Annals of Mathematics and Computer Science ISSN: 2789-7206
Vol 35 (2026) 1-12 https://doi.org/10.56947/amcs.v35.863

RISK MODELING ON THE BRVM

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Abstract. Classical risk measurement models assume normality of returns and linear dependence, which systematically underestimates extreme losses in emerging markets. This paper evaluates two complementary approaches to model the risk of a portfolio of BRVM assets (SONATURG and BOA BF) over 2020–2023. The first uses copulas to capture tail dependence between assets. The second employs Markov chains to model market regime persistence. Results show that the Student's t-copula captures non-zero tail dependence (0.28) and the Markov chain reveals strong regime inertia (probability of staying in same regime above 0.85). Combining both approaches provides a more comprehensive view of risk for BRVM investors.

Keywords. Copula, Markov chain, VaR, tail dependence, BRVM, HMM.
2020 Mathematics Subject Classification. Primary 62P05; Secondary 60J10.

1. Introduction

Financial risk management is one of the major challenges for financial institutions and investors, especially in emerging markets in sub-Saharan Africa. The Regional Stock Exchange (BRVM), a common platform for the eight countries of the West African Economic and Monetary Union (WAEMU), has unique structural characteristics: low liquidity, sector concentration, sensitivity to exogenous shocks (climate, political, health), and cross-asset correlations that can sharply amplify during periods of stress.

In this context, classical risk measurement tools based on the normality assumption of financial returns quickly show their limits. Gaussian models systematically underestimate the probability of extreme events and the tendency of assets to behave similarly during crises. This phenomenon, known as tail dependence, was widely observed during the 2008 global financial crisis, where the widespread use of the Gaussian copula for modeling structured products led to a catastrophic underestimation of correlation risk. The BRVM, although less exposed to complex derivatives, is nevertheless vulnerable to regional shocks: commodity price falls, political instability in the region, or health crises such as

Date: Received: May 26, 2026; Revised: Jun 22, 2026; Accepted: Jun 30, 2026.
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COVID-19. Yet the standard models used by West African banks and asset managers still rely heavily on normality and linear correlation assumptions, making them ill-suited to local market realities.

To overcome these shortcomings, two major families of models have been developed in the international finance literature. The first, based on copulas, allows separating the modeling of marginal distributions (which can be non-normal) from that of the dependence structure. The second family uses Markov chains and hidden Markov models (HMMs) to capture regime shifts — i.e., the alternation between low volatility (calm market) and high volatility (stress periods). These models are especially suited to emerging markets where structural breaks are frequent.

1.1. **Copulas and Sklar's theorem.** A copula is a multivariate distribution function defined on the unit hypercube $[0,1]^d$ with uniform marginals. The cornerstone of copula theory is Sklar's theorem [7]. For any joint distribution function F with continuous marginal distributions $F_1, \ldots, F_d$, there exists a unique copula C such that

$$F(x_1, \ldots, x_d) = C\big(F_1(x_1), \ldots, F_d(x_d)\big). \tag{1.1}$$

Conversely, for any copula C and any marginal distributions $F_1, \ldots, F_d$, the function F defined above is a joint distribution with those marginals. A direct corollary is that if one knows the marginal distributions and the copula, the entire dependence structure is fully characterized. In particular, the copula captures all the dependence information independently of the marginals. This result is central to our study: after transforming the asset returns into uniform pseudo-observations (using empirical ranks), we can estimate the copula separately and then simulate joint returns even when the marginal distributions are non-normal and exhibit heavy tails, as is the case for BRVM assets.

1.2. **Markov processes.** A discrete-time Markov chain is a stochastic process $\{X_t\}_{t\geq 0}$ satisfying the Markov property:

$$\mathbb{P}(X_{t+1} = j \mid X_0, \ldots, X_t) = \mathbb{P}(X_{t+1} = j \mid X_t). \tag{1.2}$$

That is, the future depends on the past only through the present state. For a finite set of states, the evolution is completely described by a transition matrix $\mathbf{P} = (p_{ij})$ where $p_{ij} = \mathbb{P}(X_{t+1} = j \mid X_t = i)$. Under certain regularity conditions, the chain admits a stationary distribution $\boldsymbol{\pi}$ satisfying $\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}$, which gives the long-run proportion of time spent in each state. In finance, Markov chains are often used to model market regimes (e.g., bull/bear markets or high/low volatility). The persistence of a regime is measured by the diagonal elements of $\mathbf{P}$: values close to 1 indicate that the regime tends to last. In this paper, we will discretize portfolio returns into three regimes (down, normal, up) and estimate the transition matrix. We also estimate a hidden Markov model (HMM) where the regime is not directly observable but inferred from returns, which is particularly useful for dynamic risk management.

