_n \Phi \varkappa_n, \quad n \in \mathbb{I}^+, \end{cases} \tag{1.1}$$

where $\{\vartheta_n\}, \{\varrho_n\}$ are sequences in (0,1).

In 2014, J. Srivastava [30] introduced Picard-S iterative scheme as follows:

$$\begin{cases} \varkappa_1 \in \mathcal{Q}, \\ \varkappa_{n+1} = \Phi \nu_n, \\ \nu_n = (1-\vartheta_n)\Phi\varkappa_n + \vartheta_n \Phi u_n, \\ u_n = (1-\varrho_n)\varkappa_n + \varrho_n \Phi \varkappa_n, \quad n \in \mathbb{I}^+, \end{cases} \tag{1.2}$$

where $\{\vartheta_n\}, \{\varrho_n\}$ are sequences in (0,1).

In 2018, K. Ullah and M. Arshad [36] introduced M- iterative scheme as follows:

$$\begin{cases} \varkappa_1 \in \mathcal{Q}, \\ \nu_n = (1-\vartheta_n)\varkappa_n + \vartheta_n \Phi \varkappa_n, \\ u_n = \Phi \nu_n, \\ \varkappa_{n+1} = \Phi u_n, \quad n \in \mathbb{I}^+, \end{cases} \tag{1.3}$$

where $\{\vartheta_n\}$ is a sequence in (0,1).

In 2020, T. Abdeljawad et al. [5] introduced JA -iterative scheme as follows:

$$\begin{cases} \varkappa_1 \in \mathcal{Q}, \\ \nu_n = (1-\varrho_n)\varkappa_n + \varrho_n \Phi \varkappa_n, \\ u_n = \Phi \nu_n, \\ \varkappa_{n+1} = (1-\vartheta_n)\Phi\varkappa_n + \vartheta_n \Phi u_n, \quad n \in \mathbb{I}^+, \end{cases} \tag{1.4}$$

where $\{\vartheta_n\}, \{\varrho_n\}$ are sequences in (0,1).

In 2021 Kalsoom et al. [20] introduced a new iterative scheme as follows:

$$\begin{cases} \varkappa_1 \in \mathcal{Q}, \\ \varkappa_{n+1} = \Phi((1-\vartheta_n)\Phi\varkappa_n + \vartheta_n \Phi \nu_n), \\ \nu_n = (1-\varrho_n)u_n + \varrho_n \Phi u_n, \\ u_n = (1-\sigma_n)\varkappa_n + \sigma_n \Phi \varkappa_n, \quad n \in \mathbb{I}^+, \end{cases} \tag{1.5}$$

where $\{\vartheta_n\}, \{\varrho_n\}$ and $\{\sigma_n\}$ are sequences in (0,1).

In 2020, Ali and Ali [1] introduced a new iterative scheme which is called F-iterative scheme. They proved that the F-iterative scheme is faster than S [3], Picard-S [14], V. Karakaya et al. [21], Thakur et al. [32], M* [34], and M [36] iterative schemes. Additionally, they gave the solution of a delay differential equation, approximated using the F iterative scheme. The iterative scheme read as follows:

$$\begin{cases} \varkappa_1 \in \mathcal{Q}, \\ u_n = \Phi((1-\varrho_n)\varkappa_n + \varrho_n \Phi(\varkappa_n)), \\ \nu_n = \Phi(u_n), \\ \varkappa_{n+1} = \Phi(\nu_n), n \in \mathbb{I}^+, \end{cases} \tag{1.6}$$

where $\{\varrho_n\} \in (0,1)$.

Motivated by the aforementioned works, we employ the F-iterative scheme to establish weak and strong convergence results for mappings satisfying the (η, θ) -condition in uniformly convex Banach spaces. We further present a new numerical example of a mapping satisfying the (η, θ) -condition and compare the convergence performance of the proposed F-iterative scheme with several existing iterative methods. The remainder of the paper is organized as follows. Section 1 contains the introduction, preliminaries, and basic definitions. Section 2 presents the main convergence results together with illustrative examples. Sections 3 and 4 are devoted to applications in convex minimization and image restoration, respectively. Finally, Section 5 concludes the paper.

Next, we will recall some definitions that will be used in the main results

Definition [10] Let $\mathcal{B}$ be a Banach space. Then $\mathcal{B}$ is said to be uniformly convex if for each $\epsilon \in (0, 2]$ there is a $\lambda > 0$ such that for every two elements $\varkappa, \nu \in \mathcal{B}$, any elements with $\|\varkappa\| \leq 1$, $\|\nu\| \leq 1$, $\|\varkappa - \nu\| > \epsilon$ then $\frac{1}{2}\|\varkappa + \nu\| \leq (1 - \lambda)$.

Definition ([4], [31]) Let $\mathcal{Q} \neq \emptyset$ be a subset of a Banach space $\mathcal{B}$ and $\{\varkappa_n\}$ a bounded sequence in $\mathcal{B}$. Then

(1) An asymptotic radius of $\{\varkappa_n\}$ at $\varkappa \in \mathcal{Q}$ is defined as

$$r(\varkappa, \{\varkappa_n\}) = \limsup_{n \to \infty} \|\varkappa_n - \varkappa\|,$$

(2) An asymptotic radius of $\{\varkappa_n\}$ relative to $\mathcal{Q}$ is defined by

$$r(\mathcal{Q}, \{\varkappa_n\}) = \inf\{r(\varkappa, \{\varkappa_n\}) : \varkappa \in \mathcal{Q}\},$$

(3) An asymptotic radius of $\{\varkappa_n\}$ relative to $\mathcal{Q}$ is defined by

$$A(\mathcal{Q}, \{\varkappa_n\}) = \{\varkappa \in \mathcal{Q} : r(\varkappa, \{\varkappa_n\}) = r(\mathcal{Q}, \{\varkappa_n\})\}.$$

The set $A(\mathcal{Q}, \{\varkappa_n\})$ is singleton.

