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NECESSARY AND SUFFICIENT CONDITIONS FOR THE EXISTENCE OF COMMON FIXED POINTS OF CONTINUOUS SELF-MAPPINGS IN COMPLETE MENGER SPACES

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Abstract. The purpose of this paper is to establish necessary and sufficient conditions for existence of a common fixed point of three maps f, g and h in a complete Menger space under a general contractive condition. Our main result is generalization of the result of Ismat Beg and Mujahid Abbas [7].

1. Introduction and preliminaries

In the literature of fixed point theory majority of results are on metric spaces. It is quite natural to extend the research in the direction of some newly investigated spaces such as partially ordered metric space, Sacks spaces and probabilistic metric spaces(pms). In the pms where the probable distance between two points can be measured. In this content one can see in the literature on these spaces with good number of publications. Few important aspects of fixed point theorems in probabilistic metric spaces can see in [9]. In 1991,Mishra [12] extended the notion of compatibility to PM-space. Cho et al.[15] studied the notion of compatible mappings of type (A) in Menger spaces which is equivalent to the concept of compatible mappings under some conditions.

Jungck and Rhoades [4] studied fixed point results for occasionally weakly compatible mappings. Doric et al. [3] have shown that the condition of occasionally weak compatibility reduces to weak compatibility in the presence of a unique point of coincidence (or a unique common fixed point) of the given pair of mappings. Recently many authors [2],[5],[6],[8],[13] and [14] have established a number of interesting fixed point theorems for mappings on various classes of Menger spaces.

In this paper deals with the necessary and sufficient condition for a common fixed point of three self maps f, g and h, in which the pairs (f, h) and (g, h) are occasionally weakly compatible maps. Moreover, this result does not require the

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continuity of the mapping f and g. Our main result is generalization of the result of Ismat Beg and Mujahid Abbas [7].

Definition 1.1. [1] A mapping $F : \mathbb{R} \to [0,1]$ is called a distribution function if it satisfies the conditions (i) F is nondecreasing, F is left continuous, with $\inf\{F(t) : t \in \mathbb{R}\} = 0$ and $\sup\{F(t) : t \in \mathbb{R}\} = 1$. The Heaviside function H is a distribution function defined by

$$H(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ 1 & \text{if } x > 0. \end{cases}$$

Definition 1.2. [1] Let X be a non-empty set and let L denote the set of all distribution function defined on X, i.e., $L = \{F_{x,y} : x, y \in X\}$. An ordered pair (X, F) is called a probabilistic metric spaces, where F is a mapping from $X \times X$ into L, if for every pair $(x, y) \in X$, a distribution function $F_{x,y}$ is assumed to satisfy the following four conditions:

(1): $F_{x,y}(u) = 1$, forall $u > 0$,
(2): $F_{x,y}(u) = F_{y,x}(u)$,
(3): $F_{x,y}(u) = 0$.
(4): If $F_{x,y}(u_1) = 1$ and $F_{x,y}(u_2) = 1$, then $F_{x,y}(u_1 + u_2) = 1$ for all $x, y, z \in X$ and $u_1, u_2 \geq 0$.

Definition 1.3. [1] A T-norm is function $t : [0,1] \times [0,1] \to [0,1]$ satisfying the following conditions:

(T1): $t(a, 1) = a,\ t(0, 0) = 0$,
(T2): $t(a, b) = t(b, a)$,
(T3): $t(c, d) \geq t(a, b)$ for $c \geq a,\ d \geq b\ (a, b, c, d \in [0, 1])$.
(T4): $t(t(a, b), c) = t(a, t(b, c))$ for all a, b, c in $[0, 1]$.

Example of basic Δ -norm :

1: The minimum t-norm : $t_{m(a,b)} = min\{a, b\}$,
2: The product t-norm :$t_{p(a,b)} = a.b$
3: The Likasiewicz t-norm : $t_L(a, b) = max\{a + b - 1, 0\}$.
4: The weakest t-norm, the Drastic product:

$$t_D(a, b) = \begin{cases} min(x, y) & \text{if } max(x, y) = 1, \\ 1 & \text{if otherwise} \end{cases}$$

.

Definition 1.4. [1] A Menger probabilistic metric space (X, F, t) in ordered triple, where t is a t-norm, and (X, F) is a probabilistic metric space satisfying the condition $F_{x,z}(u_1+u_2) \geq t(F_{x,z}(u_1), F_{x,z}(u_2))$ forall $x, y, z \in X$ and $u_1, u_2 \geq 0$. This inequality is known as Menger's triangle inequality.

