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FIXED POINT THEOREM ON SOFT $\mathcal{M}-$ FUZZY METRIC SPACES

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ABSTRACT. In this paper we introduced soft $\mathcal{M}-$ fuzzy metric spaces. As an important result, we give a sufficient condition for a sequence to be Cauchy in the soft $\mathcal{M}-$ fuzzy metric spaces. Thus we simplify the proofs of fixed point theorem in the soft $\mathcal{M}-$ fuzzy metric spaces with the well-known contraction conditions.

1. INTRODUCTION

Molodtsov [6] introduced soft sets as a mathematical tool to handle uncertainity associated with real world data based promlems and established the fundamental results of this new concept. The soft set is a parametrized family of subsets of the universal set. The parameter set may be any collection of cohice, sentences, natural numbers etc. Hence, soft set theory has compelling applications in several diverse fields. Later, many authors generalized soft set theory [1, 7, 5, 10, 13] . An interesting and important generalization of it were given by Maji et al. [8]. They made a theoritical study of the soft set theory and presented an application of soft sets in a decision making problem.

Very recently, Erduran et al. [4] gave a new definition which is called soft fuzzy metric spaces, by using soft points of soft sets and soft real numbers of soft real sets. For this notion they gave definition soft t-norm $\tilde{}\tilde{*}$ for the triangle inequality of soft fuzzy metric and gave some properties of soft t-norm.

In this paper, we deal the notation of a countable expansion of the soft $\mathtt{t}-$norm, we demonstrate a valuable lemma in the soft $\mathcal{M}-$ fuzzy metric space setting that guarantee that a sequence $\{\tilde{\sigma}^{n}_{e_n}\}$ is a Cauchy sequence. Utilizing this lemma, we improve the evidences of some understand fixed point theorem. We present some of them in the principle part of the paper.

2. PRELIMINARIES

In this paper, $\mathcal{X}, E$ and $P(\mathcal{X})$ are mean by an universe set, non-empty set of all parameters and power set of $\mathcal{X}$ respectively.

Definition 2.1. [4] Let $\mathcal{X}$ be the initial universe set and E be the set of parameters. A pair $(\mathcal{F}, E)$ is called a soft set over $\mathcal{X}$ if $\mathcal{F}$ is a mapping $\mathcal{F} : E \to P(\mathcal{X})$.

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Definition 2.2. [4] Let $\mathcal{X}$ be the initial universe set, E be the set of parameters and $A \subset E$. Then the mapping $\mathcal{F} : E \to P(\mathcal{X})$ defined by $e \in A,\ \mathcal{F}(e) \neq \phi$, $e \notin A,\ \mathcal{F}(e) = \phi$.

Definition 2.3. [4] A soft set $(\mathcal{F}, E)$ over $\mathcal{X}$ is said to be a null soft set denoted by Φ if for all $e \in E,\ \mathcal{F}(e) = \phi$.

Definition 2.4. [4] A soft set $(\mathcal{F}, E)$ over $\mathcal{X}$ is said to be a absolute soft set denoted by $\tilde{\mathcal{X}}$ if for all $e \in E,\ \mathcal{F}(e) = \mathcal{X}$.

Definition 2.5. [4] Let R be the set of real numbers, $\mathfrak{B}(R)$ the collection of all non-empty bounded subsets of R and E a set of parameters. Then a mapping $\mathcal{F} : E \to \mathfrak{B}(R)$ is called a soft real set. It is denoted by $(\mathcal{F}, E)$ and $R(E)$ denotes the set of all soft real sets. Also, $R(E)^*$ denotes the set of all non-negative soft real sets ($(\mathcal{F}, E)$ is said to be a non-negative soft real set if $\mathcal{F}(\lambda)$ is a subset of the set of non-negative real numbers for each $\lambda \in E$).