Definition [24] A Banach space $\mathcal{B}$ is said to satisfy Opial's condition if any sequence $\{\varkappa_n\} \in \mathcal{B}, \{\varkappa_n\} \rightharpoonup u$,

$$\liminf_{n\to\infty} \|\varkappa_n - u\| < \liminf_{n\to\infty} \|\varkappa_n - v\|,$$

for all $v \in \mathcal{B}$ with $u \neq v$.

Proposition 1.1. [28] *Let $\mathcal{Q}$ be a nonempty subset of Banach space $\mathcal{B}$. Let $\Phi : \mathcal{Q} \to \mathcal{B}$ satisfy the $(\eta, \theta)-$ condition on $\mathcal{Q}$ and for $\eta \in [0, 1]$,*

(i) $\|\Phi(\varkappa) - \Phi^2(\varkappa)\| \leq \|\varkappa - \Phi(\varkappa)\|$ *if* $\eta = \theta = 0$,

(ii) at least one of the following (a) or (b) holds:

(a) $\frac{m}{2}\|\varkappa - \Phi(\varkappa)\| \leq \|\varkappa - \nu\|$

(b) $\frac{m}{2}\|\varkappa - \Phi^2(\varkappa)\| \leq \|\Phi(\varkappa) - \nu\|$.

The condition (a) implies

$$\|\Phi(\varkappa) - \Phi(\nu)\| \leq \left(\frac{(1-\frac{m}{2}) + \theta}{1-\theta}\right)\|\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)\|\varkappa - \Phi(\varkappa)\|.$$

The condition (b) implies

$$\|\Phi^2\varkappa - \Phi(\varkappa)\| \leq \left(\frac{(1-\frac{m}{2}) + \theta}{1-\theta}\right)\|\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)\|\Phi(\varkappa) - \Phi^2(\varkappa)\|.$$

$$(iii) \quad \|\nu - \Phi(\nu)\| \leq \left(\frac{3-m+\theta}{1-\theta}\right)\|\Phi(\varkappa) - \varkappa\| + \left(\frac{(1-\frac{m}{2}) + \theta}{1-\theta}\right)\|\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)[2\|\Phi(\varkappa) - \varkappa\| + \|\nu - \Phi\nu\| + \|\varkappa - \Phi(\nu)\| + 2\|\Phi(\varkappa) - \Phi^2(\varkappa)\|.$$

Lemma 1.1. [28] *Let $\mathcal{Q}$ be a nonempty closed convex subset of Banach space $\mathcal{B}$, let $\Phi : \mathcal{Q} \to \mathcal{Q}$ be a mapping satisfying $(\eta, \theta)-$ condition. If $\varkappa^*$ is a fixed point of Φ on $\mathcal{Q}$, then for all $\varkappa \in \mathcal{Q}$, the following inequality holds $\|\varkappa^* - \Phi(\varkappa)\| \leq \|\varkappa^* - \varkappa\|$.*

Lemma 1.2. [28] *Assume $\mathcal{Q}$ is weakly compact and $\mathcal{B}$ satisfies Opial condition. Let Φ be a self- mapping on $\mathcal{Q}$ satisfying the $(\eta, \theta)-$condition. If $\{\varkappa_n\}$ is a sequence in X such that*

(1) $\{\varkappa_n\}$ *converges to* $\varkappa^*$,

(2) $\lim_{n\to\infty} \|\Phi\varkappa_n - \varkappa_n\| = 0$,

then $\Phi\varkappa^* = \varkappa^*$.

2. MAIN RESULTS

In this section, we start with the following result, which is important for proving the main result. Specifically, we utilize the F-iterative scheme to approximate the fixed point that satisfies the condition (η, θ) as follows:

Lemma 2.1. *Let $\mathcal{Q}$ be a nonempty closed convex subsets of a Banach space $\mathcal{B}$ and $\Phi : \mathcal{Q} \to \mathcal{Q}$ satisfies the $(\eta, \theta)-$condition with $Fix(\Phi) \neq \emptyset$. If $\{\varkappa_n\}$ is a sequence generated by (1.6), then $\lim_{n\to\infty} \|\varkappa_n - \varkappa^*\|$ exists for each $\varkappa^* \in Fix(\Phi)$.*

Proof. Let $\varkappa^* \in Fix(\Phi)$, using (1.6) and Lemma 1.1, we obtain

$$\begin{aligned}\|u_n - \varkappa^*\| &= \|\Phi((1-\varrho_n)\varkappa_n + \varrho_n\Phi(\varkappa_n)) - \varkappa^*\| \\ &\leq \|(1-\varrho_n)\varkappa_n + \varrho_n\Phi(\varkappa_n) - \varkappa^*\| \\ &= \|\varkappa_n - \varkappa^*\|.\end{aligned} \tag{2.1}$$

Also, using (1.6), (2.1) and Lemma 1.1, we obtain

$$\begin{aligned}\|\nu_n - \varkappa^*\| &= \|\Phi(u_n) - \varkappa^*\| \\ &\leq \|u_n - \varkappa^*\| \\ &= \|\varkappa_n - \varkappa^*\|.\end{aligned} \tag{2.2}$$

Lastly, using (1.6), (2.2) and Lemma 1.1, we obtain

$$\begin{aligned}\|\varkappa_{n+1} - \varkappa^*\| &= \|\Phi(\nu_n) - \varkappa^*\| \\ &\leq \|\nu_n - \varkappa^*\| \\ &= \|\varkappa_n - \varkappa^*\|.\end{aligned} \tag{2.3}$$

This show that $\{\|\varkappa_n - \varkappa^*\|\}$ is bounded and non-increasing for all $\varkappa^* \in Fix(\Phi)$. Thus $\{\varkappa_n\}$ is bounded and $\lim_{n\to\infty} \|\varkappa_n - \varkappa^*\|$ exists. □

Theorem 2.1. *Let $\mathcal{Q}$ be a nonempty closed convex subset of a uniformly convex Banach space $\mathcal{B}$, and let $\Phi : \mathcal{Q} \to \mathcal{Q}$ satisfy the (η, θ)-condition. If $\{\varkappa_n\}$ is a sequence generated by (1.6). Then $Fix(\Phi) \neq \emptyset$ iff $\{\varkappa_n\}$ is bounded and $\lim_{n\to\infty} \|\Phi(\varkappa_n) - \varkappa_n\| = 0$.*

Proof. Assume that $\varkappa^* \in Fix(\Phi)$. It is derived from Lemma 2.1 that $\{\varkappa_n\}$ is bounded and $\lim_{n\to\infty} \|\varkappa_n - \varkappa^*\|$ exists for all $\varkappa^* \in Fix(\Phi)$.

