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SHARE PRICE DYNAMICS OF LISTED COMPANIES ON THE DHAKA STOCK EXCHANGE USING GEOMETRIC BROWNIAN MOTION

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Abstract. The stock prices of publicly traded companies exhibit continuous and random fluctuations over time, necessitating the inclusion of a stochastic term in dynamic models to accurately capture this behavior. This study applies the geometric Brownian motion (GBM) model to analyse the stock prices of 20 randomly selected companies listed on the Dhaka Stock Exchange (DSE). The GBM model was resolved through Monte Carlo simulation to forecast stock prices over a trading horizon of approximately 30 to 35 days. Using historical data from the first four months of 2024, we predicted the share prices for the subsequent one-and-a-half months. The comparison between forecast and actual prices demonstrated a high level of concordance, with a mean absolute percentage error (MAPE) of less than 8%. These findings underscore the efficacy of the model in providing robust predictions of share prices for selected companies.

1. Introduction

The dynamic behavior of stock prices has had a profound impact on numerous economies, particularly within emerging markets in Africa, Asia, and the Americas. These markets have attracted considerable attention because of their unique performance characteristics, piquing the interest of traders, exchange officials, and academics alike. Consequently, the existing literature on predictive analysis has been predominantly concentrated on these burgeoning markets. Predictive modeling, a commonly used method for mathematically forecasting market trends, finds extensive applications in various disciplines, including social sciences, economics, and finance. In economics, it is used primarily to predict stock prices.

Bachelier [5] was a pioneer in utilizing stochastic processes to forecast stock price behavior. He developed the first mathematical model for predicting stock prices, which he validated using future prices and options. According to Bachelier [5], stock price dynamics adhere to a Brownian motion framework that does not

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account for the time value of money. This model was subsequently refined to ensure that stock prices followed a log-normal distribution [13, 22, 23]. Samuelson's refinement is now recognised as the Geometric Brownian Motion (GBM) model.

Fama [8] characterized stock price behavior as a random walk, asserting that stock prices inherently reflect business-related information. Positive information enhances investor confidence, thereby increasing stock demand and prices, while negative information diminishes confidence, reducing demand and prices. Since information arrives randomly, stock prices change in a random fashion, supporting the random walk hypothesis. Moreover, the continuous flow of information implies immediate impacts on stock prices, suggesting that tomorrow's price changes are contingent on new information, independent of today's data [8]. He proposed that information is assimilated at three levels in an efficient market. He defined an efficient market as one comprising numerous profit-maximizing competitors striving to predict future security values, utilizing readily available current information accessible to all. Fama [8] delineated three levels of market efficiency: the weak form, semi-strong form, and strong form, collectively known as the Efficient Market Hypothesis (EMH). Fama [9] empirically tested these three forms of EMH. First, he assessed the weak form, concluding that price adjustments are based on historical stock price information. Next, he examined the semi-strong form, evaluating whether prices are influenced by publicly available information, such as annual earnings announcements, takeovers, and stock splits. Finally, he tested the strong form, investigating whether prices are affected by groups with exclusive access to pertinent information. Fama's empirical analysis led to the conclusion that stock market prices adhere to a random walk.

According to Meyler et al. [19], the Autoregressive Integrated Moving Average (ARIMA) model is a well-known statistical method for forecasting based on historical data. Despite its popularity, ARIMA faces limitations, particularly concerning non-stationarity and seasonality, as noted by Tambi [27]. In contrast, artificial neural networks (ANNs) have emerged as highly accurate models for forecasting, pattern recognition, and image processing [14, 15, 16]. These models, part of machine learning or soft computing methods, have gained significant traction in economics, finance, and economic forecasting over the past decade due to their data-driven, self-adaptive nature and ability to function as universal approximators [14]. Once trained, ANNs can generalize and make predictions even with inconsistent input data. White [28] conducted a seminal study on neural network models and stock price predictions for IBM common stock, demonstrating highly optimistic predictions. According to Hassan et al. [11], a fusion model combining hidden Markov models (HMMs), genetic algorithms (GAs), and artificial neural networks (ANNs) can effectively predict financial market movements. Hassan and Nath [12] concluded that this fusion tool outperforms a single HMM model and is comparable to the ARIMA model. Merh et al. [18] developed a neural network model with a three-layer feedforward architecture and autoregression to predict future stock price valuations, revealing that ARIMA models outperform ANN models.

