Annals of Mathematics and Computer Science
ISSN: 2789-7206
Vol 4 (2021) 14-27

# BAYESIAN GARCH MODELS FOR NIGERIA COVID-19 DATA

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Abstract. After the initial acceptance the author must prepare the manuscript according to the journal's format. Authors should use the following style file as a template. The abstract should be short and concise. Due to the high demand for accurate volatility forecasts, there has been an immense interest amongst both practitioners and researchers to model time varying volatility series. Unlike many researches on GARCH models, this paper centres on the use of generalized autoregressive conditional heteroskedasticity (GARCH) models for medical data. With GARCH models, issues of no (conditional) heteroskedasticity, no error autocorrelation, linearity, and parameter constancy are adequately tested. Bayesian estimation method is used to estimate the parameters of GARCH models based on daily COVID-19 total cases and new cases in Nigeria, for the period (28th February, 2020) when the index case came to Nigeria through to when the second wave (19th January, 2021) of the COVID-19 cases in Nigeria began to rise. Taking advantage of Bayesian approach to estimating the parameters of a model, two Bayesian generalized autoregressive conditional heteroskedasticity (GARCH) models were used in capturing the volatility dynamics of the daily total cases and daily new cases of COVID-19 in Nigeria based on their conditional distributions. From analysis, diagnosis tests, results revealed that the Bayesian Student-t GARCH (1, 1) model performed better than the Bayesian Normal GARCH (1, 1) model.

## 1. Introduction and preliminaries

Accurate volatility forecasts are of immense interests amongst both practitioners and researchers of time varying volatility data. Modelling time volatility data is a challenging task. Multitude of adaptions (or generalized) autoregressive conditional heteroskedasticity (GARCH) models are available in literatures for capturing these volatility characteristics (Bollerslev, 1986; Nelson, 1991; Dellaportas and Vrontos, 2007; Black, 1976; Ding et al., 2001 and Bai et al., 2003). Volatility clustering (i.e. periods of low volatility alternate with more agitated periods) is the main advantage of GARCH models. However, more researches on changing volatility (i.e., conditional variance) using time series models are still ongoing following the conception of autoregressive conditional heteroskedasticity

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*Date*: Received: Sep 28 2021; Accepted: Dec 2, 2021.

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2010 *Mathematics Subject Classification.* Primary 46L55; Secondary 44B20.

*Key words and phrases.* ARCH Model, Bayesian Inference, COVID-19, GARCH Model, Nigeria.

(ARCH) model. ARCH models have grown into a household of empirical models for volatility forecasting in the last two decades (Engel, 2002a; Osiewalski and Pipien, 2004).

. With GARCH models, testing the adequacy of the estimated models for no (conditional) heteroskedasticity, no error autocorrelation, linearity, and parameter constancy etc. are no longer necessary except misspecifications (Bollerslev, 1986; Engel, 2002b). GARCH models for health related data is not common in contemporary research as many researchers stop at using the autoregressive (AR) models and the autoregressive integrated moving average (ARIMA) models for health related data. This paper centres on the use of GARCH models for medical data.

. With some limitations, the frequentist utilizes the maximum likelihood (ML) estimation technique as a scheme of inference for modelling the GARCH models. From a theoretical point of view, the ML estimators benefit from being asymptotically optimal under some certain conditions (Bollerslev et al., 1994; Ankarali et al., 2020). However, these limitations do not apply when Bayesian methods are used. Firstly, any constraints on the model parameters can be incorporated in the modelling through appropriate prior specifications. Secondly, appropriate Markov Chain Monte Carlo (MCMC) procedures can explore the joint posterior distribution of the model parameters. These techniques avoid local maxima (i.e., convergence to wrong values or non-convergence) encountered via ML estimation of sophisticated GARCH models. Third, exact distributions of nonlinear functions of the model parameters can be obtained at low cost by simulating from the joint posterior distribution (Ardia, 2010).

. The popularity of Bayesian inference in the field of statistics is predominantly due to the increase in machines computation power and that information can be added a priori, leading to potentially improved estimates. One of the major drawbacks of the frequentist approach to estimating the parameters of a model is that the method of maximum likelihood cannot estimate models that are not in closed form. Thus, with the advent of the Monte-Carlo Markov chain (MCMC) method, numerical computations of the Bayesian inference have gained relevance (Gilks *et al.*, 1995; Neal, 2000). Many authors have worked on Bayesian methods applied on GARCH models (Chen *et al.*, 2009; Bauwens and Lubrano, 2002; Ausin *et al.*, 2014; Giannikis *et al.*, 2008). Ausín and Galeano (2007) modelled the errors of a GARCH model with a mixture of two Gaussian distributions; Ardia and Hoogerheide (2010), did an extensive work on the asymmetric GARCH model with Student-t innovations. The work of Nakatsuma (2000), Asai (2006) and Baurwens *et al.*, (2006) give more details about Bayesian GARCH models.