Let κ be a real value,

$$\lim_{n\to\infty} \|\varkappa_n - \varkappa^*\| = \kappa. \tag{2.4}$$

By the proof of Lemma 2.1 together with (2.4), we have

$$\limsup_{n\to\infty} \|\nu_n - \varkappa^*\| \leq \limsup_{n\to\infty} \|\varkappa_n - \varkappa^*\| = \kappa. \tag{2.5}$$

By Lemma 1.1, we have

$$\|\Phi(\varkappa_n) - \varkappa^*\| \leq \|\varkappa_n - \varkappa^*\| \Rightarrow \limsup_{n\to\infty} \|\Phi(\varkappa_n) - \varkappa^*\| \leq \limsup_{n\to\infty} \|\varkappa_n - \varkappa^*\| = \kappa. \tag{2.6}$$

Again by the proof of Lemma 2.1 together with (2.5), we have

$$\liminf_{n\to\infty} \|\varkappa_{n+1} - \varkappa^*\| \leq \liminf_{n\to\infty} \|\nu_n - \varkappa^*\| = \kappa. \tag{2.7}$$

Accordingly from (2.5) and (2.7), we obtain that, $\kappa = \|\nu_n - \varkappa^*\|$. That is,

$$\begin{aligned}\kappa = \lim_{n\to\infty} \|\nu_n - \varkappa^*\| &= \lim_{n\to\infty} \|\Phi((1-\varrho_n)\varkappa_n + \varrho_n\Phi(\varkappa_n)) - \varkappa^*\| \\ &\leq \lim_{n\to\infty} \|(1-\varrho_n)(\varkappa_n - \varkappa^*) + \varrho_n(\Phi(\varkappa_n) - \varkappa^*)\|.\end{aligned}$$

Hence,

$$\kappa = \lim_{n\to\infty} \|(1-\varrho_n)(\varkappa_n - \varkappa^*) + \varrho_n(\Phi(\varkappa_n) - \varkappa^*)\|. \tag{2.8}$$

Thus, by Lemma 1.2, we have $\lim_{n\to\infty} \|\Phi(\varkappa_n) - \varkappa_n\| = 0$.

Conversely, let $\varkappa^* \in A(\mathcal{Q}, \{\varkappa_n\})$. Now applying the Proposition 1.1 (iii), for $\eta = \frac{m}{2}$, $m \in [0,1]$,

$$\begin{aligned}
\|\varkappa_n - \Phi(\varkappa^*)\| &\le \left(\frac{3-m+\theta}{1-\theta}\right)\|\Phi(\varkappa_n) - \varkappa_n\| + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\varkappa_n - \varkappa^*\| \\
&\quad + \left(\frac{\theta}{1-\theta}\right)[2\|\Phi(\varkappa_n) - \varkappa_n\| + \|\varkappa^* - \Phi(\varkappa^*)\| + \|\varkappa_n - \Phi(\varkappa^*)\| \\
&\quad + 2\|\Phi(\varkappa_n) - \Phi^2(\varkappa_n)\|] \\
&\le \left(\frac{3-m+\theta}{1-\theta}\right)\|\Phi\varkappa_n - \varkappa_n\| + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\varkappa_n - \varkappa^*\| \\
&\quad + \left(\frac{\theta}{1-\theta}\right)[2\|\Phi(\varkappa_n) - \varkappa_n\| + \|\varkappa^* - \Phi(\varkappa^*)\| \\
&\quad + \|\varkappa_n - \Phi(\varkappa^*)\| + 2\|\varkappa_n - \Phi(\varkappa_n)\|].
\end{aligned}$$

By Proposition 1.1

$$\begin{aligned}
(1-2\theta)\limsup_{n\to\infty}\|\varkappa_n - \Phi(\varkappa^*)\| &\le (1-\eta+2\theta)\limsup_{n\to\infty}\|\varkappa_n - \varkappa^*\| \\
\Rightarrow \limsup_{n\to\infty}\|\varkappa_n - \Phi(\varkappa^*)\| &\le \left(\frac{1-\eta+2\theta}{1-2\theta}\right)\limsup_{n\to\infty}\|\varkappa_n - \varkappa^*\| \\
&\le \|\varkappa_n - \varkappa^*\|.
\end{aligned}$$

$$\left(as \left(\frac{1-\eta+2\theta}{1-2\theta}\right) \le 1, \quad for \quad 4\theta \le \eta = \frac{m}{2}\right).$$

$$\Rightarrow r(\Phi(\varkappa^*), \{\varkappa_n\}) \le r(\varkappa^*, \{\varkappa_n\}).$$

So $\Phi(\varkappa^*) \in A(\mathcal{Q}, \{\varkappa_n\})$. As the set $A(\mathcal{Q}, \{\varkappa_n\})$ has only one element, it follows that $\Phi(\varkappa^*) = \varkappa^*$. □

Theorem 2.2. *Suppose $\mathcal{B}$ is a uniformly convex Banach space which satisfies Opial condition [24], and $\mathcal{Q} \subset \mathcal{B}$ is convex nonempty closed subset if $\Phi : \mathcal{Q} \to \mathcal{Q}$ satisfies (η,θ)- condition with $Fix(\Phi) \ne \emptyset$ and $\{\varkappa_n\}$ is a sequence defined by iteration scheme (1.6). Then $\{\varkappa_n\}$ converges weakly to a fixed point of Φ.*