Geometric Brownian Motion (GBM) is another prevalent method for predicting stock prices in stochastic modeling. The GBM model posits that the returns of a particular stock are independent and normally distributed over a given period [6], accounting for specific levels of volatility and drift. However, Estember [7] points out that the assumption of constant variability and drift in GBM does not always hold in real-world scenarios. Agustini et al. [10] developed a Brownian motion-based forecasting model using the Jakarta Corporate stock price index, achieving a mean absolute percentage error rate of 20%, indicating high forecast accuracy. Rathnayaka et al. [21] created a forecasting model based on GBM and compared its forecasts with those of a traditional ARIMA model. Using data from the Colombo Stock Exchange in Sri Lanka, they found that the GMB model's predictions were more accurate and effective than those of the traditional model.

The literature reveals diverse opinions regarding the performance of different forecasting models. Hence, further comparative studies on stochastic modeling are necessary to establish consistency in stock price prediction methodologies. This paper analyses stock price behavior and develops a stochastic model for forecasting. Ten randomly selected companies listed on the Dhaka Stock Exchange (DSE) were studied to develop a stochastic model for stock price estimation and prediction. The study integrates insights from mathematics researchers, financial engineers, and economists, demonstrating the efficacy of stochastic models in predicting share prices. It also enhances risk control and investment profitability. By developing and testing a model on the DSE, the study proves the effectiveness of stochastic models in forecasting share prices.

2. Model Formulation

The Brownian motion stochastic process emerged in the 18th century as an effort to describe the irregular motions of microscopic pollen grains suspended in a drop of water. Today, the Brownian motion process and its many generalizations and extensions are prevalent in numerous and diverse areas of pure and applied science. These areas include economics, communication theory, biology, management science, mathematical finance, and statistics.

2.1. Brownian Motion.

The Brownian motion, denoted by $B(t)$ is a stochastic process used to describe the random behaviour of a stochastic process and has the following properties [2].

(1) **Independence of increments**: $B(t) - B(s), t \geq s$ is independent of the past values $B(u)$ for $u \leq s$.
(2) **Normal increments**: $B(t) - B(s)$ has Normal distribution with mean 0 and variance $t - s$, that is, $B(t) - B(s) \sim N(0, t - s)$.
(3) **Continuity of paths**: $B(t), t \geq 0$ are continuous functions of t.

Louis Bachelier [4] used the arithmetic Brownian motion as the first model of stock prices. He assumed that the discount rate in the stock price $S(t)$ satisfies the following differential equation:

$$dS(t) = \mu dt + \sigma dB(t) \tag{2.1}$$

Where $B(t)$ is the standard Brownian motion or Wiener process, μ is the return on the stock price, σ is the volatility of the stock price. Equation (2.1) is called arithmetic Brownian motion. This model laid the foundation for later developments in the stochastic modeling of financial markets.

The solution to Equation (2.1) is

$$S(t) = S(0) + \mu t + \sigma B(t) \tag{2.2}$$

The solution to the arithmetic Brownian motion model can lead to a negative stock price with positive probability, which is not realistic in practice. To address this issue, the model was redefined by Osborne [23], who stated that the return of the stock follows a log-normal distribution. This redefinition led to the introduction of the geometric Brownian motion (GBM) model by Samuelson [24].

2.2. **Geometric Brownian Motion (GMB).** Consider the dynamics of the share price process of a company denoted by $S(t)$, the return of the share price by μ, then the return of the share price of a company defined by

$$Return = \frac{change\,in\,price}{original\,price} \tag{2.3}$$

Consider a small subsequent time interval $(t, t + \Delta t)$ during which $S(t)$ becomes $S(t + \Delta t) = S(t) + \Delta S(t))$. The return on the energy price between time t and $(t + \Delta t)$ is given by

$$Return = \frac{S(t + \Delta t) - S(t)}{S(t)} = \frac{\Delta S(t)}{S(t)}$$

It represents the daily return on a company's stock price. Stock returns can be modeled as a two-asset portfolio consisting of a risk-free asset, such as a bond, and a risky asset, such as a derivative. If the return on the risk-free asset is μ, then its return over a very short time horizon dt will be μdt. However, the return on the risky asset is uncertain, and this uncertainty or randomness is captured by the Brownian motion $B(t)$.

$$\frac{\Delta S(t)}{S(t)} = \mu \Delta t + \sigma dB(t) \tag{2.4}$$

$$\frac{dS(t)}{S(t)} = \mu \Delta t + \sigma dB(t)$$

or

$$dS(t) = \mu S(t)dt + \sigma S(t)dB(t) \tag{2.5}$$

where σ is the standard deviation of stock returns and depends on random changes in asset prices due to external influences such as unexpected news. Equation (2.5) is called geometric Brownian motion. In integral form

$$S\left(t\right) = \sigma \int_0^t S(u)du + \sigma \int_0^t S(u)dB(u) \tag{2.6}$$

S(t) is said to be an Ito process.