Definition 1.5. [1] Let (X, F, Δ) be a PM-space. Then,

1: a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$, if for every $\epsilon > 0$ and $0 < \lambda < 1$ there exists a positive integer $\mathbb{Z}^+$ such that $F_{x_n,x}(\epsilon) > 1 - \lambda$ whenever $n \geq \mathbb{Z}^+$;

2: a sequence $\{x_n\}$ in X is called a Cauchy sequence if for every $\epsilon > 0$ and $\lambda > 0$ we can find a positive integer $\mathbb{Z}^+$ such that $F_{x_n,x_m}(\epsilon) > 1 - \lambda$ whenever $n, m \geq \mathbb{Z}^+$;

3: a Manger PM-space is said to be complete if every Cauchy sequence is convergent to a point in X:

Definition 1.6. [12] Let (X, F, T) be a Menger space such that the t-norm T is continuous and (A, S) be mappings from X into itself. Then, the pair (A, S) is said to be

i: *compatible*[12], if

$$\lim_{n\to\infty} F_{AS_{x_n}, SA_{x_n}}(t) = 1,$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \ for \ some \ z \in X.$$

ii: *weakly compatible*[12], if they commute at their coincidence points.

iii: *compatible maps of type(A)*[15], if

$$\lim_{n\to\infty} F_{SA_{x_n}, AA_{x_n}}(t) = 1, \ and \ \lim_{n\to\infty} F_{AS_{x_n}, SS_{x_n}}(t) = 1,$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

for some $z \in X$.

Definition 1.7. [4] Two self mappings A and S of a Menger space (X, F, T) are occasionally weakly compatible (owc) if and only if there is a point $x \in X$ which is a coincidence point of A and S at which A and S commute.

Definition 1.8. (Altering distance function [10]). The function $\Phi : [0, +\infty) \to [0, +\infty)$ is called an altering distance function if the following properties are satisfied:

1: Φ is continuous and non-decreasing,

2: $\Phi(t) = 0$ if and only if $t = 0$.

Proposition 1.9. [15] *Let (X, F, T) be a Menger space such that T-norm t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$,and $A, S : X \to X$ be continuous mappings. If the pair (A, S) is compatible, then they are weakly compatible.*

Proposition 1.10. [15] *Let (X, F, T) be a Menger space such that T-norm t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$,and $A, S : X \to X$ be continuous mappings. If the pair (A, S) are compatible, then they are compatible maps of type (A).*

Proposition 1.11. [15] *Let (X, F, T) be a Menger space such that T-norm t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$,and $A, S : X \to X$ be compatible maps of type (A). If one of the pair (A, S) is continuous, then the pair (A, S) is compatible.*

Proposition 1.12. [15] *Let (X, F, T) be a Menger space such that T-norm t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$,and $A, S : X \to X$ be mappings. Let the pair (A, S) be compatible maps of type (A). Then we have*

1: $\lim_{n\to\infty} SAx_n = Az$ *if S is continuous at z.*
2: $ASz = SAz$ *and* $Az = Sz$ *if A and S are continuous at z.*

Lemma 1.13. [11] *Let $\{y_n\}$ be a sequence in X with $T(a, b) = mina, b$ for all $a, b \in [0, 1]$. If there exist a constant $k \in (0, 1)$ such that for every $n \in N$and every $\epsilon > 0$ we have*

$$F_{y_{n+1},y_n}(k\epsilon) \geq F_{y_n,y_{n-1}}(\epsilon),$$

then $\{y_n\}$ is a Cauchy sequence in X.

2. Main Results

Theorem 2.1. *Let (X, F, T) be a complete Menger space and f, g be two selfmaps on X. The mappings f and g have a common fixed point in X if and only if there exists a continuous mapping $h : X \to f(X) \cap g(X)$ and the pairs (f, h) and (g, h) are owc and satisfying the condition*

$$F_{hx,hy}(kt) \geq \varphi\Bigg(min\Bigg\{F_{hx,fx}(t), F_{hy,gy}(t), F_{fx,gy}(t), F_{hx,gy}(t), F_{hy,fx}(2t)\Bigg\}\Bigg), \quad (2.1)$$

for all $x, y \in X$, $k \in (0, 1)$, $\varphi \in \Phi$ and $t > 0$. Indeed f, g and h have a unique common fixed point in X.