. In this paper, Bayesian estimation method is used to estimate the parameters of GARCH models based on daily COVID-19 total cases and new cases in Nigeria, to see how volatile the new cases are as against the total number of cases in Nigeria and to see which of the conditional distributions best capture the volatility of the total number of COVID-19 cases and the new cases for the period under study using their standard errors. The data employed for this analysis are the daily total confirmed cases and daily new confirmed cases of coronavirus (COVID-19) in Nigeria when the index case came to Nigeria through to when the second wave of the COVID-19 cases in Nigeria began to rise (from 28th February, 2020 to 19th January, 2021).

## 2. Methodology

**2.1. Bayesian Estimation.** Let $\theta$ represents the parameter of a model such that $\theta$ can be a scalar, a vector or matrix. In Bayesian estimation, $\theta$ is considered as a random variable and is characterized by a prior density $p(\theta)$ as our belief in $\theta$ without the data y (in frequentist approach, $\theta$ is not a random variable). Using Bayes rule, $p(\theta)$ is incorporated such that:

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{\int p(y|\theta)p(\theta)d\theta} \tag{2.1}$$

where $p(\theta|y)$ is the posterior density (our belief in when the data $y$ is given) and $p(y|\theta)$ is likelihood function (the probability that the data could be generated by the model with parameter values $\theta$ ). Note that both the prior and likelihood are necessary requirements for Bayesian estimation (Ogundeji and Adeleke, 2019). To evaluate the integration in equation 1, Monte-Carlo method is used. The parameter $\theta$ is defined in the context of particular model (without a model, $\theta$ is meaningless). Let us assume that the model is $m$. Then, Bayes rule is given by

$$p(\theta|y,m) = \frac{p(y|\theta,m)p(\theta|m)}{\int p(y|\theta,m)p(\theta|m)d\theta} \tag{2.2}$$

It is needful to specify the model, when there is more than one model to consider. Also, a model is not fully defined until a range for the parameter values for $\theta$ is specified. The Bayes rule can be used to compare different models.

**2.2. ARCH and GARCH.** Autoregressive conditional heteroskedacity (ARCH) model was formulated in 1982 by an economist Robert Engle to address the problem of forecasting volatility in asset prices. His model assumed the variation of financial returns was not constant over time but are autocorrelated, or conditional to/dependent on each other. For instance, one can see in stock returns where periods of volatility in returns tends to cluster together. The ARCH model (Engle 1982) is given as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i e_{i-1}^2. \tag{2.3}$$

Where $\alpha_0 > 0$ and $\alpha_i \geq 0$, $i = 1, 2, ..., q$. The problem with ARCH model is that it takes a long lag to capture the empirical characteristics. Bollerslev (1986) generalizes the pure ARCH model to the general autoregressive conditional heteroskedacity (GARCH) model. The GARCH models are the most commonly used in the literature for modelling volatility, estimating Value-at-Risk (VaR) and expected shortfall. An innovational process $\{e_i\}$ is a GARCH $(p, q)$ model if

i. $e_i = \sigma_i a_i$, where $a_i$'s are independent and identically distributed $N(0.1)$.
ii.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i e_{i-1}^2 + \sum_{i=1}^{p} \beta_j \sigma_{i-j}^2 \tag{2.4}$$

where $\alpha_0 > 0, \alpha_i \geq 0, i = 1, 2, ..., q$ and $\beta_j \geq 0,\ j = 1, 2, ..., p$ for the positivity of the conditional variance. The GARCH model also called the symmetric GARCH model developed by Bollerslev and Taylor (1986) assumes that the dynamic behaviour of the conditional variance is given by the conditional variance equation:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \quad \epsilon_i|I_{i-1}\ (0, \sigma_t^2) \tag{2.5}$$

where the conditional variance and volatility are conditional on the information set, $I_{i-1}$. The GARCH conditional volatility is the annualized square root of this conditional variance. Since the conditional variances at different points in time are related, the process is not identically distributed and neither is it independent, because the second conditional moments, i.e. the conditional variances, at different points in time are related. The GARCH $(p, q)$ offers flexibility for taking into account, both autoregressive and moving average components, in the equation of variance. The generalization is similar in spirit to that of moving average (MA) models to autoregressive moving average (ARMA) models. Intuitively, consider the pure ARCH $(\infty)$ model.