In particular, if $(\mathcal{F}, E)$ is a singleton soft set, then identifying $(\mathcal{F}, E)$ with the corresponding soft element, we will call it a soft real number. $R(E)$ denotes the set of all soft real numbers. We use notations $\tilde{r},\ \tilde{s},\ \tilde{t}$ to denote soft real numbers.

Definition 2.6. [4] For two soft real numbers $\tilde{r},\ \tilde{s} \in R(E)$ we define

(1) $\tilde{r} \tilde{\leq} \tilde{s}$ if $\tilde{r}(\lambda) \leq \tilde{s}(\lambda)$, for all $\lambda \in E$,
(2) $\tilde{r} \tilde{\geq} \tilde{s}$ if $\tilde{r}(\lambda) \geq \tilde{s}(\lambda)$, for all $\lambda \in E$,
(3) $\tilde{r} \tilde{<} \tilde{s}$ if $\tilde{r}(\lambda) < \tilde{s}(\lambda)$, for all $\lambda \in E$,
(4) $\tilde{r} \tilde{>} \tilde{s}$ if $\tilde{r}(\lambda) > \tilde{s}(\lambda)$, for all $\lambda \in E$.

Definition 2.7. [11] A soft set $(\mathcal{F}, A)$ over $\mathcal{X}$ is said to be a soft point, if there exist $x \in \mathcal{X}$ and $e \in E$ such that $\mathcal{F}(e) = \{x\}$ and $\mathcal{F}(e^{'}) = \phi$ some $x \in \mathcal{X}$ and $\forall e^{'} \in E\{e\}$. It will be denoted by $\tilde{x}_e$.

Definition 2.8. [11] A soft point $\tilde{x}_e$ is said to be belongs to a soft set $(\mathcal{G}, A)$, if for the element $e \in E,\ \mathcal{F}(e) = \{x\} \subset \mathcal{G}(e)$. We write $\tilde{x}_e \tilde{\in} (\mathcal{G}, A)$.

Definition 2.9. [11] Two soft points $\tilde{x}_{e_1}, \tilde{y}_{e_2}$ are said to be equal if $e_1 = e_2$ and $x = y$. If $\tilde{x}_{e_1} \neq \tilde{y}_{e_2} \Leftrightarrow x \neq y$ or $e_1 \neq e_2$.

3. Soft $\mathcal{M}-$ Fuzzy Metric Spaces

Let $\tilde{\mathcal{X}}$ be the absolute soft set, E be a non-empty of parameters and $SP(\tilde{\mathcal{X}})$ be the collection of all soft points of $\tilde{\mathcal{X}}$. Let $R(E)^*$ denote the set of all non-negative soft real numbers. Soft real numbers in the $[0, 1]$ and $(0, \infty)$ are indicated by $[0, 1](E)$ and $(0, \infty)(E)$ respectively.

Definition 3.1. [4] A binary operation $\tilde{*} : [0, 1](E) \to [0, 1](E)$ is a continuous soft $\mathtt{t}-$ norm fulfills the accompanying conditions:

(i) $\tilde{*}$ is associative and commutative,
(ii) $\tilde{*}$ is continuous,
(iii) $\tilde{\sigma} \tilde{*} \bar{1} = \tilde{\sigma}$ for all $\tilde{\sigma} \tilde{\in} [0, 1](E)$,
(iv) $\tilde{\sigma} \tilde{*} \tilde{\varsigma} \tilde{\leq} \tilde{\alpha} \tilde{*} \tilde{\beta}$ for $\tilde{\sigma}, \tilde{\varsigma}, \tilde{\alpha}, \tilde{\beta} \tilde{\in} [0, 1](E)$ with the end goal that $\tilde{\sigma} \tilde{\leq} \tilde{\alpha}$ and $\tilde{\varsigma} \tilde{\leq} \tilde{\beta}$.