Proof. Since $\mathcal{B}$ is uniformly convex, it must be reflexive. Now, according to the Theorem 2.1, $\{\varkappa_n\}$ is bounded. Then it has a weakly convergent subsequences $\{\varkappa_{n_i}\}$ and $\{\varkappa_{n_j}\}$ of $\{\varkappa_n\}$ to some point $a \in \mathcal{Q}$. In the view of Theorem 2.1, $\lim_{n\to\infty} \|\Phi\varkappa_{n_i} - \varkappa_n\| = 0$. Hence, applying Lemma 1.2, assume $\varkappa^* \in Fix(\Phi)$. Then, $\lim_{n\to\infty} \|\varkappa_n - \varkappa^*\|$ exists. We show that $\{\varkappa_n\}$ has a unique weak subsequential limit in $Fix(\Phi)$. If one assumes that this claim is not valid, then he must have a subsequence, which we may denote by $\{\varkappa_{n_j}\}$ of $\{\varkappa_n\}$ such that it will converge weakly to a point $b \in \mathcal{Q}$, and we follow in the same manner. By

Lemma 2.1, the $\lim_{n\to\infty}\|\varkappa_n - b\|$ exists such that $a \neq b$, then by the Opial's condition [24], we have

$$\begin{aligned}
\lim_{n\to\infty}\|\varkappa_n - a\| &= \lim_{n_i\to\infty}\|\varkappa_{n_i} - a\| \\
&< \lim_{n_i\to\infty}\|\varkappa_{n_i} - b\| \\
&= \lim_{n\to\infty}\|\varkappa_n - b\| \\
&= \lim_{n_j\to\infty}\|\varkappa_{n_j} - b\| \\
&< \lim_{n_j\to\infty}\|\varkappa_{n_j} - a\| \\
&= \lim_{n\to\infty}\|\varkappa_n - a\|.
\end{aligned}$$

This is a contradiction, so $a = b$. Hence $\{\varkappa_n\}$ converges weakly to a fixed point of $Fix(\Phi)$ and this completes the proof. □

Theorem 2.3. *Suppose $\mathcal{B}$ be a uniformly convex Banach space and $\mathcal{Q} \subset \mathcal{B}$ is convex nonempty closed subset and $\Phi : \mathcal{Q} \to \mathcal{Q}$ satisfies (η, θ)- condition with $Fix(\Phi) \neq \emptyset$ and $\{\varkappa_n\}$ is a sequence defined by iteration process (1.6) and $\liminf_{n\to\infty} d(\varkappa_n, Fix(\Phi)) = 0$. Then $\{\varkappa_n\}$ converges strongly to a fixed point of Φ.*

Proof. By Lemma 2.1, $\lim_{n\to\infty}\|\varkappa_n - \varkappa^*\|$ exists, for every fixed point of Φ, it follows that $\liminf_{n\to\infty} d(\varkappa_n, Fix(\Phi))$ exists. Accordingly,

$$\liminf_{n\to\infty} d(\varkappa_n, Fix(\Phi)) = 0. \tag{2.9}$$

The above limit provided as two subsequences $\{\varkappa_{n_k}\}$ and $\{\varkappa_k\}$ of $\{\varkappa_n\}$, respectively, in the following way

$$\|\varkappa_{n_k}\| \leq \|\varkappa_{n_k}\| \leq \frac{1}{2^k}, \tag{2.10}$$

it follows that

$$\begin{aligned}
\|\varkappa_{k+1} - \varkappa_k\| &\leq \|\varkappa_{k+1} - \varkappa_{n_{k+1}}\| + \|\varkappa_{n_{k+1}} - \varkappa_k\| \\
&\leq \frac{1}{2^{k+1}} + \frac{1}{2^k} \leq \frac{1}{2^{k-1}} \to 0,\ as\ k \to \infty,
\end{aligned}$$

consequently, we obtained

$$\|\varkappa_{k+1} - \varkappa_{n_{k+1}}\| = 0,$$

which show that $\{\varkappa_k\}$ is Cauchy sequence in $Fix(\Phi)$ and so it converges to an element $\varkappa^*$. By Lemma 2.1, $\lim_{n\to\infty}\|\varkappa_n - \varkappa^*\|$ exists and hence $\{\varkappa_n\}$ converges strongly to a fixed point of Φ. □

First, we construct an example of mapping that satisfies the $(\eta, \theta)-$ condition but not condition (C). We then use this example to compare the quality of $F-$iteration process with the leading Agarwal et al., Srivastava, Ullah and Arshad, Abdeljawad et al., and Kalsoom et al. iterative schemes.

Example 2.1. Let $\mathcal{Q} = [3,5]$ be endowed with absolute valued norm. Define a mapping $\Phi : \mathcal{Q} \to \mathcal{Q}$ by

$$\Phi(\varkappa) = \begin{cases} \frac{9+\varkappa}{4}, & if\ \varkappa \neq 5 \\ 3, & if\ \varkappa = 5. \end{cases}$$

It is easy to see that Φ does not satisfy the condition (C). Choose $\eta = 0.8,\ \theta = 0.2$, we prove that Φ satisfies the $(\eta,\theta)-$ condition.
Case-I: For $\varkappa, \nu \in [3,5)$, we have

$$\begin{aligned}
\left(\frac{1-\eta+\theta}{1-\theta}\right)|\varkappa-\nu|+\left(\frac{\theta}{1-\theta}\right)[|\varkappa-\Phi(\nu)|+|\nu-\Phi(\varkappa)|]&\\
&= \frac{1}{2}|\varkappa-\nu| + \frac{1}{4}\Big[|\varkappa-\Phi(\nu)| + |\nu-\Phi(\varkappa)|\Big]\\
&= \frac{1}{2}||\varkappa-\nu| + \frac{1}{4}\left(\left|\varkappa-\left(\frac{9+\nu}{4}\right)\right| + \left|\nu-\left(\frac{9+\varkappa}{4}\right)\right|\right)\\
&= \frac{3}{4}|\varkappa-\nu|\\
&\geq \frac{1}{4}|\varkappa-\nu|\\
&= |\Phi(\varkappa)-\Phi(\nu)|.
\end{aligned}$$