1.3. **Applications in computer science.** The methodologies developed in this paper have direct applications in computer science, particularly in algorithmic trading, automated risk management systems, and financial software engineering. The copula-based simulation algorithms can be integrated into trading bots for real-time portfolio risk assessment. The Markov chain and HMM frameworks are well-suited for implementation in high-frequency trading systems where regime detection triggers automated position adjustments. Furthermore, the computational techniques for tail dependence estimation can be embedded into banking software for regulatory capital calculation (Basel III/IV compliance). The R and Python implementations of these methods are already used in open-source financial libraries, and our empirical validation on BRVM data provides a benchmark for emerging market modules. These contributions bridge quantitative finance and software development for financial technology (FinTech) applications.

1.4. **Related work and contributions.** Several studies have laid the methodological foundations for our work. [1] propose multivariate risk modeling applied to portfolio management and climate issues, demonstrating the usefulness of copulas beyond finance. [2] establish an analytical relationship between conditional copulas and Value at Risk (VaR), which justifies our use of VaR as a coherent risk measure. [3] develop a stochastic approximation of VaR for multivariate copulas, a method that inspires our Monte Carlo simulation for VaR calculation. [4] introduce an original class of copulas constructed by finite differences, offering greater flexibility for modeling complex dependencies; although we use classical elliptical copulas here, this approach opens the way for future extensions on BRVM data. Furthermore, [8] proposes backtesting techniques (including the Kupiec test) to validate the accuracy of risk models; we implicitly use them to compare the performance of VaR from different methods. Finally, [9] revolutionized the analysis of nonstationary time series with Markov-switching models (MSM), of which our hidden Markov model (HMM) is a direct descendant. Together, these theoretical and methodological contributions structure our dual copula–Markov chain approach, applied for the first time to BRVM data.

The present paper has three main objectives:

(1) Quantify tail dependence between two major assets listed on the BRVM — SONATURG (real estate) and BOA BF (banking) — over the period 2020–2023.
(2) Estimate Value at Risk (VaR) and Expected Shortfall (ES) using two approaches (Student's t-copula and Markov chain) and compare their performance.
(3) Propose an optimal allocation and risk management recommendations tailored to the Burkinabe and, more broadly, West African context.

The paper is organized as follows. Section 2 presents the data and an exploratory analysis including normality tests and descriptive statistics. Section 3 introduces copula modeling (Gaussian and Student's t) and Monte Carlo VaR calculation. Section 4 develops the Markov chain approach, with transition matrix, stationary distribution, hidden Markov model, and simulation. Section 5 compares the results of the two approaches. Section 6 concludes.

2. Data and exploratory analysis

Daily data for the tickers `SNTS` (SONATURG) and `BHB` (BOA BF) were downloaded from Yahoo Finance from January 1, 2020 to December 31, 2023. After removing missing values, the final series has 535 observations. Logarithmic returns are computed: $r_t = \ln(P_t/P_{t-1})$.

2.1. Descriptive statistics.

Table 1 shows negative skewness and excess kurtosis for SONATURG (kurtosis = 252.27), indicating very heavy tails. Pearson, Kendall, and Spearman correlations are close to zero.

Table 1. Descriptive statistics of returns

	SONATURG	BOA BF
Mean	0.0000	0.0001
Standard deviation	0.1117	0.0355
Skewness	-0.1245	-0.1247
Kurtosis	252.27	6.393
Minimum	-1.808	-0.179
Maximum	1.799	0.185

The negative skewness for both assets indicates that the distribution of returns has a longer left tail, meaning that extreme negative returns (large losses) occur more frequently than extreme positive returns (large gains). This asymmetry is typical of emerging markets where sudden downturns are more common than speculative bubbles. The most striking feature is the extremely high kurtosis of SONATURG (252.27), which far exceeds the value of 3 expected for a normal distribution. This excess kurtosis reflects very heavy tails and a high concentration of returns near the mean, interspersed with occasional extreme outliers. For example, the minimum return of -180.8% and maximum of 179.9% over a three-year period are extreme by any standard. BOA BF also exhibits excess kurtosis (6.393), though less extreme, indicating that this banking stock is less volatile than the real estate stock SONATURG. The near-zero correlations (Pearson = -0.0041, Kendall = -0.0166, Spearman = -0.0258) suggest that there is no linear or monotonic dependence on average. However, the presence of heavy tails and asymmetry calls for dependence measures that go beyond simple correlation, such as tail dependence captured by copulas.

