

RADIATIVE MHD FLOW WITH VISCOELASTIC SLIP IN CHANNEL THROUGH SEMI-ANALYTICAL APPROACH

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ABSTRACT. The study examines heat transfer in viscoelastic fluid flow through an axisymmetric porous channel under the effects of velocity slip, thermal radiation, and a magnetic field. The governing equations are transformed into self-similar form and these equations are solved using Optimal Homotopy Analysis Method. The influence of slip, porosity, viscoelasticity, thermal conductivity, radiation, and magnetic parameters on velocity and temperature profiles is analyzed. Results shows that increasing slip condition near the wall reduces the temperature. High Reynolds number increases fluid velocity and reduces thermal boundary layer thickness, whereas small Reynolds number and high slip produces the thicker thermal layers.

Keywords. Porous wall; Velocity slip; Magnetic field; Thermal radiation; Optimal Homotopy Analysis Method (OHAM);

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1. INTRODUCTION

Maxwell [1] introduced the foundational theory of molecular motion in gases through the dynamical theory of gases, establishing a statistical approach that laid the groundwork for modern fluid dynamics. Later, Oldroyd [2] extended this foundation by formulating rheological equations of state, specifically addressing the behaviour of viscoelastic fluids, it exhibits both viscous and elastic characteristics which are essential for accurately modelling polymeric and biological flows. The substantial numerical advancement, viscoelastic fluids are the type of non-Newtonian fluid it shows both viscous and elastic behaviour. In these fluids, the thickness (viscosity) becomes lower when the fluid is stirred or stretched at the same rate for some time. Examples of viscoelastic fluid include polymer solutions and melts such as nylon, gelatine and many fluids used in the industries. Non-Newtonian fluid flow in axisymmetric geometries has drawn a lot of attention due to its vast applications in engineering, biomedical and industrial systems. The processing of polymers, physiological fluxes and transportation of porous media are frequently using viscoelastic fluids, in order to design the effectiveness of the

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systems. The heat and mass transfer are essential to realize the flow and thermal behaviour of such fluids. Viscoelastic fluid in a channels with porous walls have been studied frequently in the past, with the traditional no-slip boundary conditions. This assumption might not be true in many real-world situations, however microfluidics, rarefied gas dynamics and polymer-coated surfaces, where the fluid might show some slide at the boundary. Slip conditions, such as Navier slip model, greatly affect the fluid flow and heat transfer characteristics and provides more realistic depiction of physical processes close to the wall. In addition to bridging the gap between classical and microscale analyses, this extended model provides a more flexible framework for researching the mass and heat transfer in intricate fluid systems with realistic boundary conditions as discussed by Chhabra and Richardson [3] and Barnes [4]. Bujurke et al. [5] used computer extended series method to analyze laminar boundary layer flow in a narrow channel with a varying gap. Bujurke et al. [6] analysed the laminar boundary layer flow in a channel with one porous wall and shows the effect of wall permeability on flow behavior. Omokhuale and Uwanta [7] examined the heat and mass transfer in viscoelastic fluid flow under the influence of an induced magnetic field, and shows the interplay between magnetic parameters and flow resistance, particularly important in industrial magnetohydrodynamic (MHD) applications. Awati et al. [8] applied the Homotopy Analysis Method (HAM) to solve the lubrication problem of a long porous slider and demonstrated that the method provides accurate convergent analytical solutions for nonlinear lubrication flow equations. Awati et al. [9] presented the unsteady boundary layer flow and heat transfer of nanofluid over permeable stretching/shrinking surfaces have been effectively analyzed using advanced numerical techniques such as the shifted Chebyshev collocation method. Further, Awati et al. [10] investigated the semi-numerical analysis of MHD nanofluid flow and heat transfer over a non-isothermal porous medium in the presence of chemical reaction by using a semi-analytical approach. Swain and Mahanthesh [11] analysed, how nanoparticle aggregation affects flow and thermal characteristics in a radiating MHD nano-liquid under slip conditions. Shakeri et al. [12] employed the variational iteration method to analyse heat transfer behaviour of a non-Newtonian fluid flow in an axisymmetric porous channel. By incorporating slip boundary conditions along with radiation and magnetic field effects, the present investigation extends the previous studies on heat transfer and viscoelastic fluid flow in an axisymmetric porous channel. The similarity transformations are used to simplify the governing nonlinear partial differential equations, and these equations are solved by using Optimal Homotopy Analysis Method (OHAM). OHAM is an efficient in solving highly nonlinear problems without requiring small parameter and the solutions are obtained analytically. The impact of important parameters on temperature and velocity fields is methodically investigated. These characteristics include porosity, thermal conductivity, slip coefficient, and viscoelastic parameter. Marinca and Herisanu [13] introduced OHAM as an improvement to existing semi-analytical techniques, the method demonstrates that, it offers better convergence and accuracy than the standard perturbation technique. Extending this work, the authors also analysed steady flow of a fourth-grade fluid past a porous plate, emphasizing the method's