$$\sigma_t^2 = \beta_0 + \beta_1 e_{i-1}^2 + \beta_2 e_{i-2}^2 + \dots \tag{2.6}$$

If $\beta_i = \beta_1^i$ for $i \geq 1$, then we have

$$\begin{aligned} \sigma_i^2 &= \beta_0 + [\beta_1 + \beta_1^2 B + \beta_1^3 B^2 + \dots]e_{i-1}^2 \\ &= \beta_0 + \beta_i[1 + \beta_i B + \beta_i^2 B^2 + \dots]e_{i-1}^2 \\ &\beta_0 + \frac{\beta_1}{\beta_i B} e_{i-1}^2 \end{aligned}$$

which implies

$$1 - \beta_i B\sigma_i^2 = (1 + \beta_i B)\beta_0 + \beta_i e_{i-1}^2 \tag{2.7}$$

where $\alpha_0 = \beta_0(1 - \beta_1)$. The above equation is in the form of GARCH $(1, 1)$ model, even though the coefficients of $e_{i-1}^2$ and $\sigma_{i-1}^2$ are identical.

## 3. Results and Discussion

3.1. **Data Description and Exploration.** The data employed for this analysis are the daily total confirmed cases and daily new confirmed cases of coronavirus (COVID-19) in Nigeria from 28th February, 2020 to 19th January, 2021; obtained from the website of the World Health Organization (WHO). The graphical exploration of the data using time plots is displayed in Figure 1

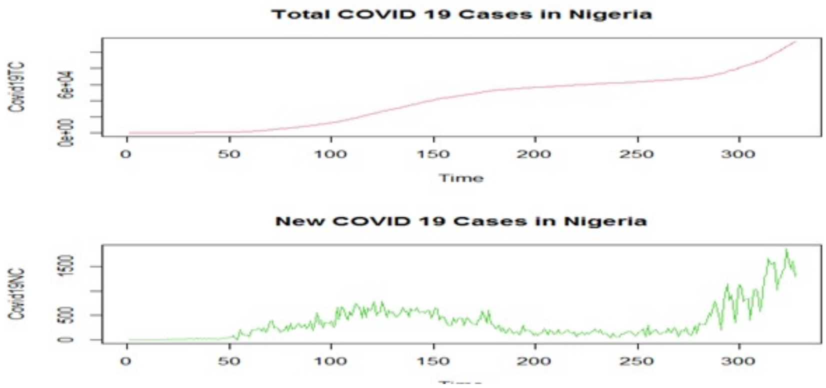


Figure 1. Time Plot of Daily Confirmed Total Cases and New Cases in Nigeria

Figure 1 display shows the gradual increase of daily total confirmed cases and daily new confirmed cases of coronavirus (COVID-19) in Nigeria during the first wave (early period) of the pandemic. However, the second wave (later period) of the pandemic is better illustrated in the daily new confirmed cases of COVID-19. Basic summary statistics are given in Table 1.

Table 1. Basic Summary Statistics of Coronavirus (COVID-19) in Nigeria

| | **Min.** | **Mean** | **Standard Deviation** | **Skewness** | **Excess Kurtosis** | **Max.** |
|---|---|---|---|---|---|---|
| **Total Cases** | 1.00 | 40172.24 | 30784.91 | 0.1195021 | −1.066736 | 113305 |
| **New Cases** | 1.00 | 346.4985 | 353.4592 | 1.755879 | 3.303492 | 1464 |

. To test for stationarity, Augmented Dickey fuller test was conducted for both daily total confirmed cases and daily new confirmed cases. For the total confirmed cases, the data was not stationary at all lags and for all conditions as the p-values were greater than 0.05. For the new confirmed cases, the data was stationary at lag zero for only drift and trends only. The results are shown in Table 2.

. To achieve stationarity of the data for daily total confirmed cases, the log of the difference of the data was taken and the plot of this is seen in Figure 2.