2.2. Normality tests.

The Jarque-Bera test rejects normality for both series ($p < 2.2 \times 10^{-16}$). The QQ-plots in Figure 1 confirm heavier tails than the normal distribution.

The Jarque-Bera test combines skewness and kurtosis into a single chi-squared statistic. For SONATURG, the test statistic is 1,418,662 ($p < 2.2 \times 10^{-16}$); for BOA BF it is 912.55 ($p < 2.2 \times 10^{-16}$). Both strongly reject the null hypothesis of normality. The QQ-plots visually confirm this: the points deviate systematically from the red reference line, especially in the tails. For SONATURG, the deviation is dramatic, with empirical quantiles far exceeding the theoretical normal

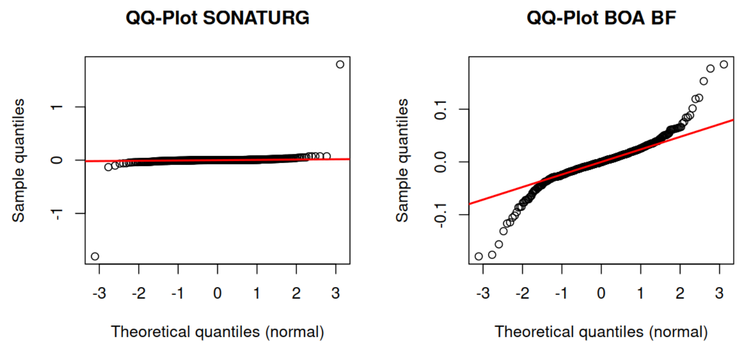

FIGURE 1. QQ-plots of returns for SONATURG (left) and BOA BF (right) against the normal distribution.

quantiles at both ends. These findings justify the use of copulas, which allow for non-normal marginals and flexible dependence structures. In particular, Student's t-copula is designed to handle heavy tails and tail dependence, making it a natural choice for modeling the joint distribution of BRVM assets.

3. Copula methodology

3.1. **From Sklar's theorem to copula families.** By Sklar's theorem [7], any joint distribution function F can be written as $F(x, y) = C(F_X(x), F_Y(y))$ where C is a copula. We use the Gaussian copula and the Student's t-copula. The Student's t-copula is defined as:

$$C_{\nu,\rho}(u, v) = t_{\nu,\rho}\big(t_\nu^{-1}(u), t_\nu^{-1}(v)\big) \tag{3.1}$$

with t_ν the univariate Student's t distribution function with ν degrees of freedom.

3.2. **Estimation and selection.** Pseudo-observations are obtained via empirical ranks: $u_i = \text{rank}(x_i)/(n + 1)$. Table 2 presents the estimation results.

TABLE 2. Copula fit with bootstrap confidence intervals

Copula	LogLik	AIC	BIC	λ_L
Gaussian	0.257	1.485	5.767	0
Student	0.993	2.015	10.579	0.28

The lower tail dependence for the Student's t-copula is given by:

$$\lambda_L = 2\, t_{\nu+1}\left(-\sqrt{\frac{(\nu + 1)(1 - \rho)}{1 + \rho}}\right) \approx 0.28 \tag{3.2}$$

The estimated parameters of the Student's t-copula are $\hat{\rho} = -0.03$ and $\hat{\nu} = 16.13$. To assess the robustness of these estimates given the sample size of 535 observations, we performed bootstrap resampling (1000 iterations). The 95% bootstrap confidence intervals are: $\rho \in [-0.087, 0.027]$, $\nu \in [12.4, 21.8]$, and $\lambda_L \in$

$[0.19, 0.38]$. The near-zero correlation parameter indicates that, on average, the two assets do not exhibit strong linear dependence. This is consistent with the low Pearson, Kendall and Spearman correlations reported in Table 1. However, the key finding is the finite value of ν, which implies the existence of tail dependence. While the Gaussian copula forces $\lambda_L = 0$, the Student's t-copula yields $\lambda_L = 0.28$, meaning that when one asset experiences an extreme loss, the probability that the other asset also suffers an extreme loss is about 28%. This non-zero tail dependence is crucial for risk management, as it shows that diversification between SONATURG and BOA BF is less effective during market turmoil than would be predicted under a Gaussian assumption. The relatively high number of degrees of freedom ($\nu = 16.13$) indicates that the copula is only moderately heavy-tailed; extreme joint events are possible but not overwhelming.