effectiveness in capturing the behaviour of non-Newtonian fluids in porous media. In another application, OHAM is successfully used for solving thin film flow problems, highlighting its practical applicability to surface coating and lubrication processes. The results showcased as fast convergence and accurate solution without linearization or discretization. Liao [14] explained OHAM approach for the solution of highly nonlinear ordinary differential equations. For instance, Meghashree et al. [15] analyzed magnetic field-dependent viscosity effects over a rotating disk. Sil [16] presented an exact solution for MHD micropolar fluid flow through a porous medium using a hodographic approach. Wusu and Olabanjo [17] applied the Nelder–Mead optimization algorithm to solve mixed boundary value problem for ordinary differential equations, demonstrating its effectiveness for handling derivative boundary constraints without relying on gradient-based techniques.

The present study also has an important implications in the field of computer science, particularly in computational modeling, simulation, and optimization of complex fluid systems. The semi-analytical approach employed in this work provides efficient and accurate solution that can be integrated into numerical algorithms for simulating MHD flows in engineering and technological applications. Such models are highly relevant in the design of cooling systems for electronic devices, microfluidic channels, and porous media-based heat exchangers, where precise control of heat transfer and fluid behavior is essential. Furthermore, the incorporation of viscoelastic effect, thermal radiation, and velocity slip enhances the realism of simulations used in computational fluid dynamics (CFD) software, thereby improving predictive capabilities. These advancements contribute to the development of high-performance computing techniques, data-driven modeling, and algorithmic optimization in fluid mechanics problems, which are of significant interest in modern computational science and engineering. OHAM is useful in computer science for solving complex nonlinear mathematical models that arise in scientific computing and numerical analysis. It provides accurate approximate solutions with good convergence and reduced computational effort. The method can be applied in areas such as simulation, optimization and computational modeling.

2. MATHEMATICAL FORMULATION

2.1. Flow analysis. Consider a steady two-dimensional incompressible non-Newtonian viscoelastic fluid flow on the turbine disc for cooling surface. The disc is separated by a distance L as shown in Figure 1. The surface of lower disc is parallel to r -axis which is heated externally with temperature T_w , while temperature of upper disc is T_0 with $T_w > T_0$ and z -axis is perpendicular to it. The disc at a distance L is porous and non-Newtonian fluid (namely Maxwell fluid) is injected uniformly with velocity v_w . The similarity variables are proposed for the governing flow and heat transfer equations by Kurtcebe and Erim [18], Sepasgozar et al. [19] and Mirzaei et al. [20]. Under these suppositions, the continuity, momentum and energy equations with thermal radiation for neglecting the viscous dissipation term becomes