TABLE 2. Stationarity Test Raw Data (Total Cases & New Cases)

| | | **Lag** | **ADF** | **P-Value** |
|---|---|---|---|---|
| *Total Cases* | *No drift, no trend* | 0 | 22.91 | 0.99 |
| | *With drift, no trend* | 0 | 11.18 | 0.99 |
| | *With drift, With trend* | 0 | 4.249 | 0.99 |
| *New Cases* | *No drift, No trend* | 0 | $-1.676$ | 0.09 |
| | *With Drift , No trend* | 0 | $-2.92$ | 0.05 |
| | *With drift, with trend* | 0 | $-3.63$ | 0.03 |
| *Total Cases* | *No drift, no Trend* | 1 | 3.09 | 0.99 |
| | *With drift, no trend* | 1 | 2.07 | 0.99 |
| | *With Drift, with trend* | 1 | $-0.20$ | 0.99 |
| *New Cases* | *No drift, no trend* | 1 | $-0.76$ | 0.41 |
| | *With drift, no trend* | 1 | $-1.86$ | 0.38 |
| | *With drift, with trend* | 1 | $-2.53$ | 0.35 |

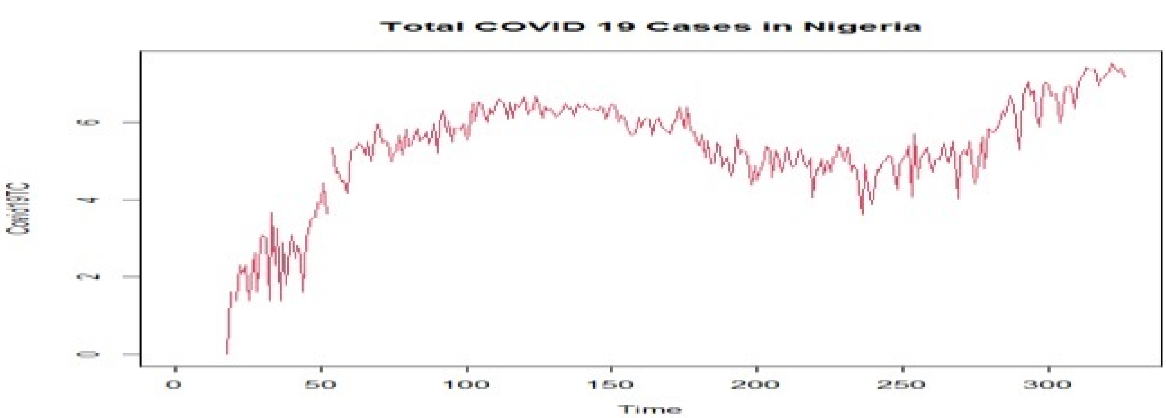


FIGURE 2. Time Plot of Differenced Data of Daily Total Confirmed COVID-19

The confirmation that the differenced data of daily total confirmed data is stationary is given by Augmented Dickey Fuller test:
$H_{0(\textbf{Total cases})}$: There is no stationarity in the daily total cases of COVID-19 differenced dataset
$H_{1(\textbf{Total cases})}$: There is stationarity in the daily total cases of COVID-19 differ-

enced dataset
. At 5% level of significance, test rejects $H_{0(Total\ cases)}$ and conclude that there is stationarity in the daily total cases of COVID-19 differenced dataset. The result of this is shown in Table 3.

Table 3. Stationarity for Differenced Data of Daily Total & New Cases of COVID-19

| **Variables** | | **Lag** | **ADF** | **P-Value** |
|---|---|---|---|---|
| **Total Cases** | *No drift, no trend* | 0 | $-0.292$ | 0.560 |
| | *With drift, no trend* | 0 | $-4.65$ | 0.0100 |
| | *With drift and trend* | 0 | $-4.96$ | 0.0100 |
| | *No drift, no trend* | 1 | 0.470 | 0.779 |
| | *With drift, no trend* | 1 | $-4.07$ | 0.0100 |
| | *With drift and trend* | 1 | $-3.99$ | 0.0100 |

3.2. **Model Identification.** To begin model identification, the data are fitted using ARIMA models then the autoregressive conditional heteroscedacity Lagrange multiplier (ARCH-LM) test is conducted. Thus, the ARIMA models used to fit the data follow a moving average model. The result of the ARIMA model is shown in Table 4.