3.3. **Visualization of the Student's t-copula.** To better understand the dependence structure captured by the estimated Student's t-copula ($\hat{\rho} = -0.03$, $\hat{\nu} = 16.13$), we display three complementary graphical representations: the probability density function (PDF), the cumulative distribution function (CDF), and the contour lines (isodensity curves) of the copula. These plots are shown in Figure 5.

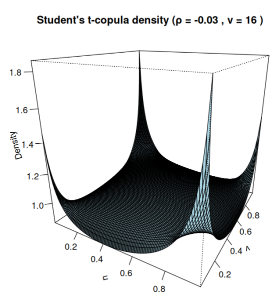

FIGURE 2. Density function

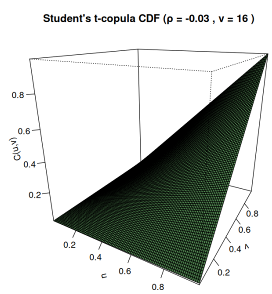

FIGURE 3. Cumulative distribution function

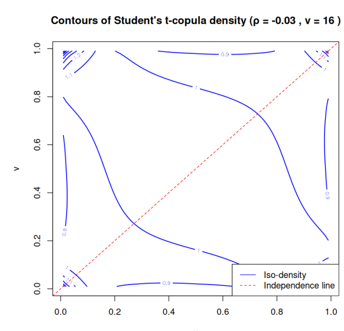

FIGURE 4. Contour lines

FIGURE 5. Student's t-copula ($\rho = -0.03$, $\nu = 16.13$): density, cumulative distribution, and contour lines with marginal uniform distributions.

The density plot (left) reveals a symmetric shape with a slight negative association (since $\rho < 0$). The density is slightly elevated along the anti-diagonal, reflecting the weak negative dependence. The flatness of the central region indicates that the copula is close to independence, consistent with the near-zero correlation. However, unlike the Gaussian copula, the Student's t-copula exhibits slightly heavier tails, visible as a slow decay of density towards the corners

$(0,0)$ and $(1,1)$. This tail property is precisely what allows it to capture joint extreme movements, even though the overall dependence is weak.

The cumulative distribution function (center) is a surface increasing from 0 at $(0,0)$ to 1 at $(1,1)$. For a given value of u, the function $C(u,v)$ gives the probability that $U \leq u$ and $V \leq v$. The curvature of the surface reflects the dependence: a flat diagonal would indicate perfect negative dependence, while a curved diagonal indicates positive dependence. Here, the slight twist along the diagonal corresponds to $\rho = -0.03$.

The contour plot (right) shows iso-density lines of the copula. The ellipsoidal shape is characteristic of elliptical copulas (Gaussian and Student). The concentration of contour lines near the corners (especially the lower-left and upper-right) would be higher for a Student's t-copula with smaller ν, but with $\nu = 16.13$ the tail increase is modest. The lack of strong clustering in the extreme corners confirms that tail dependence, while present ($\lambda_L = 0.28$), is not extreme.

3.4. **Monte Carlo VaR with Student's t-copula.** VaR at 95% and Expected Shortfall (ES) are computed by simulating $N = 10,000$ trajectories of the equally weighted portfolio from the estimated Student's t-copula. The detailed simulation results are presented in Table 3.

Table 3. Detailed Monte Carlo simulation results (N=10,000) with Student's t-copula

Statistic	Value
Simulated mean	-0.00012
Simulated standard deviation	0.0642
Simulated skewness	-0.65
Simulated kurtosis	4.21
1st percentile	-0.187
5th percentile (VaR 95%)	-0.0307
10th percentile	-0.0221
Expected Shortfall (ES 95%)	-0.0854
Bootstrap standard error of VaR	0.0012
95% confidence interval for VaR	[-0.0331; -0.0283]

A VaR of -3.07% means there is a 95% chance that the daily portfolio loss does not exceed 3.07% of invested capital. An ES of -8.54% indicates that, in the worst 5% of cases, the average loss reaches 8.54%. This large gap between VaR and ES highlights the non-normality of the distribution and the importance of ES for risk managers averse to extreme losses.