2.3. **Analytic Solution to GBM.** To solve the Equation (2.5) analytically, we proceed as follows: Let

$$f(t) = \ln S\left(t\right), \text{then } f^{'} = \frac{1}{S(t)} \text{ and } f^{''}(S(t)) = \frac{1}{S(t)^2}$$

By Ito formula for Ito process

$$d(\ln S(t)) = f^{'} dS(t) + \frac{1}{2} f^{''} \sigma^2(t)dt$$

Hence,

$$\begin{aligned} d(\ln S(t)) &= \frac{1}{S(t)} dS(t) + \frac{1}{2}\left(-\frac{1}{S(t)^2}\right)\sigma^2(t)S(t)^2 dt \\ d(\ln S(t)) &= \frac{1}{S(t)}(\mu S(t)dt + \sigma S(t)dB(t)) - \frac{1}{2}\sigma^2 dt \\ d(\ln S(t)) &= \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dB(t) \end{aligned}$$

Integrating both sides gives solution can be found as

$$S(t) = S(0)exp\Big((\mu - \frac{1}{2}\sigma^2)t + \sigma B(t)\Big) \tag{2.7}$$

Equation (2.7), clearly shows that the value of $S(t)$ is always positive and hence it is a suitable to model the share price of energy or any other financial derivatives.

2.4. **Simulation Procedure.** As described earlier in the properties of Brownian motion, the increment in Brownian motion $B\left(t+1\right) - B\left(t\right)$ is normally distributed with mean 0 and variance 1. Hence it follows that the probability distribution of the change in the value of share price from now to the coming year is standard normally distributed with mean 0 and variance 1:

$$B(t+1) - B(t) \sim N(0,1)$$

Similarly, the probability distribution of the change in the value of share price between year 1 and year 2 is also normally distributed with mean 0 and variance 1:

$$B(t+2) - B(t+1) \sim N(0,1)$$

Thus, the probability distribution of change in the value of share price in two years is the sum of the two normal distributions each with mean 0 and variance 1. This means that in two years the probability distribution of change in the value of energy price is normally distributed with mean $\mu = 0 + 0 = 0$ and variance $\sigma^2 = 1 + 1 = 2$:

$$B(t+2) - B(t) \sim N(0, 2)$$

Hence in two years the probability distribution of the change in $S(t)$ is normally distributed with mean 0 and standard deviation $\sqrt{2}$. That is,

$$S(t) - S(0) \sim N(0, \sqrt{2})$$

. More generally, over T years, the probability distribution of the share price change is

$$S(t) - S(0) \sim N(0, \sqrt{T})$$

. In a short time period Δt, the change in the value of the stock price is normally distributed with mean 0 and standard deviation . Thus

$$S(t) - S(0) \sim N(0, \sqrt{\Delta t})$$

Let $\xi(t)$ denote the randomness captured by the Brownian motion $B(t).\xi(t)$ is known as the White Noise process and is defined as the derivative of the Brownian motion $B(t)$ such that

$$\xi(t) = \frac{dB(t)}{dt} = B'(t) \tag{2.8}$$

Therefore, from Equation (2.5), we can express the change in stock price $S(t)$ in terms of $\xi(t)$:

$$dS(t) = \mu S(t)dt + \sigma S(t)\xi(t)dt \tag{2.9}$$

In discrete form, in the short time period Δt, Equation (2.9) can be written as

$$\Delta S(t) = \mu S(t)\Delta t + \sigma S(t)\xi(t)\sqrt{\Delta t} \tag{2.10}$$

We will use Equation (2.10) to simulate future share prices of companies listed on DSE. A Monte Carlo simulation of the share price will be based on sampling random outcomes for the process. A price path for the share price can be simulated by sampling repeatedly for $\xi(t)$ from N(0,1) and substituting in Equation (2.10). We use Excel to produce a random sample between 0 and 1.