Table 4. ARIMA model Identification for the Total cases and New cases of COVID-19

| | **MA(1)** | **MA(2)** | $Drift$ | $\hat{\sigma}^2$ |
|---|---|---|---|---|
| **Total Cases** | $-0.4208$ | $-0.1973$ | – | 4039.32 |
| $ARIMA(0,2,2)$ | (0.0531) | (0.0496) | | |
| **New Cases** | $-0.4334$ | $-0.2059$ | 4.5110 | 0.0009759 |
| $ARIMA(0,1,2)$ | (0.0529) | (0.0496) | (2.3941) | |

To test for Autoregressive Conditional Heteroscedacity Lagrange multiplier (ARCH-LM)
$H_{0(Total\ cases)}$: There is no heteroscedacity in the ARIMA model for the total cases
$H_{1(Total\ cases)}$: There is heteroscedacity in the ARIMA model for the total cases
$H_{0(New\ cases)}$: There is no heteroscedacity in the ARIMA model for the new cases
$H_{1(New\ cases)}$: There is heteroscedacity in the ARIMA model for the new cases
. At 5% level of significance, reject H0(Total cases) & $H_0$(New cases) based on the result in Table 5 and conclude that the ARIMA model for total cases and new cases is heteroscedastic.

TABLE 5. ARCH-LM Test for the Total cases and New cases of COVID-19

| | *Order* | **PQ Test (P-value)** | **LM Test (P-value)** |
|---|---|---|---|
| *Total Cases* | 4 | 123(p¡.001) | 155.56(p¡.001) |
| *New Cases* | 4 | 123(p¡.001) | 155.56(p¡.001) |

3.3. **Parameter Estimation of the Bayesian GARCH Model.** Following the work of Ardia (2009), the estimate of parameters of the Bayes GARCH models for both for the student-t innovation and the normal innovation. The difference between the two methods of estimation is that the student-t innovation is heavy tailed while the normal innovation is normally distributed. Bayesian GARCH model utilizes the Monte-Carlo Markov (MCMC) which uses various sampler and chains of iteration, however, this work used the Metropolis Hasting sampler. Thus, the number of iteration is what justifies whether it is a t distribution or it is a normal distribution. In the implementation of the student-t distribution as the conditional distribution, a higher number of iteration will definitely yield a heavier tailed distribution while a lower number of iteration will yield a normally distributed random variables. Note that when the value of $\alpha_1$ (reaction parameter) is high, it implies that the there is a tendency to have a higher wave of the disease infection (higher volatility) and when the value of is low implies that it possesses lower volatility. For the parameter (persistent parameter), when it falls between 0.85 and 0.98 implies a lower value of $\alpha$ and a lower value of $\beta$ implies a higher value of $\alpha_1$ and in this case, will imply that the disease will be spikier that is subject to higher volatility. This is the case of the standard GARCH model given as GARCH (1, 1).

TABLE 6. Estimation of Bayesian GARCH Model (1, 1) with Student-t Distribution

| **Variables** | $\alpha_0$ | $\alpha_1$ | $\beta$ | $\nu$ |
|---|---|---|---|---|
| *Total Cases* | 1.5803 | 0.9659 | 0.1374 | 201.7108 |
| | (0.166625) | (0.006745) | (0.005317) | (11.485142) |
| *New Cases* | 0.3711 | 0.5421 | 0.5962 | 111.6844 |
| | (0.010062) | (0.006242) | (0.003829) | (12.140189) |

. Now, from the result in Table 6, it is seen that $\alpha$ (the reaction parameter) is 0.9659 for total cases and 0.5421 for new cases. These values, according to Alexander (2008) indicate a very high volatility. This implies that the COVID-19 total cases and new cases are given to higher volatility which means that there is a tendency for a spike to occur in terms of the total number of cases and that of new cases. $\beta$ (persistent parameter) for total cases is 0.1374 and that of new cases is 0.5962 and $\nu$ which is the conditional distribution parameter is 201.7108 for total cases and 111.6844 for new cases of COVID-19 spread in Nigeria.

Table 7. Estimation of Bayesian GARCH Model (1, 1) with Normal Distribution

| **Variables** | $\alpha_0$ | $\alpha_1$ | $\beta$ | $\nu$ |
|---|---|---|---|---|
| *Total Cases* | 0.6333 | 0.5877 | 0.3915 | 484.0071 |
| | (0.1205) | (0.1157) | (0.1449) | (13.0986) |
| *New Cases* | 0.04732 | 0.19778 | 0.67127 | 247.00451 |
| | (0.01558) | (0.04083) | (0.01200) | (61.38459) |

. Table 7 shows the parameter estimates for total cases and new cases of COVID-19 with normal innovation. For total cases of COVID-19, it is seen that $\alpha$(reaction parameter) is 0.5877, $\beta$ (persistent parameter) is 0.3915 and $\nu$ which is the conditional distribution parameter is 484.0071 and the errors of the parameters of the model is close to zero. For new cases of COVID-19, it is seen that $\alpha$ (reaction parameter) is 0.23092, $\beta$ (persistent parameter) is 0.66634 and $\nu$ which is the conditional distribution parameter is 299.00268 and the errors of the parameters of the model is close to zero.