Proof. **Necessary condition:** Suppose $fz = z = gz$ for some $z \in X$. Define a mapping by $hx = z$ for all x in X. Suppose h is continuous mapping of X into $f(X)\cap g(X)$ and h it commutes with f and g, conclusively and thus the pairs (f, h) and (g, h) are satisfies required conditions. Further, for any $k \in (0, 1), \varphi \in \Phi$ and $t > 0$, then inequality (2.1)

$$F_{hx,hy}(kt) = F_{z,z}(kt) = 1 \geq \varphi\Bigg(min\Bigg\{F_{hy,fx}(t), F_{hy,gy}(t), F_{fx,gy}(t)F_{hx,gy}(t), F_{hy,fx}(2t)\Bigg\}\Bigg),$$

holds good.
Sufficient condition:- Let x_0 be an arbitrary point in X. Suppose $h(X) \subseteq g(X)$, let for $n \leq N$ then there exists a point $x_1 \in X$ such that $hx_0 = gx_1 = y_1$(say). Again for $x_1 \in X$ there exists a point $x_3 \in X$ such that $hx_1 = fx_2 = y_2$. Continue in this way, we can have

$$y_{2n-1} = hx_{2n-2} = gx_{2n-1}\ ;\ y_{2n} = hx_{2n-1} = fx_{2n}. \quad (2.2)$$

Putting $x = x_{2n}, y = x_{2n+1}$ in (2.1), then we have

$$\begin{aligned} F_{hx_{2n},hx_{2n+1}}(kt) \quad &\geq \quad \varphi\Bigg(min\Bigg\{F_{hx_{2n},fx_{2n}}(t), F_{hx_{2n+1},gx_{2n+1}}(t), F_{fx_{2n},gx_{2n+1}}(t), F_{hx_{2n},gx_{2n+1}}(t), \\ &\qquad F_{hx_{2n+1},fx_{2n}}(2t)\Bigg\}\Bigg). \end{aligned}$$

Using (2.2), we have

$$\begin{aligned}F_{y_{2n+1},y_{2n+2}}\quad(kt) &\geq \varphi\Big(min\Big\{F_{y_{2n+1},y_{2n}}(t), F_{y_{2n+2},y_{2n+1}}(t), F_{y_{2n},y_{2n+1}}(t), F_{y_{2n+1},y_{2n+1}}(t),\\ &\quad F_{y_{2n+2},y_{2n}}(2t)\Big\}\Big),\\ &\geq \varphi\Big(min\Big\{F_{y_{2n},y_{2n+1}}(t), \Delta(F_{y_{2n+2},y_{2n+1}}(t), F_{y_{2n+1},y_{2n}}(t))\Big\}\Big),\\ &= \varphi\Big(min\Big\{F_{y_{2n+2},y_{2n+1}}(t), F_{y_{2n+1},y_{2n}}(t)\Big\}\Big).\end{aligned}$$

If $min\{F_{y_{2n+2},y_{2n+1}}(t), F_{y_{2n+1},y_{2n}}(t)\}$ is $F_{y_{2n+2},y_{2n+1}}(t)$, then by the property of φ, we get

$F_{y_{2n+1},y_{2n+2}}(kt) \geq \varphi(F_{y_{2n+1},y_{2n+2}}(t)) > F_{y_{2n+1},y_{2n+2}}(t)$, a contradiction.
Thus

$$F_{y_{2n+1},y_{2n+2}}(kt) \geq \varphi(F_{y_{2n+1},y_{2n}}(t)) > F_{y_{2n+1},y_{2n}}(t). \tag{2.3}$$

Similarly, we get

$$F_{y_{2n+2},y_{2n+3}}(kt) \geq \varphi(F_{y_{2n+2},y_{2n+1}}(t)) > F_{y_{2n+2},y_{2n+1}}(t). \tag{2.4}$$

From (2.3) and (2.4), we obtain that for each $n \in N$ such that

$$F_{y_{n+1},y_{n+2}}(kt) \geq F_{y_n,y_{n+1}}(t). \tag{2.5}$$

On the application of Lemma 1.13, the sequence defined by (2.2) is a Cauchy sequence in X. The completeness of the space X implies that every Cauchy sequence is a convergent sequence in X. Therefore the sequence $\{y_n\}$ converges to a point z (say) in X. Thus

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} hx_n = \lim_{n\to\infty} fx_{2n} = \lim_{n\to\infty} gx_{2n-1} = z, \text{for some z} \in \text{X}. \tag{2.6}$$