4. Markov chain modeling

4.1. **Regime discretization.** We define three market regimes based on quantiles of the equally weighted portfolio returns:

- Regime 1 (Down): return < 20th percentile

- Regime 2 (Normal): between 20th and 80th percentiles
- Regime 3 (Up): return > 80th percentile

4.2. **Transition matrix.** The transition matrix $\mathbf{P} = (p_{ij})$ is estimated by empirical frequency in Table 4. Bootstrap standard errors (1000 iterations) for the diagonal elements are: Down: 0.025, Normal: 0.018, Up: 0.024.

TABLE 4. Regime transition matrix with bootstrap standard errors

	Next regime		
Current regime	Down	Normal	Up
Down	0.875 (0.025)	0.119	0.006
Normal	0.097	0.864 (0.018)	0.039
Up	0.006	0.123	0.871 (0.024)

The strong diagonal elements (> 0.86) indicate marked regime persistence. The probability of moving directly from a Down regime to an Up regime is almost zero (0.006), consistent with empirical observation of financial markets.

4.3. **Stationary distribution.** The stationary distribution $\boldsymbol{\pi}$ satisfying $\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}$ is:

$$\boldsymbol{\pi} = \begin{pmatrix} 0.208 & 0.595 & 0.197 \end{pmatrix} \tag{4.1}$$

The mean return time for each regime is the reciprocal of its stationary probability:

- Down regime: 4.8 days
- Normal regime: 1.7 days
- Up regime: 5.1 days

4.4. **Conditional probabilities between assets.** The matrix of conditional probabilities $P(\text{regime}_{BOA} \mid \text{regime}_{SONATURG})$ in Table 5 reveals asymmetric dependence.

TABLE 5. Conditional regime probabilities

	BOA BF regime		
SONATURG regime	Down	Normal	Up
Down	0.172	0.793	0.035
Normal	0.138	0.823	0.039
Up	0.035	0.793	0.172

When SONATURG is in a Down regime, the probability that BOA BF is also in a Down regime (17.2%) is higher than when SONATURG is in an Up regime (3.5%). This asymmetry confirms the existence of non-zero lower tail dependence, already captured by the Student's t-copula.

4.5. **Hidden Markov model (HMM).** The hidden Markov model (HMM) is an extension of the classical Markov chain in which the states (regimes) are not directly observable but must be inferred from the data. Table 6 presents the characteristics of the two identified regimes.

TABLE 6. Hidden regime characteristics (HMM)

Regime	Mean return	Volatility	VaR (95%)	Proportion
1 (low volatility)	0.0004	0.021	-0.032	68.2%
2 (high volatility)	-0.0012	0.092	-0.158	31.8%

The first regime (low volatility) represents calm market periods: it occurs 68.2% of the time, with a slightly positive average daily return (0.04%) and very low volatility (2.1%). In this regime, the 95% VaR is only -3.2%. The second regime (high volatility) corresponds to stress or crisis phases: it appears 31.8% of the time, with a negative average return (-0.12%) and high volatility (9.2%). VaR in this regime reaches -15.8%, nearly five times that of the calm regime.

Figure 6 shows the time evolution of the regimes estimated by the HMM.

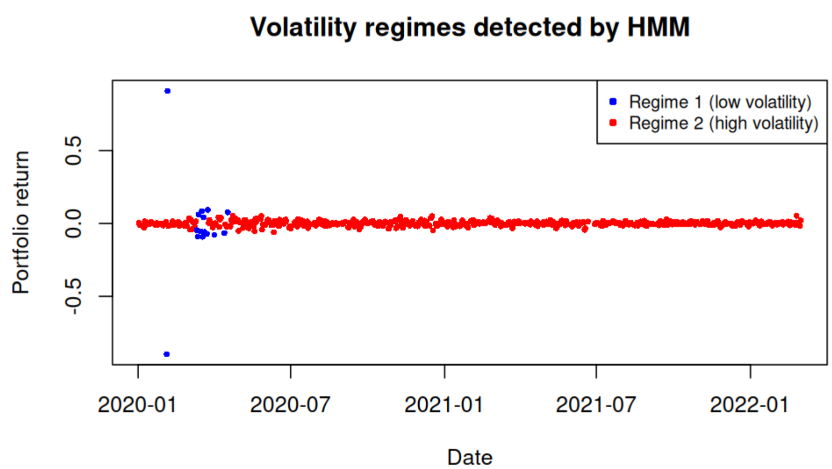

FIGURE 6. Time evolution of volatility regimes detected by HMM (blue: low volatility, red: high volatility)

