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# SOLUTION OF FISHERS EQUATION USING HOMOTOPY PERTURBATION ABOODH TRANSFORM METHOD

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Abstract. In this study, a nonlinear one dimensional reaction Diffusion Equation called Fisher's equation is solved using Aboodh transform coupled with Homotopy Perturbation Method (AHPM). This Method is applied to the general one dimensional Fishers equation. This technique is demonstrated with three unique examples with different initial conditions. The simplicity of the Aboodh transform contributes to its rapid convergence rate from approximate to exact solutions, rendering it both easy to use and reliable.

## 1. Introduction

Fisher's equation is a typical illustration of a PDE, which serves as one of the foundational model for understanding population dynamics and dispersion in a variety of biological and ecological contexts [26, 6, 18]. When examining how diffusion and local growth rates affect a population's long-term geographic distribution, this equation is crucial [19, 8].

$$\frac{\partial \phi}{\partial t} = \rho \nabla^2 \phi + k\phi(1-\phi) \tag{1.1}$$

The population density is represented by $\phi$ in this equation, time is represented by $t$ the diffusion coefficient by $\rho$, and the growth rate parameter by $k$. It exemplifies fundamental ideas in mathematical ecology and biology by showing how populations react to the interaction of geographic dispersion and local growth [9, 12].

Over time, numerous methods have been employed to solve and simplify partial differential equations, particularly those that are nonlinear. HAM [10], STDM [14], VIM [4, 5], HPM [5, 7, 17], LTDM [27], MHPM [22] HPMLT [15], AHPM [21], MAHPM [24] and LHPM [23] are a few examples. A powerful mathematical technique known as the Aboodh Homotopy Perturbation Method (AHPM) may be used to a wide range of complex differential equations in science and engineering [28, 16, 21, 1]. This methodology combines the homotopy perturbation

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method , a perturbation-based approach to approximating solutions, with the Aboodh transform, which effectively linearizes differential equations. By combining these two approaches, AHPM offers a flexible and organized framework for managing nonlinear problems, giving specialists and researchers a helpful tool to handle challenging issues. This method simplifies the analysis of complex population dynamics and reveals how factors

## 2. Materials and Methods

In this section, we consider a general nonlinear Klein-Gordon equation. We use Homotopy perturbation method, to decompose the nonlinear term, so that the solution can be obtained with iteration procedure.

2.1. **Aboodh Transform.** We employ a transform known as the Aboodh transform, developed to investigate functions in the set $A$, specifically for functions of exponential order. [1]

$$A = \{\phi(t) : \exists\, M,\ k_1, k_2 > 0\ |\phi(t)| < Me^{-ut}\} \tag{2.1}$$

Where $M < \infty\ \forall\ k_1, k_2\ \leq\ \infty$. The Aboodh transform is defined as an integral equation of the form [1].

$$\mathcal{A}\left[\phi\left(t\right)\right] = R\left(u\right) = \frac{1}{u}\int_0^\infty \phi\left(t\right)e^{-ut}dt\ \ t \geq 0\ k_1 \leq u \leq k_2 \tag{2.2}$$

Table 1. The Aboodh transform of some partial differential functions [1]

| $A[\phi(y,t)]$ | $\psi(y,u)$ |
|---|---|
| $\mathcal{A}\left[\frac{\partial\phi(x,t)}{\partial y}\right]$ | $R'\left(y,u\right)$ |
| $\mathcal{A}\left[\frac{\partial^2\phi(y,t)}{\partial y^2}\right]$ | $R''\left(y,u\right)$ |
| $\mathcal{A}\left[\frac{\partial^n\phi(y,t)}{\partial y^n}\right]$ | $R^{(n)}\left(y,u\right)$ |
| $\mathcal{A}\left[\frac{\partial\phi(y,t)}{\partial t}\right]$ | $vR\left(y,u\right) - \frac{\phi(y,0)}{u}$ |
| $\mathcal{A}\left[\frac{\partial^2\phi(y,t)}{\partial t^2}\right]$ | $= v^2R\left(y,u\right) - \frac{1}{u}\frac{\partial\phi(y,0)}{\partial t} - \phi(y,0)$ |

Table 2. Aboodh transform of Some Functions [1]

| $\phi(t)$ | $K_f\left\{\phi\left(t\right)\right\} = \psi(u)$ |
|---|---|
| $1$ | $\frac{1}{u^2}$ |
| $t$ | $\frac{1}{u^3}$ |
| $t^n$ | $\frac{n!}{u^{2n+1}}$ |
| $e^{at}$ | $\frac{1}{u^2-au}$ |
| $sin\left(at\right)$ | $\frac{a}{u(u^2+a^2)}$ |
| $\cos(at)$ | $\frac{1}{u^2+a^2}$ |