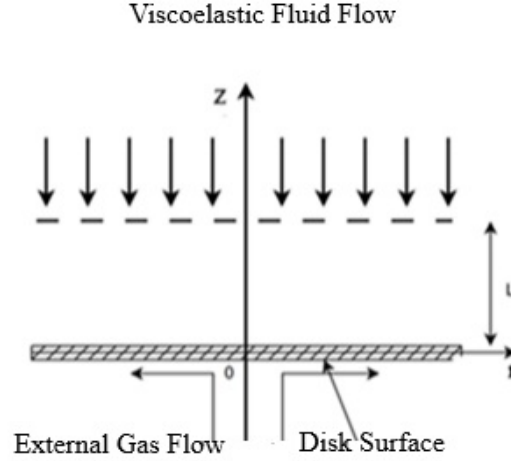


FIGURE 1. Schematic geometry of the flow problem.

$$\frac{\partial(ru_r)}{\partial r} + \frac{\partial(ru_z)}{\partial z} = 0, \quad (2.1)$$

r -direction:

$$\rho(u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_z}{\partial z}) = -\frac{P}{r} + (\frac{\tau_{rr}}{\partial r} + \frac{\tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r}) - \sigma B_0^2 u, \quad (2.2)$$

z -direction:

$$\rho(u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z}) = -\frac{P}{r} + (\frac{\tau_{zr}}{\partial r} + \frac{\tau_{zz}}{\partial z} + \frac{\tau_{zr}}{r}), \quad (2.3)$$

where $-\sigma B_0^2 u$ is the Lorentz force, which opposes the fluid flow in the x -direction, and $\tau_{rr}, \tau_{rz}, \tau_{zr}, \tau_{zz}$ are the components of the stress matrix. The analytical model under the above assumptions leads to the following boundary conditions

$$z = 0, \quad u_r = L_s \frac{\partial u_r}{\partial z}, \quad T = T_w, \quad z = L, \quad u_r = 0, u_z = -V, \quad T = T_0. \quad (2.4)$$

where u_r, u_z are the velocity components in r and z directions respectively, and V is the injection velocity, ρ, P are the density and pressure. The stress components τ_{ij} can be expressed in the form of $\tau_{ij} = \phi_0 I + \phi_1 A + \phi_2 B + \phi_3 A^2 + \dots$, where I is the unit matrix, A and B are the symmetric matrices defined by $A = \|\frac{\delta v_i}{\delta x_j} + \frac{\delta v_j}{\delta x_i}\|$, $B = \|\frac{\delta a_i}{\delta x_j} + \frac{\delta a_j}{\delta x_i} + 2\frac{\delta v_m}{\delta x_i} \frac{\delta v_m}{\delta x_j}\|$ and $\phi_k (k = 0, 1, 2, 3)$ are the polynomial invariants of A, B, A^2 . The study is restricted to second order fluid for which $\phi_k (k = 0, 1, 2, 3)$ are constant and $\phi_k (k = 4, 5, \dots)$ are zero. So that, the stress components are as follows

$$\tau_{rr} = \phi_1 A_{rr} + \phi_2 A_{rr}^2 + \phi_3 B_{rr},$$

$$\tau_{zz} = \phi_1 A_{zz} + \phi_2 A_{zz}^2 + \phi_3 B_{zz},$$

$$\tau_{\theta\theta} = \phi_1 A_{\theta\theta} + \phi_2 A_{\theta\theta}^2 + \phi_3 B_{\theta\theta},$$

where ϕ_1, ϕ_2, ϕ_3 are material dependent viscoelastic parameters. The following

similarity transformations are considered for transforming the nonlinear partial differential equations (NPDEs) of the model into self-similar equations

$$\eta = \frac{z}{L}, \quad f(\eta) = \frac{\psi}{Vr^2}, \quad u_r = \frac{V_r}{L} f'(\eta), \quad u_z = -2V f(\eta). \quad (2.5)$$