Basic example of a continuous soft $\mathtt{t}-$ norm are $\tilde{\sigma} \tilde{*} \tilde{\varsigma} = \min\{\tilde{\sigma}, \tilde{\varsigma}\},\ \tilde{\sigma} \tilde{*} \tilde{\varsigma} = \tilde{\sigma} \odot \tilde{\varsigma}$ and $\tilde{\sigma} \tilde{*} \tilde{\varsigma} = max\{\tilde{\sigma} \oplus \tilde{\varsigma} \ominus \bar{1}, \bar{0}\}$.

Definition 3.2. Let $\tilde{*}$ be a soft $\mathtt{t}-$ norm, and let $\tilde{*}_n; [0,1](E) \to [0,1](E),\ n \in \mathbb{N}$ be defined in the following way

$$\tilde{*}_1(\tilde{\sigma}) = \tilde{\sigma}\ \tilde{*}\ \tilde{\sigma},\ \tilde{*}_{n+1} = \tilde{*}_n(\tilde{\sigma})\ \tilde{*}\ \tilde{\sigma}, \quad n \in \mathbb{N},\ \tilde{\sigma}\ \tilde{\in}\ [0,1](E).$$

A soft $\mathtt{t}-$ norm $\tilde{*}$ is said to be soft $\mathcal{H}-$ type if the family $\{\tilde{*}_n(\tilde{\sigma})\}_{n \in \mathbb{N}}$ is equicontinuous at $\tilde{\sigma} = \bar{1}$.

A trivial example of soft $\mathtt{t}-$ norm of soft $\mathcal{H}-$ type is $\tilde{\sigma}\ \tilde{*}\ \tilde{\varsigma} = \min\{\tilde{\sigma}, \tilde{\varsigma}\}$.

Each soft $\mathtt{t}-$ norm $\tilde{*}$ can be extended by associativity in a unique way to an $n-$ ary operation taking for $(\tilde{\sigma}_1, \tilde{\sigma}_2, \ldots, \tilde{\sigma}_n)\ \tilde{\in}\ [0,1](E)^n$ the values

$$\tilde{*}^1_{\iota=1}\tilde{\sigma}_\iota = \tilde{\sigma}_1,\ \ \tilde{*}^n_{\iota=1}\tilde{\sigma}_\iota = \tilde{*}^{n-1}_{\iota=1}\tilde{\sigma}_\iota\ \tilde{*}\ \tilde{\sigma}_n = \tilde{\sigma}_1\ \tilde{*}\ \tilde{\sigma}_2\ \tilde{*}\ \ldots \tilde{\sigma}_n.$$

A soft $\mathtt{t}-$ norm $\tilde{*}$ can be extended to a countable infinite operation taking for any sequence $(\tilde{\sigma}_n)_{n\in\mathbb{N}}$ from $[0,1](E)$ the value

$$\tilde{*}^\infty_{\iota=1}\tilde{\sigma}_\iota = \lim_{n\to\infty}\ \tilde{*}^n_{\iota=1}\tilde{\sigma}_\iota.$$

The sequence $(\tilde{*}^\infty_{\iota=1}\tilde{\sigma}_\iota)_{n\in\mathbb{N}}$ is non-increasing and bounded from below and hence the limit $\tilde{*}^\infty_{\iota=1}\tilde{\sigma}_\iota$ exists.

In the fixed point theory it is of interest to investigate the classes of soft $\mathtt{t}-$ norms $\tilde{*}$ and sequences $(\tilde{\sigma}_n)$ from the interval $[0,1](E)$ with the end goal that $\lim\limits_{n\to\infty} \tilde{\sigma}_n = \bar{1}$ and $\lim\limits_{n\to\infty} \tilde{*}^\infty_{\iota=n}\tilde{\sigma}_\iota = \lim\limits_{n\to\infty} \tilde{*}^\infty_{\iota=1}\tilde{\sigma}_{n+\iota} = \bar{1}$.