Table 8. Model Diagnostic of Bayesian GARCH Model (1, 1) with Student-t Distribution

| | *Standard Deviation* | *Naive Standard Error* | *Time Series Error* |
|---|---|---|---|
| *Total Cases* | 121.8497 | 1.2185 | 11.6638 |
| *New Cases* | 96.0302 | 0.9603 | 12.1603 |

Table 9. Standard Deviation and Error values of Bayesian GARCH Model with Normal Distribution

| | *Standard Deviation* | *Naive Standard Deviation* | *Time Series Error* |
|---|---|---|---|
| *Total Cases* | 79.3325 | 3.9666 | 13.47 |
| *New Cases* | 193.3123 | 9.6656 | 61.4610 |

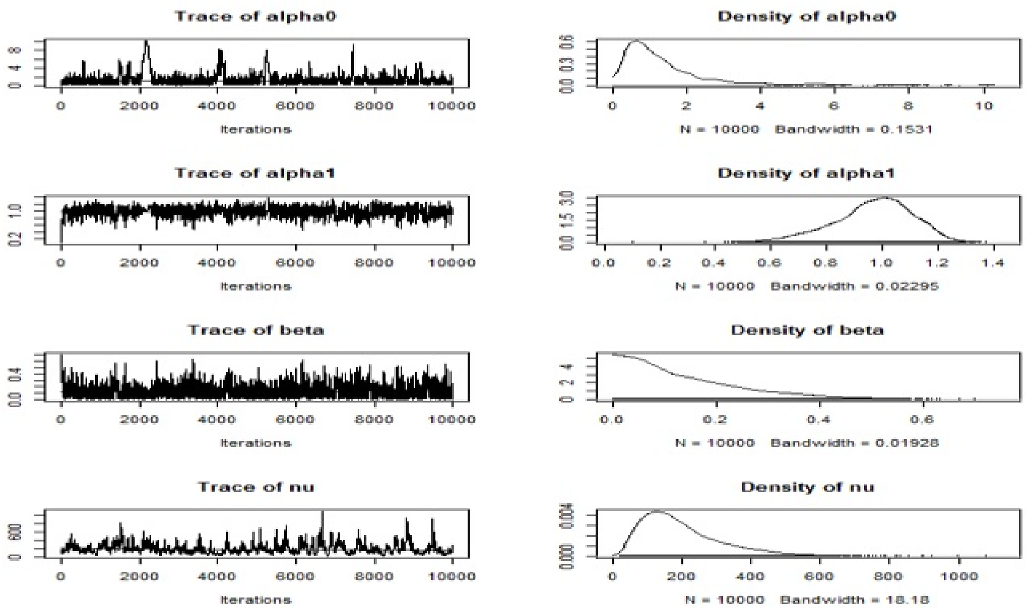


Figure 3. Plot of the Total cases of COVID-19 with Student-t Innovations.

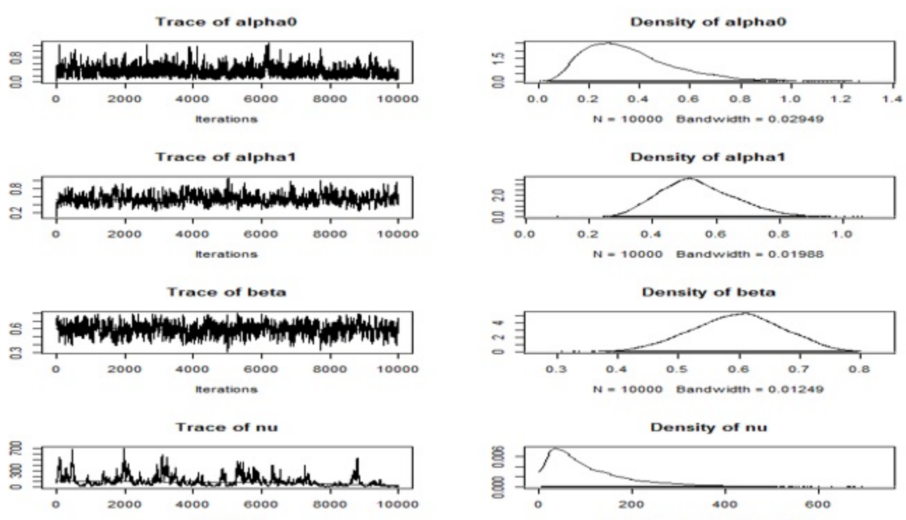


Figure 4. Plot of the New cases of COVID-19 with Student-t Innovations

. The work of Alexander and Barbosa (2008) and Ardia (2009) explains the level of volatility in the COVID-19 total cases and new cases in Nigeria. The results of the student t distribution and the normal distribution show that the ARCH parameter had a very high volatility with a value of 0.9659 for total cases and 0.5421 for new cases respectively (see Table 6). This implies that the COVID-19 total