2.5. **Parameter Estimation (Volatility σ and Drift μ).** To develop the algorithm for the simulation process, we need to estimate two parameters: the volatility (σ) and the drift (μ) of the share price for the selected companies. Both parameters will be calculated using daily time units. The daily closing prices for the first four months of 2024, sourced from Investing.com, will be used to compute the daily returns of share prices for ten selected companies on the DSE. From this data, the average daily return (μ) will be determined. Additionally, the standard deviation of the daily returns (volatility, σ) will be computed. With μ and σ known, we can simulate the daily price paths of individual companies for the next one and a half months. Assuming no price changes from the last trading day of April 2024 to the first trading day of May 2024, the initial stock price $S(0)$ for the simulation is the closing price on the last trading day of April

2024. The procedure for the Monte Carlo simulation process is summarized as follows:

(1) Compute the daily returns of stock for the first four months of 2024 as follows

$$\mu_i = \ln\left(\frac{s\left(t_i\right)}{S\left(t_{i-1}\right)}\right) \tag{2.11}$$

and mean stock price return as

$$\hat{\mu}_i = \frac{1}{n}\sum_{i=1}^{n} \ln\left(\frac{S\left(t_i\right)}{S\left(t_{i-1}\right)}\right) \tag{2.12}$$

(2) Compute the standard deviation (volatility , σ) of daily returns

$$S(t) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} \mu_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n} \mu_i\right)^2} \tag{2.13}$$

(3) Determine the time interval Δt, where $\Delta t = \frac{1}{trading\,days}$

(4) Simulate the price path using the equation

$$\ln \Delta S(t) = \mu S(t)\Delta t + \sigma S(t)\xi(t)\sqrt{\Delta t} \tag{2.14}$$

For each company, several price paths are simulated by generating different sets of random numbers. This approach captures various realizations of price paths, from which the mean is taken as the realized price path for each individual company. By the Central Limit Theorem, it follows that this mean path approximates the true price path of the company.