Definition 3.3. A mapping $\tilde{\mathcal{M}} : SP(\tilde{\mathcal{X}}) \times SP(\tilde{\mathcal{X}}) \times SP(\tilde{\mathcal{X}}) \times (0,\infty)(E) \to [0,1](E)$ is said to be a soft $\mathcal{M}-$ fuzzy metric on the soft set $\tilde{\mathcal{X}}$ if $\tilde{\mathcal{M}}$ satisfies the following conditions: for every $\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \tilde{a}_e\ \tilde{\in}\ \tilde{\mathcal{X}}$ and $\tilde{t}, \tilde{s}\ \tilde{>}\ \bar{0}$,

($\mathcal{SFM}$-1) $\tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \tilde{\mathtt{t}})\ \tilde{>}\ \bar{0}$,

($\mathcal{SFM}$-2) $\tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \tilde{\mathtt{t}}) = \bar{1}$ if and only if $\tilde{\sigma}_{e_r} = \tilde{\varsigma}_{e_s} = \tilde{\eta}_{e_t}$,

($\mathcal{SFM}$-3) $\tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \tilde{\mathtt{t}}) = \tilde{\mathcal{M}}(p\{\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}\}, \tilde{\mathtt{t}})$ where p is permutation function.

($\mathcal{SFM}$-4) $\tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \tilde{\mathtt{t}} \oplus \tilde{\mathtt{s}})\ \tilde{\geq}\ \tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{a}_e, \tilde{\mathtt{t}})\ \tilde{*}\ \tilde{\mathcal{M}}(\tilde{a}_e, \tilde{\eta}_{e_t}, \tilde{\eta}_{e_t}, \tilde{\mathtt{s}})$,

($\mathcal{SFM}$-5) $\tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \cdot) : (0,\infty)(E) \to [0,1](E)$ is continuous.

The soft set $\tilde{\mathcal{X}}$ with a soft $\mathcal{M}-$ fuzzy metric $\tilde{\mathcal{M}}$ on $\tilde{\mathcal{X}}$ is called a soft $\mathcal{M}-$ fuzzy metric space ($S_{\mathcal{M}}FMS$) and denoted by $(\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$.

Definition 3.4. Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$ be a $S_{\mathcal{M}}FMS$, then

(i) A sequence $\{\tilde{\sigma}^n_{e_n}\}$ in $\tilde{\mathcal{X}}$ is said to be convergent to $\tilde{\sigma}_e$ if for each $\tilde{\mathtt{t}}\ \tilde{>}\ \bar{0}$, $\tilde{\mathcal{M}}(\tilde{\sigma}_e, \tilde{\sigma}_e, \tilde{\sigma}^n_{e_n}, \tilde{\mathtt{t}}) \to \bar{1}$ as $n \to \infty$.

(ii) A sequence $\{\tilde{\sigma}^n_{e_n}\}$ in $\tilde{\mathcal{X}}$ is said to be a Cauchy sequence if for each $\bar{0}\ \tilde{<}\ \tilde{\varepsilon}\ \tilde{<}\ \bar{1}$ and $\tilde{\mathtt{t}}\ \tilde{>}\ \bar{0}$, there exists $n_0 \in \mathbb{N}$ with the end goal that $\tilde{\mathcal{M}}(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^m_{e_m}, \tilde{\mathtt{t}})\ \tilde{>}\ \bar{1} \ominus \tilde{\varepsilon}$ for each $n, m \geq n_0$.

(iii) A soft $\mathcal{M}-$ fuzzy metric spaces are said to be complete if every Cauchy sequence is convergent.

Definition 3.5. A $S_{\mathcal{M}}FMS$ $(\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$ is said to be symmetric if $\tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\varsigma}_{e_s}, \tilde{\mathtt{t}}) = \tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\mathtt{t}})$ for all $\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}\ \tilde{\in}\ \tilde{\mathcal{X}}$ and for each $\tilde{\mathtt{t}}\ \tilde{>}\ \bar{0}$.