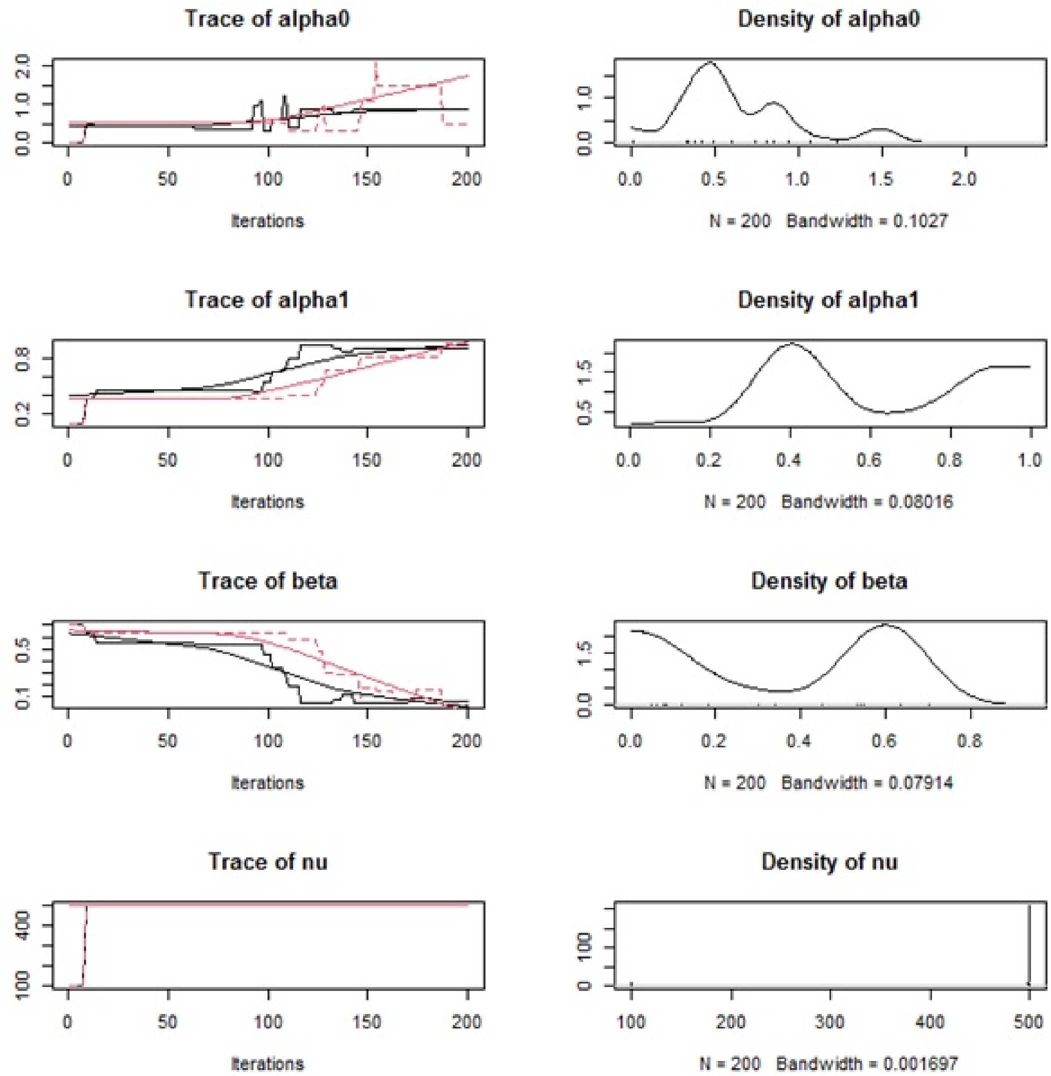


FIGURE 5. Plot of the Total cases of COVID-19 with Normal Innovations

cases and new cases in Nigeria are on the rise. In addition, the GARCH parameter $\beta$ (persistent parameter) shows that the volatility of the COVID-19 total cases and new cases in Nigeria dies out quickly. Compared to the Bayesian GARCH (1,1) model with student-t distribution, the Bayesian GARCH model (1,1) with normal distribution had ARCH parameter with lower values and GARCH parameter higher value but the conditional variances of the parameters are high (Table 7). This implies that the Bayesian GARCH model (1,1) with normal distribution is not suitable for the data. Also, the standard error for Bayesian GARCH model (1, 1) with normal distribution has a higher value compared with the student-t distribution (see Table 8 and 9). From the results obtained favour the Bayesian GARCH (1,1) model with student-t distribution as the reaction parameter had higher volatility at lower iterations and was stable at higher iteration values.