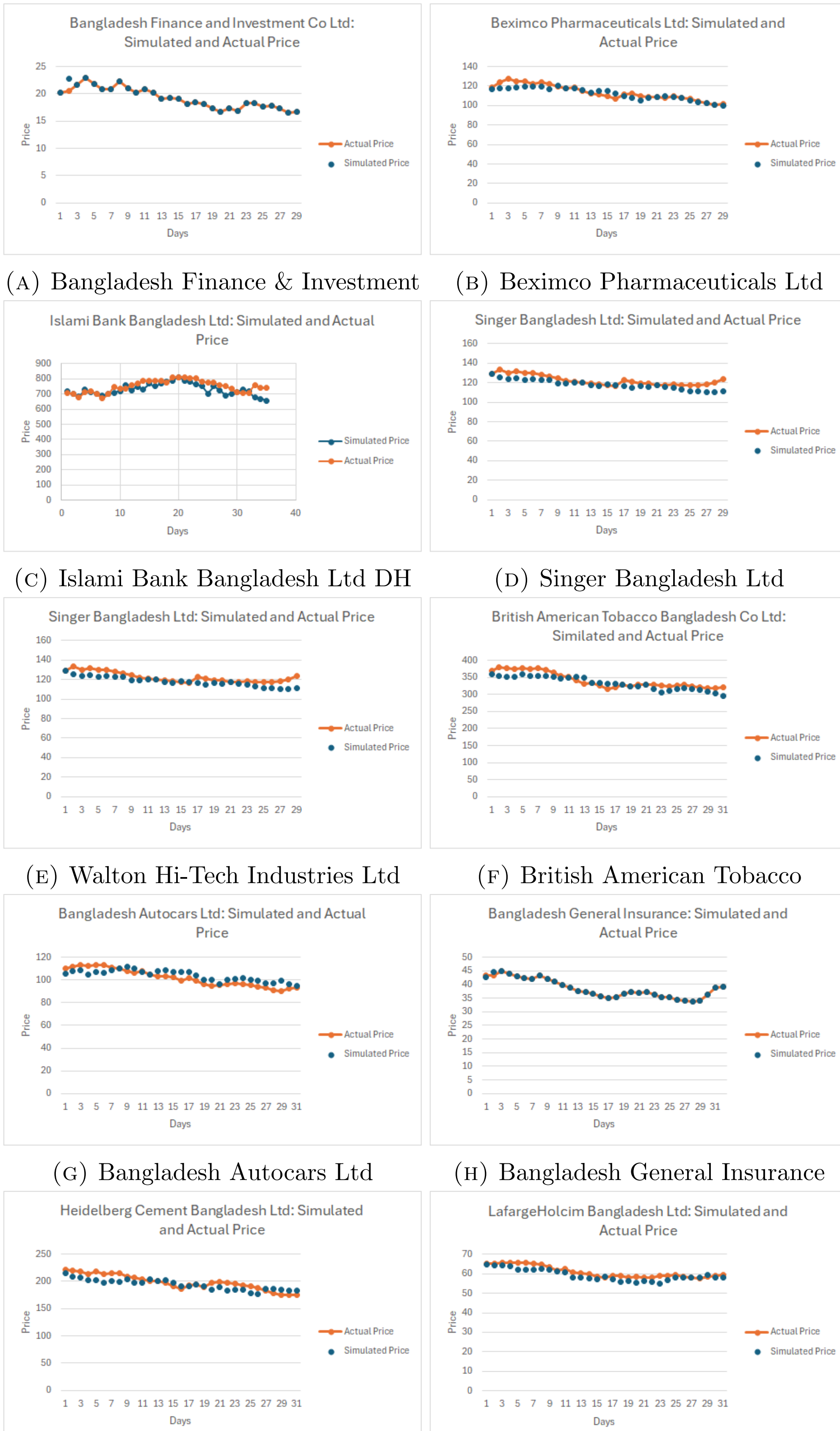
3. Results

The average daily returns and volatilities for the selected 20 companies listed on the DSE are presented in Table 1. These values, calculated from the first four months of 2024, are used to predict prices for the next 30 to 35 trading days. Price paths for all companies were generated using Monte Carlo simulation in Excel and then compared with actual prices. The comparison between simulated and actual prices for the 20 companies is shown in Figure 1-2. The figures demonstrate that the simulated prices closely matched the actual prices.

To evaluate the accuracy of the predictions, we calculated the mean absolute percentage error (MAPE) between the simulated and actual prices. The MAPE value is computed using the formula:

$$MAPE = \frac{1}{N}\sum_{i=1}^{n}\left|\frac{A_i - F_i}{A_i}\right|$$

where A_i is the actual price, F_i is the simulated price and N is the number of prediction. To assess the MAPE value, we referred to the scale provided by Abidin and Jaffar [1], shown in Table 2. According to this scale, Table 1 indicates that the forecasting accuracy for all companies are very good.



(A) Bangladesh Finance & Investment

(B) Beximco Pharmaceuticals Ltd

(C) Islami Bank Bangladesh Ltd DH

(D) Singer Bangladesh Ltd

(E) Walton Hi-Tech Industries Ltd

(F) British American Tobacco

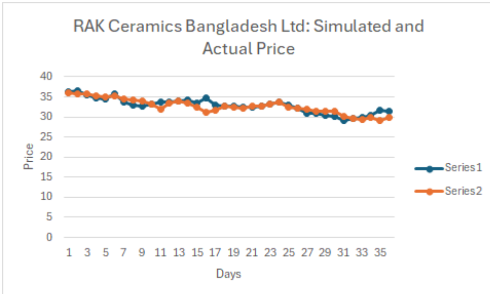
(G) Bangladesh Autocars Ltd

(H) Bangladesh General Insurance

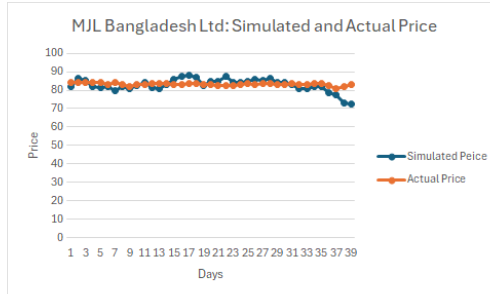
(I) Heidelberg Cement Bangladesh Ltd

(J) LafargeHolcim Bangladesh Ltd

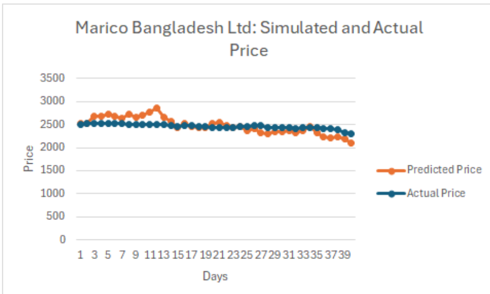
FIGURE 1. Comparison of actual and simulated prices for selected companies.