Suppose h is continuous and the pair (f, h) is owc, then by the proposition (1.12), implies that

$$\lim_{n\to\infty} fh_{x_{2n}} = hz, \text{for some z} \in \text{X}. \tag{2.7}$$

Now, we show that $hz = z$, if $hz \neq z$, replace $x = h_{x_{2n}}$ and $y = x_{2n-1}$ in (2.1), so we have

$$\begin{aligned}F_{hh_{x_{2n}},hx_{2n-1}}(kt) \quad\geq\quad & \varphi\Big(min\Big\{F_{hhx_{2n},fhx_{2n}}(t), F_{hx_{2n-1},gx_{2n-1}}(t), F_{fhx_{2n},gx_{2n-1}}(t),\\ & F_{hhx_{2n},gx_{2n-1}}(t), F_{hx_{2n-1},fhx_{2n}}(2t)\Big\}\Big).\end{aligned} \tag{2.8}$$

On letting $n \to \infty$, then shall get

$$\begin{aligned} F_{hz,z}\quad(kt) &\geq \varphi\Big(min\Big\{F_{hz,hz}(t), F_{z,z}(t), F_{hz,z}(t), F_{hz,z}(t), F_{z,hz}(2t)\Big\}\Big), \\ &\geq \varphi\Big(min\Big\{1, 1, F_{hz,z}(t), F_{hz,z}(t), F_{z,hz}(2t)\Big\}\Big) \\ &= \varphi\Big(F_{hz,z}(t)\Big) > F_{hz,z}(t), \end{aligned}$$

a contradiction. Thus

$$hz = z. \tag{2.9}$$

Again using $h(X) \subseteq f(X)$, then there exists a point $u \in X$ such that

$$fu = hz = z. \tag{2.10}$$

To show that $fu = hu$, we shall put $x = fu$ and $y = x_{2n-1}$ in (2.1), so have

$$\begin{aligned} F_{hu,hx_{2n-1}}(kt) \quad \geq \quad & \varphi\Big(min\Big\{F_{hu,fu}(t), F_{hx_{2n-1},gx_{2n-1}}(t), F_{fu,gx_{2n-1}}(t), F_{hu,gx_{2n-1}}(t), \\ & F_{hx_{2n-1},fu}(2t)\Big\}\Big). \end{aligned}$$

Taking as $n \to \infty$ in the above inequality, we obtain that

$$F_{hu,z}(kt) \geq \varphi\Big(min\Big\{F_{hu,fu}(t), F_{z,z}(t), F_{fu,z}(t), F_{hu,z}(t), F_{z,fu}(2t)\Big\}\Big).$$

Using the property of φ,

$$\begin{aligned} F_{hu,z}\quad(kt) &\geq \varphi\Big(min\Big\{F_{hu,z}(t), F_{z,z}(t), F_{z,z}(t), F_{hu,z}(t), F_{z,z}(2t)\Big\}\Big) \\ &= \varphi\Big(F_{hu,z}(t)\Big) > F_{hu,z}(t), \end{aligned}$$

a contradiction, then we get

$$hu = fu = z. \tag{2.11}$$

Since (f, h) is owc, then we get

$$\begin{aligned} hu = \quad fu \quad &\Rightarrow hfu = fhu. \\ &\Rightarrow hz = fz = z. \qquad\qquad (By\ (2.9), (2.10)\ and\ 2.11)) \end{aligned}$$

Hence z is common fixed point of h and f.
Finally, we show that z is a fixed point of g also. Since $h(X) \subseteq g(X)$, then there exists point $v \in X$ such that

$$gv = hz(= z). \quad (2.12)$$

Now, we show that $hv = gv$. We consider $x = x_{2n}$ and $y = v$ in (2.1), we get

$$F_{hx_{2n},hv}(kt) \quad \geq \quad \varphi\Big(min\Big\{F_{hx_{2n},fx_{2n}}(t), F_{hv,gv}(t), F_{fx_{2n},gv}(t), F_{hx_{2n},gv}(t), F_{hv,fx_{2n}}(2t)\Big\}\Big).$$