## 3. Aboodh Homotopy Perturbation Method (AHPM)

The general partial differential equation is of the form [25]:

$$L\phi(y,t) + R\phi(y,t) + N\phi(y,t) = f(y,t) \tag{3.1}$$

| | |
|---|---|
| $L$ | Linear Operator |
| $R$ | Coefficient of the Laplace operator |
| $N$ | Coefficient of Nonlinear Term |
| $f(y,t)$ | Source term |

Taking the Aboodh transform of both sides of equation (3.1),

$$\mathcal{A}\{L\phi(y,t)\} = \mathcal{A}\{f(y,t)\} - \mathcal{A}\{R\phi(y,t) + N\phi(y,t)\}, \tag{3.2}$$

$$u\mathcal{A}\{\phi(y,t)\} - \frac{\phi(y,0)}{u} = \mathcal{A}\{f(y,t)\} - \mathcal{A}\{R\phi(y,t) + N\phi(y,t)\}, \tag{3.3}$$

$$u\mathcal{A}\{\phi(y,t)\} = \frac{\phi(y,0)}{u} + \mathcal{A}\{f(y,t)\} - \mathcal{A}\{R\phi(y,t) + N\phi(y,t)\} \tag{3.4}$$

Multiply equation (3.4) by $\frac{1}{u}$

$$\mathcal{A}\{\phi(y,t)\} = \frac{1}{u}\left[\frac{\phi(y,0)}{u} + \mathcal{A}\{f(y,t)\}\right] - \frac{1}{u}\mathcal{A}\{R\phi(y,t) + N\phi(y,t) \tag{3.5}$$

Taking the Aboodh inverse transform of (3.5), we obtain

$$\mathcal{A}^{-1}\{\mathcal{A}\{\phi(y,t)\} = \mathcal{A}^{-1}\left\{\frac{1}{u}\left[\frac{\phi(y,0)}{u} + \mathcal{A}\{f(y,t)\}\right]\right\} - \mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\{R\phi(y,t) + N\phi(y,t)\}\right\} \tag{3.6}$$

Let $G(y,t) = \mathcal{A}^{-1}\left\{\frac{1}{u}\left[\frac{\phi(y,0)}{u} + \mathcal{A}\{f(y,t)\}\right]\right\}$. Then, we have

$$\phi(y,t) = G(y,t) - \mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\{R\phi(y,t) + N\phi(y,t)\}\right\} \tag{3.7}$$

Since the approximation is independent of$t$, so all it derivatives with respect to $t$ are zero. Then, we obtain

$$\phi_{r+1}(y,t) = G(y,t)) - \mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\{R\phi_m(y,t) + N\phi_m(y,t)\}\right\} \tag{3.8}$$

Applying Homotopy Perturbation Method to equation (3.8). Assume equation (3.8) can also be expressed as

$$\begin{aligned} \phi_m(y,t) &= \phi_0(y,t) + p\phi_1(y,t) + p^2\phi_2(y,t) + p^3\phi(y,t) + \dots \\ \phi(y,t) &= \sum_{r=0}^{\infty} p^m\phi_m(y,t) \end{aligned} \tag{3.9}$$

Taking into consideration the nonlinear term, it can be simplified as

$$N\phi(y,t) = \sum_{r=0}^{\infty} p^m H_m(\phi) \tag{3.10}$$

Where He is the polynomial $\phi$ that denotes the nonlinear terms, and$H_m(\phi)$. The initial terms are given by

$$\begin{aligned} H_0(\phi) &= \phi_0\phi_0 \\ H_1(\phi) &= \phi_0\phi_1 + \phi_1\phi_0 \\ H_2(\phi) &= \phi_0\phi_2 + \phi_1\phi_1 + \phi_2\phi_0 \\ H_3(\phi) &= \phi_0\phi_3 + \phi_1\phi_2 + \phi_2\phi_1 + \phi_3\phi_3 \end{aligned} \tag{3.11}$$