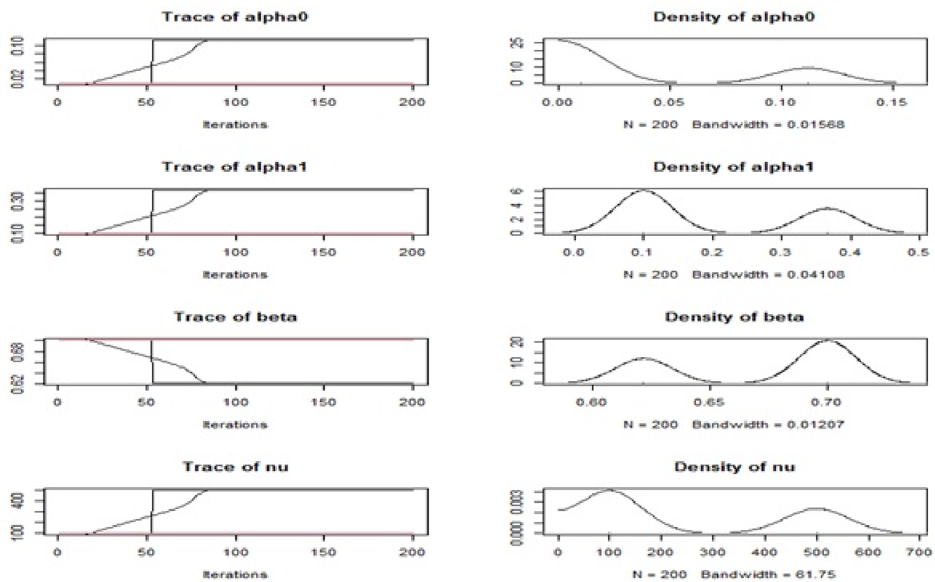


FIGURE 6. Plot of the New cases of COVID-19 with Normal Innovations.

This is same for the conditional distribution parameter for the GARCH parameter which is stable throughout the iteration (Figure 3 and 4). This is the reason that the GARCH parameter accounted for a lower value compared to the ARCH parameter and the conditional distribution parameter for the student t distribution. The Normal innovation to the GARCH (1,1) model shows a flat curve for all parameters and did not fully capture the volatility of the COVID-19 total cases and new cases in Nigeria (Figure 5 and 6). Thus, the standard errors for the t-distribution had lower values compared to that of the normal distribution which means that based on this study, the student t distribution is a better fit compared to the Gaussian distribution as regard the conditional distribution for this study.

## 4. CONCLUSION

Modelling the COVID-19 dataset for daily total cases and daily new cases was achieved using two conditional distributions: Student t distribution and the Gaussian or Normal distribution. Compared to the Bayesian GARCH (1,1) model with student-t distribution, the Bayesian GARCH model (1,1) with normal distribution which had ARCH parameter with lower values and GARCH parameter higher the value but the conditional variance parameter was extremely high. The study implies that the Bayesian GARCH model (1,1) with normal distribution is not suitable for the data. Also, the standard error for Bayesian GARCH model (1,1) with normal distribution has a higher value compared with the student-t distribution. The results obtained favour the Bayesian GARCH (1,1) model with student-t distribution as the reaction parameter had higher volatility at lower iterations and was stable at higher iteration values. The same applicable for the conditional distribution parameter for the GARCH parameter which is stable

throughout the iteration. This is the reason that the GARCH parameter accounted for a lower value compared to the ARCH parameter and the conditional distribution parameter for the student t distribution. The Normal innovation to the GARCH (1,1) model shows a flat curve for all parameters and did not fully capture the volatility of the COVID 19 total cases and new cases in Nigeria.
. In conclusion, the Bayesian estimation with student $t$ innovation performed better than the one with the normal innovation with standard errors being less than that of the normal innovation as regard to the conditional distribution parameter. In addition, the plots for the student t distribution captured the dynamics of the volatility in the COVID-19 data for total cases and new cases in Nigeria than that of the normal distribution.

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