Case-II: For $\varkappa \in [3,5)$, and $\nu = 5$, we have

$$\begin{aligned}
\left(\frac{1-\eta+\theta}{1-\theta}\right)|\varkappa-\nu|+\left(\frac{\theta}{1-\theta}\right)[|\varkappa-\Phi(\nu)|+|\nu-\Phi(\varkappa)|]&\\
&= \frac{1}{2}|\varkappa-5| + \frac{1}{4}\left[|\varkappa-3| + \left|5-\left(\frac{9+\varkappa}{4}\right)\right|\right]\\
&\geq \frac{1}{4}|\varkappa-3|\\
&= |\Phi(\varkappa)-\Phi(\nu)|.
\end{aligned}$$

Case-III: For $\varkappa = \nu = 5$, we have

$$\left(\frac{1-\eta+\theta}{1-\theta}\right)|\varkappa-\nu| + \left(\frac{\theta}{1-\theta}\right)[|\varkappa-\Phi(\nu)| + |\nu-\Phi(\varkappa)|] \geq 0 = |\Phi(\varkappa)-\Phi(\nu)|.$$

Hence, Φ satisfies the $(\eta,\theta)-$ condition.

Next, we discussed the convergence behaviour of some iterative schemes with a table and graphical representation.

TABLE 1. Convergence of F- iteration (1.6) for the initial point $\varkappa_1 = 3.5$ satisfying $(\eta, \theta)-$condition.

Steps	Agarwal et al.	Srivastava	Ullah and Arshad	Abdeljawad et al	Kalsoom et al.	F
1	3.50000000	3.50000000	3.50000000	3.50000000	3.50000000	3.50000000
2	3.23694728	3.03118003	3.02418333	3.09666334	3.09537622	3.00604583
3	3.08645040	3.00194704	3.00133654	3.02136389	3.02061901	3.00008353
4	3.02567703	3.00012167	3.00007960	3.00508895	3.00485859	3.00000124
5	3.00701761	3.00000760	3.00000486	3.00124256	3.00117967	3.00000002
6	3.00184838	3.00000048	3.00000030	3.00030648	3.00029002	3.00000000
7	3.00047780	3.00000003	3.00000002	3.00007597	3.00007174	3.00000000
8	3.00012221	3.00000000	3.00000000	3.00001888	3.00001781	3.00000000
9	3.00003106	3.00000000	3.00000000	3.00000470	3.00000443	3.00000000
10	3.00000786	3.00000000	3.00000000	3.00000117	3.00000110	3.00000000
11	3.00000198	3.00000000	3.00000000	3.00000029	3.00000027	3.00000000
12	3.00000050	3.00000000	3.00000000	3.00000007	3.00000007	3.00000000
13	3.00000013	3.00000000	3.00000000	3.00000002	3.00000002	3.00000000
14	3.00000003	3.00000000	3.00000000	3.00000000	3.00000000	3.00000000
15	3.00000001	3.00000000	3.00000000	3.00000000	3.00000000	3.00000000
16	3.00000000	3.00000000	3.00000000	3.00000000	3.00000000	3.00000000

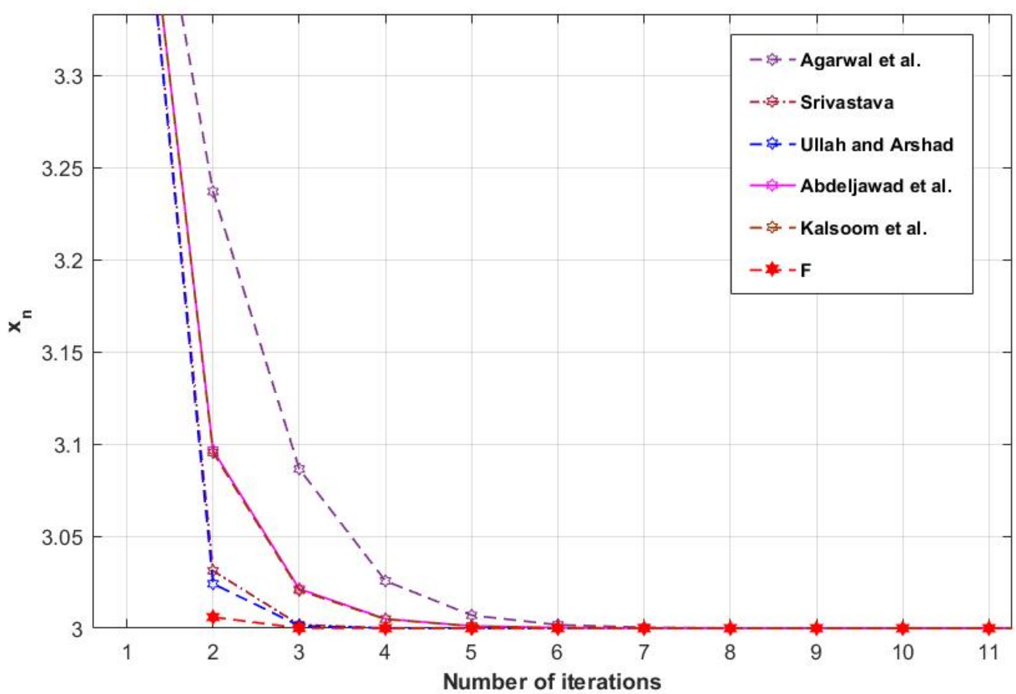

FIGURE 1. Convergence behaviour of Agarwal et al., Srivastava, Ullah and Arshad, Abdeljawad et al., Kalsoom et al. and F-iterative algorithms towards the fixed point of the mapping Φ.