Definition 3.6. Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$ be a $S_{\mathcal{M}}FMS$. A function $(\mathcal{J}, \varphi) : (\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*}) \to (\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$ is said to be a fuzzy soft contraction mapping if there exists a soft real number $\tilde{q} \tilde{\in} [0,1](E)$ such that

$$\tilde{\mathcal{M}}((\mathcal{J},\varphi)\tilde{\sigma}_{e_r}, (\mathcal{J},\varphi)\tilde{\varsigma}_{e_s}, (\mathcal{J},\varphi)\tilde{\eta}_{e_t}, \tilde{\mathsf{t}}) \tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \frac{\tilde{\mathsf{t}}}{\tilde{q}}\Big), \quad \tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t} \tilde{\in} \tilde{\mathcal{X}}.$$

4. Main Results

Lemma 4.1. *Let $\{\tilde{\sigma}^n_{e_n}\}$ be a sequence in a symmetric $S_{\mathcal{M}}FMS$ $(\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$. Let's keep that that there are $\tilde{q} \tilde{\in} (0,1)(E)$ with the end goal that*

$$\tilde{\mathcal{M}}(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \tilde{\sigma}^{n+2}_{e_n}, \tilde{\mathsf{t}}) \tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^{n-1}_{e_n}, \tilde{\sigma}^{n}_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \frac{\tilde{\mathsf{t}}}{\tilde{q}}\Big), \quad n \tilde{\in} \mathbb{N},\ \tilde{\mathsf{t}} \tilde{>} \bar{0}, \tag{4.1.1}$$

and there exists $\tilde{\sigma}^0_{e_n}, \tilde{\sigma}^1_{e_n} \tilde{\in} \tilde{\mathcal{X}}$ and $\tilde{\rho} \tilde{\in} (0,1)(E)$ with the end goal that

$$\lim_{n\to\infty} \tilde{*}_{i=n}^{\infty} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^0_{e_n}, \tilde{\sigma}^1_{e_n}, \tilde{\sigma}^1_{e_n}, \frac{\tilde{\mathsf{t}}}{\tilde{\rho}^i}\Big) = 1, \quad \tilde{t} \tilde{>} \bar{0}. \tag{4.1.2}$$

Then $\{\tilde{\sigma}^n_{e_n}\}$ is a Cauchy sequence.

Proof. Let $\tilde{\varrho} \tilde{\in} (\tilde{q}, 1)(E)$. At that point the sum $\displaystyle\sum_{i=1}^{\infty} \tilde{\varrho}^i$ is convergent, and there exists $n_0 \in \mathbb{N}$ with the end goal that $\displaystyle\sum_{i=n}^{\infty} \tilde{\varrho}^i \tilde{<} \bar{1}$ for every $n > n_0$. Let $n > m > n_0$. Because of being $\tilde{\mathcal{M}}$ is non-decreasing, for each $\tilde{\mathsf{t}} \tilde{>} \bar{0}$, we have

$$\begin{aligned}
\tilde{\mathcal{M}}(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+m}_{e_n}, \tilde{\mathsf{t}}) &\tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+m}_{e_n}, \tilde{\mathsf{t}} \sum_{i=n}^{n+m-1} \tilde{\varrho}^i\Big) \\
&\tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \tilde{\mathsf{t}}\tilde{\varrho}^p\Big) \tilde{*} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^{n+1}_{e_n}, \tilde{\sigma}^{n+m}_{e_n}, \tilde{\sigma}^{n+m}_{e_n}, \tilde{\mathsf{t}} \sum_{i=n+1}^{n+m-1} \tilde{\varrho}^i\Big) \\
&= \tilde{\mathcal{M}}\Big(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \tilde{\mathsf{t}}\tilde{\varrho}^n\Big) \tilde{*} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^{n+1}_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \tilde{\sigma}^{n+m}_{e_n}, \tilde{\mathsf{t}} \sum_{i=n+1}^{n+m-1} \tilde{\varrho}^i\Big) \\
&\tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \tilde{\mathsf{t}}\tilde{\varrho}^p\Big) \tilde{*} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^{n+1}_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \tilde{\sigma}^{n+2}_{e_n}, \tilde{\mathsf{t}}\tilde{\varrho}^{n+1}\Big) \\
&\qquad \tilde{*} \dots \tilde{*} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^{n+m-1}_{e_n}, \tilde{\sigma}^{n+m-1}_{e_n}, \tilde{\sigma}^{n+m}_{e_n}, \tilde{\mathsf{t}}\tilde{\varrho}^{n+m-1}\Big)
\end{aligned}$$