Using Equation (2.5) into Equations (2.1) and (2.2), continuity equation automatically satisfied and Equation (2.2) reduces to

$$(f')^2 - 2ff'' = -\frac{L^2}{\rho V^2 r} \frac{\partial P}{\partial r} + \frac{\phi_1}{\rho V L} f''' + \frac{\phi_2}{\rho L^2} (f''^2 - 2ff''''') + \frac{\phi_3}{\rho L^2} ((f''')^2 - 2ff''''') - \frac{\sigma B_0^2 L}{\rho V} f', \quad (2.6)$$

$$4f'' = -\frac{L^2}{\rho V^2} \frac{\partial P}{\partial z} - 2\frac{\phi_1}{\rho V L} f'' + 2\frac{\phi_2}{\rho L^2} \left(14ff'' + \frac{r^2}{L} f' f''' \right) + 4\frac{\phi_3}{\rho L^2} \left(11ff'' + f f''' + \frac{r^2}{L} f' f''' \right). \quad (2.7)$$

The pressure term can be eliminated by differentiating Equation (2.6) with respect to z and Equation (2.7) with respect to r and subtracting the resulting equations, it gives

$$f'''' + 2Rff''' - K_1 R(4ff'' + 2ff''''') - M^2 f'' = 0, \quad (2.8)$$

where $R = \frac{VL}{\nu}$, injection Reynolds number, $K_1 = \frac{\phi_2}{\rho L^2}$, is the viscoelastic parameter and assuming $\frac{\phi_3}{\rho L^2} = 0$, (a second-order fluid without the highest-order term), $M^2 = \frac{\sigma B_0^2 L^2}{\mu}$ is the square of the Hartmann number. The reduced pertinent boundary conditions become

$$f(0) = 0, f'(0) = \lambda f''(0), f(1) = 1, f'(1) = 0, \quad (2.9)$$

where $\lambda = \frac{Ls}{L}$ is the velocity slip parameter.

2.2. Heat transfer analysis. The energy equation with viscous dissipation in non-dimensional form and using Rosseland approximation, the radiative heat flux $q_r = -\frac{16\sigma T_\infty^3}{3\beta_r} \nabla T$, where σ is the Stefan-Boltzmann constant, β_r is the mean absorption coefficient and T is the absolute temperature. The total heat flux becomes

$$q = q_c + q_r = -k \nabla T - \frac{16\sigma T_\infty^3}{3\beta_r} \nabla T = -\left(k + \frac{16\sigma T_\infty^3}{3\beta_r}\right) \nabla T, \quad (2.10)$$

$$\rho c_p \left(u_r \frac{\partial T}{\partial r} + u_z \frac{\partial T}{\partial z} \right) = \nabla \cdot \left[\left(k + \frac{16\sigma T_\infty^3}{3\beta_r} \right) \nabla T \right] + \varphi, \quad (2.11)$$

$$\rho c_p \left(u_r \frac{\partial T}{\partial r} + u_z \frac{\partial T}{\partial z} \right) = k(1 + R_d) \nabla^2 T + \varphi, \quad (2.12)$$

where the radiation parameter $R_d = \frac{16\sigma T_\infty^3}{3k\beta_r}$.

The temperature of fluid is assumed in the form of $T = T_0 + \sum_{n=0}^{\infty} C_n \left(\frac{r}{L}\right)^n q_n(\eta)$, and neglecting the viscous dissipation ($\phi = 0$), the energy Equation (2.3) reduces to

$$\left(1 + \frac{4}{3}R_d\right) q_n'' - \text{Pr } R (nf'q_n - 2fq_n') = 0, \quad (2.13)$$

where $R = \frac{\rho v L}{\mu}$, is the Reynolds number $\text{Pr} = \frac{\mu c_p}{k}$ representing the Prandtl number. The relevant boundary conditions become

$$q_n(0) = 1, \quad q_n(1) = 0. \quad (2.14)$$

3. OPTIMAL HOMOTOPY ANALYSIS METHOD (OHAM)

Nonlinear differential equations frequently arises in mathematical models of fluid flow, heat transfer, and other engineering systems. Classical perturbation techniques require small parameters and limiting their applicability. Homotopy-based method such as HAM overcome this limitation by constructing a continuous deformation from an initial guess to the exact solution. However, the convergence of HAM depends strongly on the auxiliary parameters. The OHAM improves this by introducing convergence-control parameters that are optimally determined through residual error minimization.