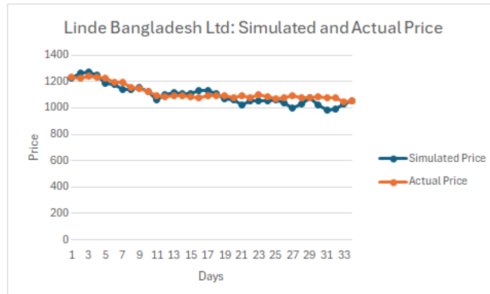
(A) Rak Ceramics Bangladesh Ltd.



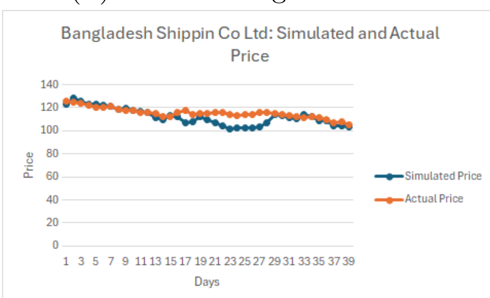
(B) MJL Bangladesh Ltd.



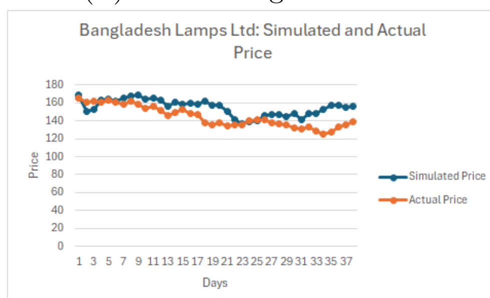
(C) Marico Bangladesh Ltd.



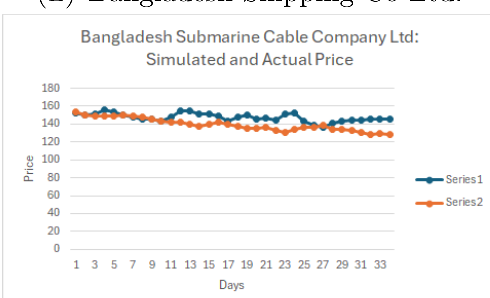
(D) Linde Bangladesh Ltd.



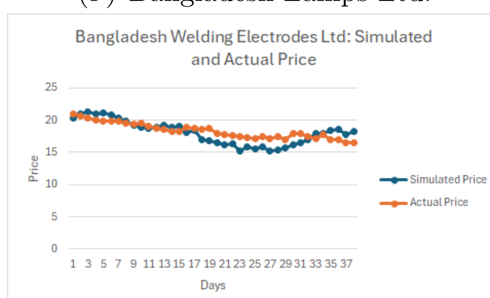
(E) Bangladesh Shipping Co Ltd.



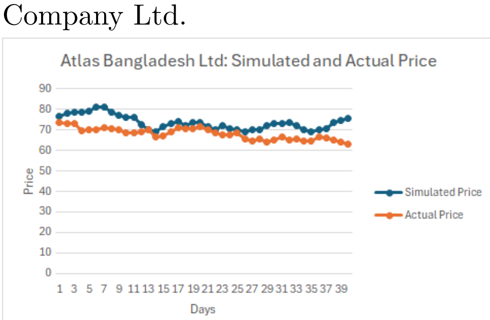
(F) Bangladesh Lamps Ltd.



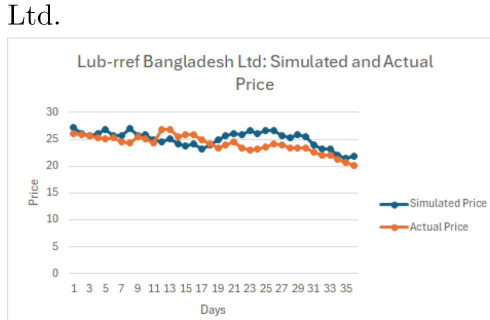
(G) Bangladesh Submarine Cable Company Ltd.



(H) Bangladesh Welding Electrodes Ltd.



(I) Atlas Bangladesh Ltd.



(J) Lubrref Bangladesh Ltd

FIGURE 2. Comparison of actual and simulated prices for selected companies.

Table 1. Mean returns and volatilities of traded shares of chosen companies on DSE.