Our next results in Section 3 are developed from Section 2 established in uniformly convex Banach spaces. Since the forward-backward operator, proximal mapping, and convex minimization framework employed in the present application are naturally formulated in real Hilbert spaces, we specialize our analysis to this setting. Since every real Hilbert space is a uniformly convex Banach space, the convergence results established in Section 2 remain valid in the Hilbert space framework. Consequently, we apply the proposed F-iterative scheme to the forward-backward operator associated with the convex minimization problem, leading to an application in image restoration.

3. APPLICATION TO CONVEX MINIMIZATION PROBLEM

In this section, we will analyze the background of a mathematical model related to the image restoration problem and use our method to address convex minimization problems, as well as the methods used to solve them. A model for the image restoration problem is formulated using a linear approach. Numerous fields, such as astronomy, deblurring, remote sensing, signal and image processing, statistical inference, optics, microscopic imaging, digital photography, and many more, include linear inverse problems.

The following model can be used to transform the image recovery problem into a one-dimensional vector:

$$B_z = c + h, \tag{3.1}$$

where the blurry operator is denoted as $B \in \mathbb{R}^{m\times n}$, the observed image is represented as $b \in \mathbb{R}^{m\times 1}$, the original image is $z \in \mathbb{R}^{n\times 1}$, and the additive noise is denoted by h. Tibshirani [33] introduced the least absolute shrinkage and selection operator (LASSO) to address the following minimization problem:

$$\min_z\{\|B_z - c\|_2^2 + \xi\|z\|_1\}, \tag{3.2}$$

where $\xi > 0$ is the regularization parameter, $\|z\|_1 = \sum_i^n |z_i|$ and $\|z\|_2 = \sum_i^n |z_i|^2$. The general minimization problem, which includes (4.1) as a special case, is described as follows:

$$\min_{z\in\mathbb{R}^n}\{\mathcal{G}(z) := \phi(z) + \chi(z)\}, \tag{3.3}$$

where χ is a non-smooth convex function and ϕ is a smooth convex loss function with a gradient that has a Lipschitz constant $L > 0$. The solution of the problems (3.3) can be described using Fermat's rule [8] in the following way, z^* is a minimizer of $(\phi + \chi)$ iff $0 \in \partial\chi(z^*) + \nabla\phi(z^*)$, where $\nabla\phi(z^*)$ and $\partial\chi(z^*)$ denote the gradient and subdifferential of ϕ and χ, respectively. Where $\partial\chi(z^*)$ is a subdifferential, which is a non-smooth function that is a generalization of a gradient. The collection of minimizers of ϕ is represented by $argmin(\phi)$, and the following iterative formula provides the classical forward-backward splitting algorithm [18] for the problem (3.3).

$$z^* = prox_{f_n\chi}(I - f_n\nabla\phi)(z^*)$$
$$= J_{f_n\partial\chi}(I - f_n\nabla\phi)(z^*), \quad f_n \in \Big(0, \frac{2}{L}\Big),\ n \in \mathbb{I}^+, \tag{3.4}$$

where $prox_\chi(x) = argmin\{\chi(s) + \frac{1}{2}\|z - s\|_2^2\}$, f_n represents the step size, and I is the identity operator. The equation (3.4) can easily be rewritten as a fixed point problem as follows:

$$z^* = \Phi(z^*). \tag{3.5}$$

Where $\Phi = prox_{f_n\chi}(I - f_n\nabla\phi)$ is referred to as the forward-backward operator. It is well established that Φ is a nonexpansive mapping when f_n is within the interval $\Big(0, \frac{2}{L}\Big)$. Furthermore, a solution to equation (3.5) is a fixed point of Φ.

Theorem 3.1. *Let $\mathcal{Q}$ be a nonempty closed convex subset of a real Hilbert space $\mathcal{H}$. Define the sequence $\{\varkappa_n\}$ as follows:*

$$\begin{cases} \varkappa_1 \in \mathcal{Q}, \\ u_n = prox_{f_n\chi}(I - f_n\nabla\phi)((1-\varrho_n)\varkappa_n + \varrho_n prox_{f_n\chi}(I - f_n\nabla\phi)(\varkappa_n)), \\ \nu_n = prox_{f_n\chi}(I - f_n\nabla\phi)(u_n), \\ \varkappa_{n+1} = prox_{f_n\chi}(I - f_n\nabla\phi)(\nu_n), \quad n \in \mathbb{I}^+, \end{cases} \tag{3.6}$$

where $\{\varrho_n\}$ is a sequence in $(0,1)$, $f \in (0, \frac{2}{L})$. Let $\phi : \mathbb{R}^n \to \mathbb{R}$ be a convex differentiable function with L-Lipschitz continuous gradient, and let $\chi : \mathbb{R}^n \to (-\infty, +\infty]$ be a proper, lower semicontinuous, convex function. Then, the sequence $\{\varkappa_n\}$ converges strongly to an element $z^* \in Argmin(\phi + \chi)$.

Proof. Let Φ be the forward-backward operator associated with ϕ and χ in relation to f, defined as $\Phi = prox_{f_n\chi}(I - f\nabla\phi)$. It follows that Φ is a nonexpansive operator, and the set of fixed points satisfies $F(\Phi) = argmin(\phi + \chi)$, as stated in Proposition 26.1 of [8]. We can derive the necessary results directly from Theorem 2.3. □

4. APPLICATION TO IMAGE RESTORATION PROBLEM

This section presents a comparative analysis of the deblurring performance of the F-iterative scheme with several well-known iterative methods, namely Agarwal et al., Srivastava, Ullah and Arshad, Abdeljawad et al., and Kalsoom et al. The restoration problem is formulated as

$$\min_{z}\{\|B_z - c\|_2^2 + \xi\|z\|_1\}, \tag{4.1}$$

where

$$\phi(z) = \|Bz - c\|_2^2$$

denotes the data fidelity term and

$$\chi(z) = \xi\|z\|_1,$$

represents the regularization term, where ξ is the regularization parameter. The operator A corresponds to the Gaussian blurring operator, while c denotes the observed blurred image. For the numerical experiment, a brain skull X-ray image of size 256×256 is used as the original image. The image is degraded using Gaussian blur with a 7×7 kernel and standard deviation $\rho = 1.5$, followed by additive Gaussian noise of variance 0.00012. The restoration quality is measured using the Peak Signal-to-Noise Ratio (PSNR).