Consider the nonlinear boundary value problem in operator form

$$L(u(t)) + N(u(t)) + g(t) = 0, \quad B(u) = 0, \quad (3.1)$$

where L is a linear operator, N is a nonlinear operator, $g(t)$ is a known function, and B represents boundary conditions.

An initial approximation $u_0(t)$ is selected such that $B[u_0(t)] = 0$. Introducing an embedding parameter $p \in [0, 1]$, the zeroth-order deformation equation is constructed as

$$(1 - p)L(\phi(t, p) - u_0(t)) = p H(p) [L(\phi(t, p)) + N(\phi(t, p)) + g(t)], \quad (3.2)$$

subject to $B(\phi(t, p)) = 0$.

At $p = 0$ and $p = 1$, we have

$$\phi(t, 0) = u_0(t), \quad \phi(t, 1) = u(t). \quad (3.3)$$

The auxiliary function is chosen as

$$H(p) = C_1 p + C_2 p^2 + \dots, \quad (3.4)$$

where C_i are convergence-control parameters.

Expanding $\phi(t, p)$ in Taylor series

$$\phi(t, p) = u_0(t) + \sum_{k=1}^{\infty} u_k(t, C_i) p^k, \quad (3.5)$$

the approximate solution of order m is

$$\tilde{u}^{(m)}(t) = u_0(t) + \sum_{k=1}^m u_k(t, C_i). \quad (3.6)$$

Substituting into Equation (3.1), the residual is defined as

$$R(t, C_i) = L(\tilde{u}^{(m)}) + N(\tilde{u}^{(m)}) + g(t). \quad (3.7)$$

The convergence-control parameters are determined by minimizing the squared residual error

$$J(C_i) = \int_a^b R^2(t, C_i) dt. \quad (3.8)$$

By applying the Galerkin approach, we get

$$\int_a^b R(t, C_i) \frac{\partial R(t, C_i)}{\partial C_i} dt = 0, \quad (3.9)$$

which yields a system of algebraic equations to determine the optimal values of C_i .

3.1. Application to the present problem. The self similar equations (2.8) and (2.13) become

$$\mathcal{N}_1[f] = f'''' + 2R f f''' - K_1 Re (4f'' f''' + 2f' f'''') = 0, \quad (3.10)$$

$$\mathcal{N}_2[g] = g'' - Pr Re (n f' g - 2f g') = 0, \quad (3.11)$$

with the boundary conditions are

$$f(0) = 0, \quad f'(0) = 0, \quad f(1) = 0, \quad f'(1) = 0, \quad (3.12)$$

$$g(0) = 0, \quad g(1) = 0. \quad (3.13)$$

Initial guesses are chosen as

$$f_0(z) = 3z^2 - 2z^3, \quad g_0(z) = 1 - z. \quad (3.14)$$

Linear operators

$$L_1[f] = \frac{d^4 f}{dz^4}, \quad L_2[g] = \frac{d^2 g}{dz^2}. \quad (3.15)$$

3.2. Computational algorithm for convergence-control parameters. To address the convergence explicitly, the optimal parameters are determined using the following procedure

- (1) Select initial approximations satisfying the boundary conditions.
- (2) Construct OHAM deformation equation and compute series up to the order m .
- (3) Form the truncated solutions $f^{(m)}(z)$ and $g^{(m)}(z)$.
- (4) Substitute into governing equations to obtain residuals $R_1(z)$ and $R_2(z)$.
- (5) Define squared residual errors

$$E_i = \int_0^1 R_i^2(z) dz, \quad i = 1, 2.$$

- (6) Minimize E_i with respect to $C_0[i]$ using

$$\frac{\partial E_i}{\partial C_0[i]} = 0.$$

- (7) Solve the resulting algebraic system of equations to obtain optimal parameters.