Name	BDFN	BXPH	IBBD	SGBD	WALT
Mean (μ)	0.009111835	-0.002639129	-0.001977856	-0.002122208	-0.007396034
Std (σ)	0.051852842	0.017955661	0.035616794	0.010686313	0.025145436
MAPE (%)	0.39	2.37	3.64	3.56	2.42

Name	BATC	BDAT	BGIC	HEID	LFAR
Mean (μ)	-0.004461228	-0.002432935	-0.002580914	-0.0008908	-0.000850619
Std (σ)	0.018917942	0.020948443	0.016800702	0.022727335	0.012610597
MAPE (%)	3.64	4.28	0.15	4.22	2.88

Name	RAKC	MJLB	MARI	LIND	BSCD
Mean (μ)	0.004002438	0.000801245	-0.000649371	0.002983313	-0.003943925
Std (σ)	0.03289209	0.006884197	0.00602797	0.016532554	0.026862725
MAPE (%)	5.05	2.67	6.02	3.17	4.63

Name	BGLA	BANA	BDWE	ATBG	LUBR
Mean (μ)	0.011165183	0.009894255	0.001714192	0.00854821	0.007796469
Std (σ)	0.038265366	0.03044912	0.029583361	0.02622094	0.029959547
MAPE (%)	7.13	7.78	6.08	7.78	7.16

4. Discussion

The geometric Brownian motion described by the linear stochastic differential equation (SDE) (2.9) is a continuous-time stochastic process. It is termed a linear SDE because the drift and volatility terms are linear functions of S [17]. Geometric Brownian Motion assumes that the share price is log-normally distributed, meaning that the logarithm of the share price at time t follows a normal distribution with a mean of μt and a variance of $\sigma^2 t$. The mathematical details based on the Ito formula are essential and should be thoroughly explored. Let us consider that the share price process adheres to the GBM model:

$$dS(t) = \mu S(t)dt + \sigma S(t)\xi(t)dB(t) \tag{4.1}$$

The first term of the above equation represents the drift, which is the average movement of the share price over each time increment. The second term represents the volatility, which introduces a random component affecting the stock's price. This volatility is responsible for the noise observed in the price curve. Using Ito formula the solution of Equation (4.1) can be written as

$$d\left(\text{In}\, S(t)\right) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dB(t) \tag{4.2}$$

TABLE 2. MAPE accuracy rating scale

MAPE Value	Forecasting Accuracy
$< 10\%$	Very good
$11\% - 20\%$	Good
$21\% - 50\%$	Within reasonable limit
$> 51\%$	Inaccurate

Equation (4.2) indicates that $\ln S(t)$ follows an Ito process with mean rate $\left(\mu - \frac{1}{2}\sigma^2\right)$ and variance σ^2. Thus, the change in share price between two future times s and t, $s < t$ is normally distributed with mean rate $\left(\mu - \frac{1}{2}\sigma^2\right)(t - s)$ and variance $\sigma^2(t - s)$. If the initial time is 0 and the future time is T, then we can write

$$\text{In}\, S(T) - \text{In}\, S(0) \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)T, \sigma\sqrt{T}\right)$$

or

$$\text{InS}(T) \sim N\left(\text{InS}(0) + \left(\mu - \frac{1}{2}\sigma^2\right)T, \quad \sigma\sqrt{T}\right)$$

Now, if the share prices follow a log-normal distribution and the mean μ and volatility σ are known, the distribution of the share prices and their density function can be derived. This allows for a detailed description of the future behavior of the share prices.

Consider for example BDFN (Bangladesh Finance and Investment Company Ltd) with initial price $S(0) = 20.20$, with an annual return of $\mu = 0.009111835$ and volatility $\sigma = 0.051852842$. The price behaviour of the BDFN in the next month can be established using the equation

$$\text{InS}(T) \sim N\left(\text{In}\, S(0) + \left(\mu - \frac{1}{2}\sigma^2\right)T, \sigma\sqrt{T}\right)$$