The PSNR formula [15] is defined by

$$PSNR(\varkappa_n) = 10\log_{10}\left(\frac{(MAX_I)^2}{MSE}\right), \tag{4.2}$$

where MAX_I denotes the maximum possible pixel intensity value. Since the images are normalized to the interval $[0, 1]$, we take $MAX_I = 1$. Therefore, the PSNR formula reduces to

$$PSNR(\varkappa_n) = 10\log_{10}\left(\frac{1}{MSE}\right). \tag{4.3}$$

The mean squared error (MSE) is defined as $MSE = \frac{1}{S}\|\varkappa_n - \widetilde{\varkappa}\|_2^2$, where $\widetilde{\varkappa}$ denotes the original image and S represents the total number of image pixels. The initial image in this experiment was blurry, and the regularization parameter was set at $\xi = 10^{-3}$. The maximum eigenvalues of the matrix B^TB were used to determine the Lipschitz constant L. The control parameters $\{\vartheta_n\}, \{\varrho_n\}, \{\sigma_n\}$ are chosen as follows: $\vartheta_n = 0.7, \varrho_n = \sigma_n = 0.55, \eta = 0.8, \theta = 0.2$. The maximum number of iterations is taken as $n = 1300$. The numerical results are shown in Fig. 2-4 and TABLE 2.