Using the BVPh package, the optimal values are obtained as

$$C_0[1] = -0.994428, \quad C_0[2] = -0.976625. \quad (3.16)$$

3.3. Convergence analysis. The convergence of the solution is verified using

$$\text{Err}^{(k)} = \left\{ \int_0^1 R_1^2 dz, \int_0^1 R_2^2 dz \right\}. \quad (3.17)$$

The error decreases rapidly from 10^{-7} to 10^{-24} , confirming the accuracy and stability of OHAM solution. The determination of convergence-control parameters via residual error minimization and Galerkin-type optimization follows standard OHAM procedures developed by Marinca et al.[13] and Liao [14].

4. RESULTS AND DISCUSSION

The radiative MHD flow and heat transfer of a viscoelastic fluid in a porous channel with velocity slip was systematically studied using OHAM. The validity and accuracy of the semi-analytical approach were established while comparing the results with finite difference solutions. Table 1 presents the OHAM results for the dimensionless stream function $f(\eta)$ and temperature profile $q_n(\eta)$ shows an excellent agreement with the benchmark numerical data for negligible deviation. In the present work, OHAM series was computed with $m = 12$ terms (12th order approximation). The squared residual error of the governing equations was minimized to below 10^{-24} for the momentum equation and 10^{-25} for the energy equation. This confirmation offers a reliable foundation for the subsequent parametric study of the system.

TABLE 1. Comparison between results of finite difference method and OHAM, when $R = 0.5, K_1 = 0.01, n = 0$

η	$f(\eta)$		$q(\eta)$	
	Finite difference [20]	OHAM	Finite difference [20]	OHAM
0.0	0	0	1	1
0.1	0.029561	0.0295612	0.884719	0.884719
0.2	0.109311	0.109309	0.769826	0.769826
0.3	0.225795	0.225794	0.656216	0.656216
0.4	0.365624	0.36562	0.545162	0.545162
0.5	0.515644	0.51564	0.438099	0.438098
0.6	0.663077	0.663156	0.336409	0.336407
0.7	0.796081	0.796124	0.241225	0.241222
0.8	0.903297	0.903307	0.153298	0.153294
0.9	0.974382	0.974386	0.072933	0.072932
1.0	1	1	0	0

Figure 2 illustrates the dimensionless velocity distributions of $f(\eta)$ and temperature profiles $q_n(\eta)$ against the similarity variable η . These profiles represent

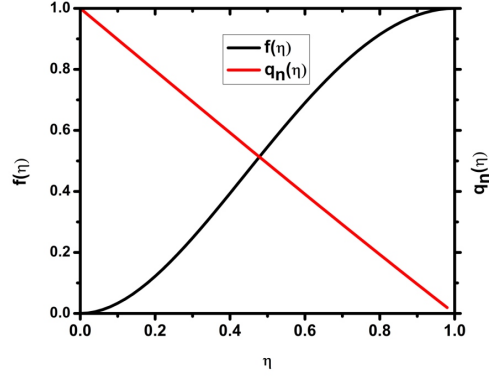


FIGURE 2. Variation of dimensionless stream function $f(\eta)$ and temperature profile $q_n(\eta)$ with respect to the similarity variable η .