On taking limits as $n \to \infty$ and using (2.6) and (2.12), we have

$$F_{z,hv} \quad (kt) \geq \varphi\Big(min\Big\{F_{z,z}(t), F_{hv,z}(t), F_{z,z}(t), F_{z,z}(t), F_{hv,z}(2t)\Big\}\Big).$$
$$= \varphi\big(F_{hv,z}(t), F_{hv,z}(2t)\big) > \varphi\big(F_{hv,z}(t)\big) > F_{hv,z}(t),$$

a contradiction. Thus

$$hv = z. \tag{2.13}$$

From (2.12) and (2.13), we get

$$hv = gv = z. \tag{2.14}$$

Since the pair (h, g) is owc and from (2.13) and (2.14), we get,

$$hv = \quad gv \quad \Rightarrow hgv = ghv.$$
$$\Rightarrow hz = gz = z.$$

This leads to that z is a common fixed point of f, g and h. Finally, we get uniqueness part easily from the inequality. □

Theorem 2.2. *Let (X, F, T) be a complete Menger space and f, g be two selfmaps on X. The mappings f and g have a common fixed point in X if and only if there exists a continuous mapping $h : X \to f(X) \cap g(X)$ and the pair (f, h) is compatible and (g, h) is weakly compatible and satisfying the condition*

$$F_{hx,hy}(kt) + q\Big(1 - max\Big\{F_{hx,gy}(t), F_{hy,gy}(t)\Big\}\Big)$$
$$\geq \varphi\Big(min\Big\{F_{hy,fx}(2t), F_{fx,gy}(t), F_{hx,fx}(t)\Big\}\Big), \tag{2.15}$$

for all $x, y \in X$, $k \in (0, 1)$, $\varphi \in \Phi$ and $t > 0$. Indeed f, g and h have a unique common fixed point in X.

Proof. Take $q = 0$ in (2.15), then it is follows as corollary of Theorem 2.1. □

The following is supporting for the Theorem 2.1

Example 2.3. Let (X, F, Δ) is a complete Menger space where $t(a, b) = min\{a, b\}$ for all $a, b \in [0, 1]$. Let $X = [0, 1]$ be the set with the metric d defined by $d(x, y) = |x - y|$ and for each $t \in [0, 1]$. Define

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|} & \text{if } t > 0 \\ 0 & \text{if } t = 0 \end{cases}$$

for all $x, y \in X$ and $t > 0$. Let f, g and h are self maps on X. Define $h(x) = \frac{1}{2}$, if $0 \leq x \leq 1$,

$$f(x) = \begin{cases} 1 - x & , \text{if } 0 \leq x \leq \frac{1}{2}, \\ \frac{1}{2} & , \text{if } \frac{1}{2} < x \leq 1, \end{cases} \quad and\ g(x) = \begin{cases} x & , \text{if } 0 \leq x \leq \frac{1}{2}, \\ \frac{1}{2} & , \text{if } \frac{1}{2} < x \leq 1, \end{cases}$$

Proof. It is clear that h is continuous mapping and $h(X) \subset f(X) \cap g(X)$ and $h : X \to f(x) \cap g(x)$. At the point $1/2$ the pairs (f, h) and $g, h)$ satisfies owc property. And condition (2.1) is also satisfies. Then the mappings f, g and h have unique common fixed point $1/2$ in X. □

Corollary 2.4. *(Theorem 2.1 of [7]). Let (X, F, T) be a complete Menger space and f, g be two selfmaps on X. The mappings f and g have a common fixed point in X if and only if there exists a continuous mapping $h : X \to f(X) \cap g(X)$ and the following conditions are satisfying;*

(a) either the pairs (f, h) is compatible and (g, h) is weakly compatible

or the pair (g, h) is compatible and (f, h) is weakly compatible and

(b) $F_{hx,hy}(kt) \geq min\left\{F_{hx,fx}(t), F_{hy,gy}(t), F_{fx,gy}(t), F_{hx,gy}(t), F_{hy,fx}(2t)\right\}$,

for all $x, y \in X$, $k \in (0, 1)$, and $t > 0$. Indeed f, g and h have a unique common fixed point in X.

Corollary 2.5. *Let (X, F, T) be a complete Menger space and f, g be two selfmaps on X. The mappings f and g have a common fixed point in X if and only if there exists a continuous mapping $h : X \to f(X) \cap g(X)$ and the following conditions are satisfying the condition (b) of corollary 2.4, and*

(a) either the pairs (f, h) is compatible maps of type (A) and (g, h) is weakly compatible

or the pair (g, h) is compatible maps of type (A) and (f, h) is weakly compatible.

Indeed f, g and h have a unique common fixed point in X.

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