By (4.1.1) we get,

$$\tilde{\mathcal{M}}(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \tilde{\sigma}^{n+2}_{e_n}, \tilde{\mathsf{t}}) \tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^0_{e_n}, \tilde{\sigma}^1_{e_n}, \tilde{\sigma}^2_{e_n}, \frac{\tilde{\mathsf{t}}}{\tilde{q}^n}\Big), \quad n \in \mathbb{N},\ \tilde{\mathsf{t}} \tilde{>} \bar{0}.$$

Because of being $n > m$, we have

$$\tilde{\mathcal{M}}(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+m}_{e_n}, \tilde{\mathfrak{t}}) \tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^0_{e_n}, \tilde{\sigma}^0_{e_n}, \tilde{\sigma}^1_{e_n}, \frac{\tilde{\mathfrak{t}}\tilde{\varrho}^n}{\tilde{q}^n}\Big) \tilde{*} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^0_{e_n}, \tilde{\sigma}^0_{e_n}, \tilde{\sigma}^1_{e_n}, \frac{\tilde{\mathfrak{t}}\tilde{\varrho}^{n+1}}{\tilde{q}^{n+1}}\Big)$$
$$\tilde{*} \dots \tilde{*} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^0_{e_n}, \tilde{\sigma}^0_{e_n}, \tilde{\sigma}^1_{e_n}, \frac{\tilde{\mathfrak{t}}\tilde{\varrho}^{n+m-1}}{\tilde{q}^{n+m-1}}\Big)$$
$$\tilde{\geq} \tilde{*}^{n+m-1}_{i=n} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^0_{e_n}, \tilde{\sigma}^0_{e_n}, \tilde{\sigma}^1_{e_n}, \frac{\tilde{\mathfrak{t}}\tilde{\varrho}^i}{\tilde{q}^i}\Big)$$
$$\tilde{\mathcal{M}}(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+m}_{e_n}, \mathfrak{t}) \tilde{\geq} \tilde{*}^{\infty}_{i=n} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^0_{e_n}, \tilde{\sigma}^0_{e_n}, \tilde{\sigma}^1_{e_n}, \frac{\tilde{\mathfrak{t}}}{\tilde{\rho}^i}\Big), \qquad \text{where } \tilde{\rho} = \frac{\tilde{q}}{\tilde{\varrho}}.$$

Because of being $\tilde{\rho} \tilde{\in} (0,1)(E)$, by (4.1.2) for this reason $\{\tilde{\sigma}^n_{e_n}\}$ is a Cauchy sequence. □

Corollary 4.2. *Let $\{\tilde{\sigma}^n_{e_n}\}$ be a sequence in a symmetric $S_{\mathcal{M}}FMS$ $(\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$ and let $\tilde{*}$ be of soft $\mathcal{H}-$ type. If there are $\bar{q} \tilde{\in} (0,1)(E)$ with the end goal that*

$$\tilde{\mathcal{M}}(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \tilde{\sigma}^{n+2}_{e_n}, \tilde{\mathfrak{t}}) \tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^{n-1}_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+1}_{e_n}, \frac{\tilde{\mathfrak{t}}}{\tilde{q}}\Big), \quad n \tilde{\in} \mathbb{N},\ \tilde{\mathfrak{t}} \tilde{>} \bar{0}.$$

Then $\{\tilde{\sigma}^n_{e_n}\}$ is a Cauchy sequence.