That is

$$\text{InS}(T) \sim N\left(\ln 20.20 + \left(0.009111835 - \frac{1}{2}(0.051852842)^2\right) \times \frac{1}{12}, \quad 0.051852842\sqrt{\frac{1}{12}}\right)$$

$$\text{lnS}(T) \sim N(3.006329894, \quad 0.01496862614)$$

A 95% confidence interval for the distribution of $\ln S(T)$ in the next month can be given as

$$\mu - 1.96\sigma < \ln S(T) < \mu + 1.96\sigma$$

$$e^{\mu - 1.9\sigma} < S(T) < e^{\mu + 1.9\sigma}$$

$$0.9116312195 < S(T) < 1.117108226$$

Thus there is 95% confidence that in the next month, the BDFN share price return will lie between 0.9116312195 and 1.117108226. that is

$$P(1.117108226 < S(T) < 1.117108226) = 0.95$$

Now if a random variable is lognormally distributed then its density is given by

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-(\ln x - \mu)^2}, \quad x \geq 0$$

and the cumulative distribution function

$$F(X) = \int_{-\infty}^{\infty} \frac{1}{x\sigma\sqrt{2\pi}} e^{-(\ln x - \mu)^2} dx$$

If the random variable X represents the BDFN share price $S(t)$ then we can write

$$f(S(t)) = \frac{1}{S(t)\sigma\sqrt{2\pi}} e^{-(\ln S(t) - \mu)^2}, \quad s \geq 0$$

For Brent oil with $\mu = 0.009111835$ and standard deviation $\sigma = 0.051852842$ and so its density function is given as

$$f(S(t)) = \frac{1}{0.051852842 S(t)\sqrt{2\pi}} e^{-(ln(S(t)) - 0.009111835)^2}$$

The cumulative distribution function is given by

$$F(S(t)) = \int_0^t \frac{1}{0.051852842\sqrt{2\pi}} e^{-(\ln S(s) - 0.009111835)^2} ds$$

To rigorously assess the accuracy of the proposed model, simulated stock prices were compared with actual stock prices. This evaluation involved testing for significant differences at a 5% significance level ($\alpha = 0.05$) between the mean share prices predicted by the model and the observed share prices. The results, summarized in Table 3, demonstrate that the model accurately captures price behavior for 80% of the selected companies.

Additionally, stock prices for a limited number of companies listed on the DSE have been predicted using machine learning and hidden Markov models [26, 25]. These studies report Mean Absolute Percentage Error (MAPE) values comparable to those observed in this study, further corroborating the reliability of the predictive framework.

Table 3 demonstrates that the model's predicted average returns align closely with the actual average returns for 16 out of 20 traded stocks, representing an 80% success rate. However, certain assumptions inherent to the model may have contributed to deviations in specific cases. For instance, the model assumes that the initial price for the prediction period is equivalent to the closing price from the final trading day in the historical dataset, an assumption that may not consistently reflect actual market conditions. Future studies could explore alternative methods for determining the initial stock price, potentially improving the robustness and accuracy of the model's predictions. Volatility estimates in this study were derived from the historical stock prices of the preceding three months. However, the accuracy of these estimates could be improved by employing more advanced methodologies, such as Exponentially Weighted Moving Average (EWMA), ARCH, GARCH, implied volatility, or machine learning techniques [15]. Another potential limitation arises from the use of trading days as time intervals rather than calendar days, implicitly assuming that volatility evolves solely during market operation hours. This simplification may introduce minor inaccuracies in reflecting true market dynamics. Nonetheless, the model demonstrates robust predictive performance and provides meaningful insights for investors, financial analysts, and researchers.

Table 3. Hypothesis test between the mean for actual and simulated prices with ($\alpha = 5\%$) Significance

Company	BDFN	BXPH	IBBD	SGBD	WALT
p-value	0.311	0.031	< 0.001	< 0.001	0.058
Decision	Significant	Significant	Significant	Significant	not significant

Company	BATC	BDAT	BGIC	HEID	LFAR
p-value	< 0.01	0.037	0.586	0.004	< 0.001
Decision	Significant	Significant	not Significant	Significant	Significant

Company	RAKC	MJLB	MARI	LIND	BSCD
p-value	< 0.01	< 0.01	0.44	0.003	< 0.001
Decision	Significant	Significant	not Significant	Significant	Significant

Company	BGLA	BANA	BDWE	ATBG	LUBR
p-value	< 0.01	< 0.001	< 0.001	0.94	< 0.001
Decision	Significant	Significant	Significant	not Significant	Significant

5. Conclusion

A robust stochastic model has been systematically developed to predict the share price dynamics of companies listed on the Dhaka Stock Exchange. Simulated price trajectories for ten leading companies were rigorously evaluated against actual market prices, demonstrating a high degree of alignment. Additionally, a detailed framework for the mathematical analysis of stock price probability distributions was presented. This research offers significant value to investors and stakeholders, particularly within the Dhaka Stock Exchange, by providing a scientifically grounded basis for informed decision-making in stock trading.

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