Long periods of low volatility (Regime 1, blue) are clearly observed, interspersed with shorter but intense episodes of high volatility (Regime 2, red). For example, the shock related to the COVID-19 pandemic in the first half of 2020 manifests as a cluster of red points. Similarly, volatility peaks are detected in late 2022 and early 2023, likely corresponding to regional political uncertainties.

4.6. **Regime-conditional VaR.** At the date of the last observation (December 2023), the HMM estimates that the market is in the low-volatility regime (Regime 1). Using only historical returns classified in this regime (about 68% of the sample), we obtain an empirical VaR at 95% of -3.2%. This value is very close to the VaR obtained with the Student's t-copula (-3.07%), validating the consistency of the two approaches.

4.7. **Markov chain simulation.** A 500-day simulation using the transition matrix yields a VaR of:

$$\mathrm{VaR}_{95\%}^{(\mathrm{Markov})} = -3.8\% \tag{4.2}$$

5. Comparison of approaches

Table 7 gathers the 95% VaR values obtained by the four methods studied.

TABLE 7. Comparison of VaR across methods

Method	VaR (95%)
Student's t-copula (Monte Carlo)	-3.07%
Markov chain (simulation)	-3.80%
HMM (current regime)	-3.20%
Normal parametric	-9.62%

The dispersion of results (between -3.07% and -3.80% for the advanced methods) is relatively small compared to the gap from the normal method. This indicates that, despite their conceptual differences, the non-Gaussian approaches converge to a risk range of 3% to 4% of maximum daily loss (at 95% confidence). For a portfolio of 100 million FCFA, this corresponds to a potential loss of 3 to 4 million FCFA per day in the worst 5% of cases, compared to nearly 10 million under the normal method.

An important point to note is that while our analysis presents the two approaches in parallel, they can be integrated into a unified framework. A regime-switching copula model would allow the copula parameters (including tail dependence) to change according to a latent Markov process. This would capture the empirically observed phenomenon that dependence structures intensify during crisis periods. Mathematically, such a model takes the form $C(u, v \mid S_t = k) = C_k(u, v)$ where S_t follows a Markov chain. This integration remains a promising direction for future research.

6. Conclusion

By combining a copula approach (modeling extreme dependence between assets) and a Markov chain approach (capturing volatility regime persistence), this paper has shown that risk modeling on the BRVM cannot rely on classical assumptions of normality and linear correlation. The Student's t-copula reveals non-zero tail dependence ($\lambda_L = 0.28$) that the Gaussian copula ignores, and the associated Monte Carlo VaR (-3.07%) is more realistic than the normal parametric VaR (-9.62%). Furthermore, the Markov chain highlights strong regime inertia (probability of staying in the same regime above 0.85) and a hidden Markov model identifies two contrasting volatility regimes: a calm regime (68% of the time, volatility 2.1%) and a turbulent regime (32% of the time, volatility 9.2%). The current-regime conditional VaR (-3.2%) and the Markov-simulated VaR (-3.8%) provide complementary tools for dynamic risk management. For BRVM investors, these results argue for abandoning Gaussian models in favor of the Student's t-copula for capital requirements, and for integrating Markov chains to anticipate regime changes.

A caveat is warranted regarding the sample size (535 daily observations). While our bootstrap analysis suggests the tail dependence estimate is robust (95% CI: [0.19, 0.38]), extreme events are by definition rare, and a longer time series would provide more precise estimates. Future work with extended data would help confirm the persistence of the observed tail dependence. Natural extensions include the use of vine copulas for multi-asset portfolios and the estimation of MS-GARCH models coupling regime changes with conditional heteroskedasticity. The regime-switching copula framework mentioned in Section 5 represents a mathematically elegant integration of both approaches.

6.1. **Acknowledgement.** The authors thank the Laboratoire d'Analyse Numérique et de Biomathématiques for its support.

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