Lemma 4.3. *If for few $\tilde{q} \tilde{\in} (0,1)(E)$ with $\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t} \tilde{\in} \tilde{\mathcal{X}}$,*

$$\tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \tilde{\mathfrak{t}}) \tilde{\geq} \tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \frac{\tilde{\mathfrak{t}}}{\tilde{q}}),\ \tilde{\mathfrak{t}} \tilde{>} \bar{0}. \tag{4.3.1}$$

Then $\tilde{\sigma}_{e_r} = \tilde{\varsigma}_{e_s} = \tilde{\eta}_{e_t}$.

Proof. By (4.3.1) we get,

$$\tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \tilde{\mathfrak{t}}) \tilde{\geq} \tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \frac{\tilde{\mathfrak{t}}}{\tilde{q}^n}),\ n \in \mathbb{N},\ \tilde{\mathfrak{t}} \tilde{>} \bar{0}.$$

$$\therefore\ \tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \tilde{\mathfrak{t}}) \tilde{\geq} \lim_{n\to\infty} \tilde{\mathcal{M}}(\tilde{\sigma}_{e_r}, \tilde{\varsigma}_{e_s}, \tilde{\eta}_{e_t}, \frac{\tilde{\mathfrak{t}}}{\tilde{q}^n}) = \bar{1},\ \tilde{\mathfrak{t}} \tilde{>} \bar{0}.$$

Hence $\tilde{\sigma}_{e_r} = \tilde{\varsigma}_{e_s} = \tilde{\eta}_{e_t}$. □

Theorem 4.4. *Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$. be a symmetric complete $S_{\mathcal{M}}FMS$. Let $(\mathcal{J}, \varphi) : (\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*}) \to (\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$ be a fuzzy soft contraction mapping and there exists $\tilde{\sigma}^0_{e_0} \tilde{\in} \tilde{\mathcal{X}}$ and $\tilde{\rho} \tilde{\in} (0,1)(E)$ with the end goal that*

$$\lim_{n\to\infty} \tilde{*}^{\infty}_{i=n} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^0_{e_0}, \tilde{\sigma}^0_{e_0}, (\mathcal{J}, \varphi)\tilde{\sigma}^0_{e_0}, \frac{\tilde{\mathfrak{t}}}{\tilde{\rho}^i}\Big) = \bar{1},\ \tilde{\mathfrak{t}} \tilde{>} \bar{0}. \tag{4.4.1}$$

Then $(\mathcal{J}, \varphi)$ has a unique fixed point on $\tilde{\mathcal{X}}$.

Proof. Let $\tilde{\sigma}^0_{e_0} \tilde{\in} \tilde{\mathcal{X}}$ with $\tilde{\sigma}^{n+1}_{e_{n+1}} = (\mathcal{J}, \varphi)\sigma^n_{e_n}$, $n \in \mathbb{N}$. Take $\tilde{\sigma}_{e_r} = \tilde{\sigma}^{n-1}_{e_{n-1}}$, $\tilde{\sigma}_{e_s} = \tilde{\sigma}^{n-1}_{e_{n-1}}$ and $\tilde{\sigma}_{e_t} = \tilde{\sigma}^n_{e_n}$ for every $n \in \mathbb{N}$ with each $\tilde{\mathfrak{t}} \tilde{>} \bar{0}$ we have

$$\tilde{\mathcal{M}}(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}^n_{e_n}, \tilde{\sigma}^{n+1}_{e_{n+1}}, \tilde{\mathfrak{t}}) \tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^{n-1}_{e_{n-1}}, \tilde{\sigma}^{n-1}_{e_{n-1}}, \tilde{\sigma}^n_{e_n}, \frac{\tilde{\mathfrak{t}}}{\tilde{q}}\Big).$$