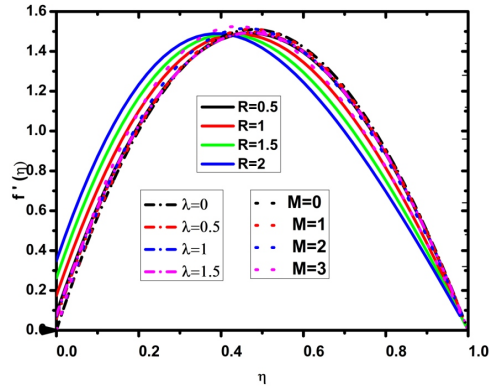


FIGURE 3. The effect of various values of Reynolds number R , λ and M on velocity components $f'(\eta)$. 1

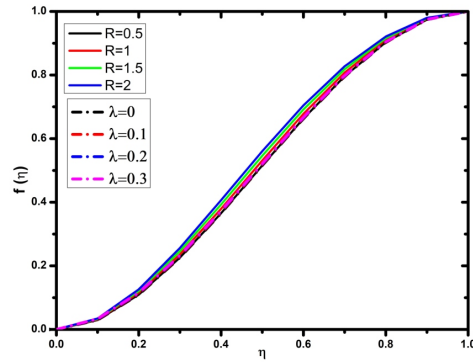


FIGURE 4. The effect of various values of Reynolds number R and λ on stream function $f(\eta)$.

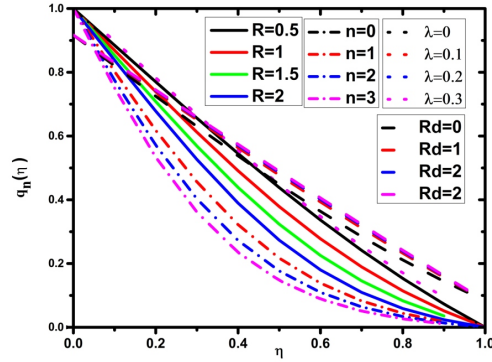


FIGURE 5. The effect of various values of Reynolds number R , λ , n and Rd on temperature distribution components $q_n(\eta)$ for $K_1 = 0.01$, $R = 2$ and $Pr = 1$.

a fundamental hydrodynamic and thermal boundary layer in the absence of parametric variations and serve as a reference for assessing the influence of various physical parameters. Figure 3 illustrates the effects of various values of Reynolds number R , Navier slip parameter λ , and magnetic parameter M on the velocity profile $f'(\eta)$. It is observed that, an increase in the Reynolds number enhances the velocity distribution due to the dominance of inertial forces over viscous forces, which accelerates the fluid motion. The Navier slip parameter λ reduces the velocity profile because increasing wall slip weakens the momentum transfer between the fluid and the channel wall. Furthermore, increasing the magnetic parameter M suppresses the velocity distribution as the applied magnetic field produces a Lorentz force that resists the fluid motion. Figure 4 presents the influence of various values of Reynolds number R and Navier slip parameter λ on the stream function $f(\eta)$. The stream function increases with increasing Reynolds number due to enhanced fluid transport, whereas higher values of Navier slip parameter reduce the flow strength because of weaker wall-fluid interaction. Figure 5 demonstrates the effects of different values of Reynolds number R , order of the heat equation n , Navier slip parameter λ , and radiation parameter Rd on the temperature distribution $q_n(\eta)$. The temperature profiles decrease with increasing Reynolds number owing to stronger convective cooling caused by intensified fluid motion. In contrast, increasing the radiation parameter raises the temperature distributions because the thermal radiation supplies additional heat energy to the fluid. The Navier slip parameter also affects thermal boundary layer by modifying the heat transport near the wall. Moreover, variations in the parameter n , which represents the order of the heat equation, significantly influence the temperature distributions by altering the thermal diffusion characteristics and energy transport behavior within the fluid system.

5. CONCLUSION

The following conclusions were drawn from the above study

- (1) The slip parameter (λ) increases near-wall velocity while reducing temperature due to lower viscous dissipation.
- (2) The Reynolds number (R) increases the peak velocity and thins the thermal boundary layer by enhancing convection.
- (3) The magnetic parameter (M) retards the flow via the Lorentz force and increases temperature through Joule heating.
- (4) The order of heat equation (n) considerably affects the temperature distributions by altering the thermal diffusion and energy transport characteristics of the fluid system.
- (5) The radiation parameter (R_d) significantly increases the temperature distributions by enhancing the radiative heat flux.

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