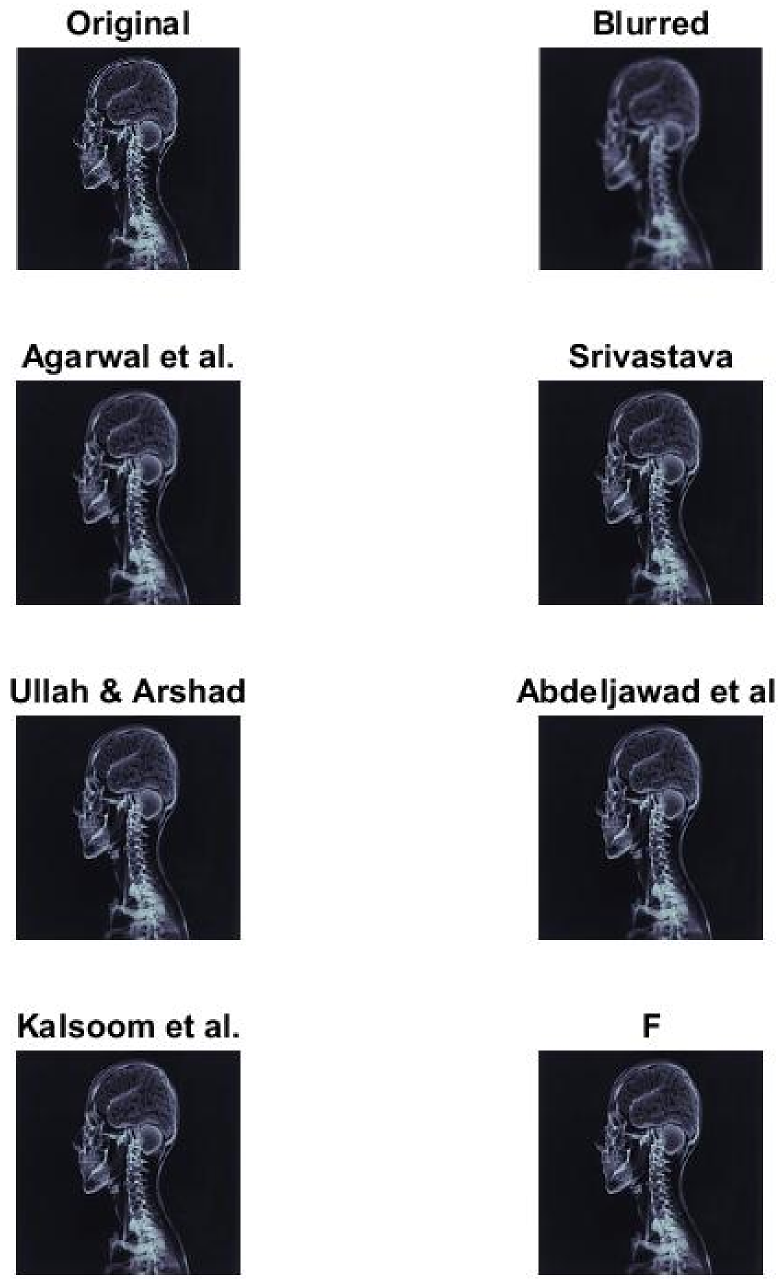

FIGURE 2. Brain skull X-ray image deblurring by various schemes

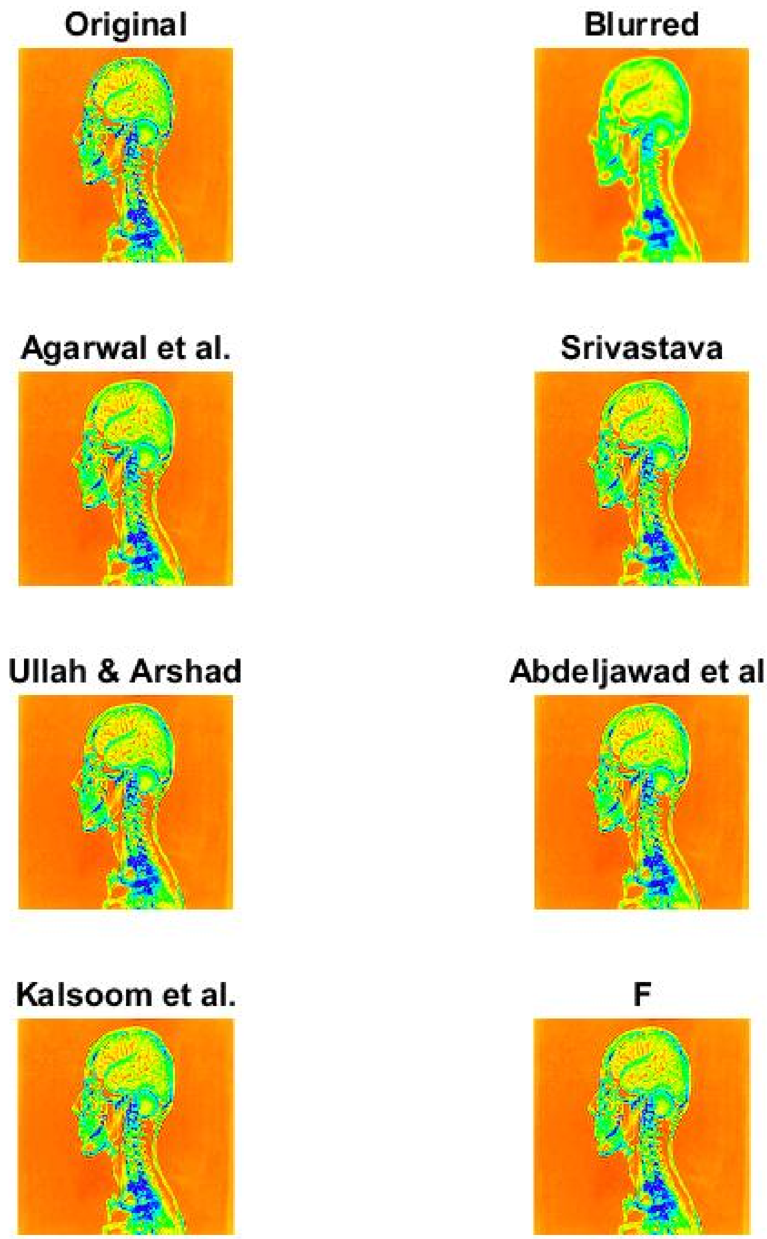

Figure 3. HSV visualization of the restored brain skull X-ray image obtained by various iterative schemes.

The numerical results demonstrate that the proposed F-iterative scheme achieves improved convergence behavior and superior PSNR performance compared with the existing iterative methods.

TABLE 2. Numerical behaviour of various methods using their PSNR values for Brain skull X-ray image

n	Agarwal et al.	Srivastava	Ullah and Arshad	Abdeljawad et al.	Kalsoom et al	F
100	28.45	29.20	29.28	29.12	29.28	29.46
300	29.09	29.85	29.94	29.77	29.94	30.15
500	29.38	30.18	30.28	30.09	30.28	30.51
700	29.58	30.42	30.52	30.32	30.52	30.77
1000	29.80	30.69	30.58	30.80	30.80	31.07

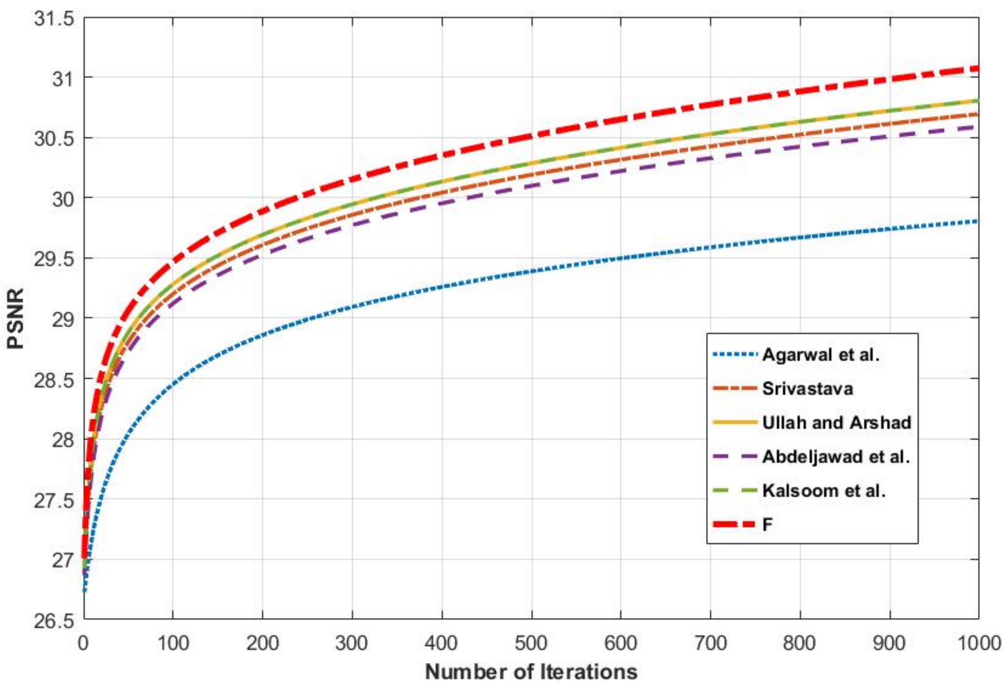

FIGURE 4. Graph Corresponding Table 2.

5. Conclusion

In this paper, we established weak and strong convergence results using F-iterative schemes for mappings with the $(\eta, \theta)-$condition in uniformly convex Banach space. A numerical example is also given to show that the F-iterative scheme converges faster than the Agarwal et al., Srivastava, Ullah and Arshad, Abdeljawad et al. and Kalsoom et al. iterative schemes under certain assumptions in mapping with the $(\eta, \theta)-$condition. The obtained numerical and visual results demonstrate that the F- iterative scheme under the generalized (η, θ)-condition is efficient and applicable to practical image restoration.

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