Using Lemma (4.1) we get $\{\tilde{\sigma}^n_{e_n}\}$ is a Cauchy sequence. Because of being $(\tilde{\mathcal{X}}, \tilde{\mathcal{M}}, \tilde{*})$ is complete, there exists $\tilde{\sigma}_e \tilde{\in} \tilde{\mathcal{X}}$ with the end goal that

$$\lim_{n\to\infty} \tilde{\sigma}^n_{e_n} = \tilde{\sigma}_e \quad \text{and} \quad \lim_{n\to\infty} \tilde{\mathcal{M}}(\tilde{\sigma}_e, \tilde{\sigma}_e, \tilde{\sigma}^n_{e_n}, \tilde{\mathfrak{t}}) = \bar{1}, \ \ \tilde{\mathfrak{t}} \tilde{>} \bar{0}. \tag{4.4.2}$$

Consider,

$$\begin{aligned}
\tilde{\mathcal{M}}((\mathcal{J},\varphi)\tilde{\sigma}_e, (\mathcal{J},\varphi)\tilde{\sigma}_e, \tilde{\sigma}_e, \tilde{\mathfrak{t}}) &\tilde{\geq} \tilde{\mathcal{M}}\Big((\mathcal{J},\varphi)\tilde{\sigma}_e, (\mathcal{J},\varphi)\tilde{\sigma}_e, \tilde{\sigma}^n_{e_n}, \frac{\tilde{\mathfrak{t}}}{2}\Big) \tilde{*} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}_e, \tilde{\sigma}_e, \frac{\tilde{\mathfrak{t}}}{2}\Big) \\
&\tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}_e, \tilde{\sigma}_e, \tilde{\sigma}^{n-1}_{e_{n-1}}, \frac{\tilde{\mathfrak{t}}}{2\tilde{q}}\Big) \tilde{*} \tilde{\mathcal{M}}\Big(\tilde{\sigma}^n_{e_n}, \tilde{\sigma}_e, \tilde{\sigma}_e, \frac{\tilde{\mathfrak{t}}}{2}\Big) \\
&\tilde{\geq} \bar{1} \tilde{*} \bar{1} \quad \text{as} \quad n \to \infty
\end{aligned}$$

$\mathcal{M}((\mathcal{J},\varphi)\tilde{\sigma}_e, (\mathcal{J},\varphi)\tilde{\sigma}_e, \tilde{\sigma}_e, \tilde{\mathfrak{t}}) \tilde{\geq} \bar{1}$.

Hence $(\mathcal{J},\varphi)\tilde{\sigma}_e = \tilde{\sigma}_e$.

Let's keep that $\tilde{\sigma}_e$ and $\tilde{\mathfrak{v}}_e$ are fixed point for $(\mathcal{J},\varphi)$. Now,

$$\tilde{\mathcal{M}}(\tilde{\sigma}_e, \tilde{\sigma}_e, \tilde{\mathfrak{v}}_e, \tilde{\mathfrak{t}}) = \tilde{\mathcal{M}}((\mathcal{J},\varphi)\tilde{\sigma}_e, (\mathcal{J},\varphi)\tilde{\sigma}_e, (\mathcal{J},\varphi)\tilde{\mathfrak{v}}_e, \tilde{\mathfrak{t}}) \tilde{\geq} \tilde{\mathcal{M}}\Big(\tilde{\sigma}_e, \tilde{\sigma}_e, \tilde{\mathfrak{v}}_e, \frac{\tilde{\mathfrak{t}}}{\tilde{q}}\Big)$$

Using Lemma (4.3) implies that $\tilde{\sigma}_e = \tilde{\mathfrak{v}}_e$. □

5. Conclusion

In this paper, we utilize a countable extension of the soft $\mathtt{t}-$ norm . Utilizing this development and contraction condition to demonstrate the sequence in soft $\mathcal{M}-$ fuzzy metric spaces is a Cauchy sequence. Fixed point theorems is additionally demonstrated by applying our outcomes.

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