Then, by substituting (3.9) and (3.10) into (3.8), we have

$$\sum_{m=0}^{\infty} p^m \phi_{r+1}(y,t) = G(y,t) + p\left(\mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\left(\sum_{m=0}^{\infty} Rp^m\phi_{ry} + \sum_{m=0}^{\infty} H_m(\phi)\right)\right\}\right) \tag{3.12}$$

$$\begin{aligned} \phi_0 + p^1\phi_1 + p^2\phi_2 + \cdots = G(y,t) - p\mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\left(\sum_{m=0}^{\infty} R\phi_{0y} + \sum_{r=0}^{\infty} H_0(\phi)\right)\right.\right. \\ \left.\left. + p\left(\sum_{m=0}^{\infty} R\phi_{1y} + \sum_{m=0}^{\infty} H_1(\phi)\right) + p^2\left(\sum_{m=0}^{\infty} R\phi_{2y} + \sum_{m=0}^{\infty} H_2(\phi)\right) + \ldots\right\}\right] \end{aligned} \tag{3.13}$$

The desired approximation is obtained by equating the corresponding exponents of $p$ in equation (3.13) such that $\phi_0(y,t)$ is defined as $\phi_0(y,t) = \alpha + t\beta$, where $\alpha$ and $\beta$ denotes the initial conditions.

$$p^0 : \phi_0(y,t) = G(y,t) = \mathcal{A}^{-1}\left\{\frac{1}{u}\left[\frac{\phi(y,0)}{u} + \mathcal{A}\left\{f(y,t)\right\}\right]\right\} \tag{3.14}$$

$$p^1 : \phi_1(y,t) = \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\left(\sum_{r=0}^{\infty} R\phi_{0y} + \sum_{r=0}^{\infty} H_0(\phi)\right)\right\}\right] \tag{3.15}$$

$$p^2 : \phi_2(y,t) = \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\left(\sum_{m=0}^{\infty} R\phi_{1y} + \sum_{m=0}^{\infty} H_1(\phi)\right)\right\}\right] \tag{3.16}$$

$$p^3 : \phi_3(y,t) = \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\left(\sum_{m=0}^{\infty} R\phi_{2y} + \sum_{m=0}^{\infty} H_2(\phi)\right)\right\}\right] \tag{3.17}$$

## 4. Main result

Here, nonlinear one dimensional Fishers Equation will be solved using AHPM

**Example 4.1.** Considering the one dimensional Fishers type equation of the form [2]

$$\phi_t - \phi_{yy} - 1\left(1-\phi\right)\phi = 0 \tag{4.1}$$

With

$$\phi\left(y,0\right) = \mu \tag{4.2}$$

**Solution**

We can rewrite equation (4.1) as

$$\frac{\partial \phi}{\partial t} - \frac{\partial^2 \phi}{\partial y^2} - 1\,(1-\phi)\,\phi = 0 \tag{4.3}$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} + \phi - \phi^2 \tag{4.4}$$

Taking the Aboodh transform of equation (4.4),

$$\mathcal{A}\left\{\frac{\partial \phi}{\partial t}\right\} = \mathcal{A}\left\{\frac{\partial^2 \phi}{\partial y^2} + \phi - \phi^2\right\} \tag{4.5}$$

$$u\mathcal{A}\{\phi(y,t)\} - \frac{\phi\,(y,0)}{u} = \mathcal{A}\left\{\frac{\partial^2 \phi}{\partial y^2} + \phi - \phi^2\right\} \tag{4.6}$$

$$u\mathcal{A}\left\{\phi(y,t)\right\} = \frac{\phi\,(y,0)}{u} + \mathcal{A}\left\{\frac{\partial^2 \phi}{\partial y^2} + \phi - \phi^2\right\} \tag{4.7}$$

Multiply equation (4.7) by $\frac{1}{u^2}$

$$\mathcal{A}\{\phi(y,t)\} = \frac{1}{u}\left[\frac{\phi\,(y,0)}{u}\right] + \frac{1}{u}\mathcal{A}\left\{\frac{\partial^2 \phi}{\partial y^2} + \phi - \phi^2\right\} \tag{4.8}$$

Taking the inverse Aboodh transform of equation (4.8), we obtain

$$\mathcal{A}^{-1}\{\mathcal{A}\left\{\phi(y,t)\right\} = \mathcal{A}^{-1}\left\{\frac{1}{u}\left[\frac{\phi\,(y,0)}{u}\right]\right\} + \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\frac{\partial^2 \phi}{\partial y^2} + \phi - \phi^2\right\}\right] \tag{4.9}$$

Since $\phi(y,0) = \mu$. Then, we have

$$\phi(y,t) = \mathcal{A}^{-1}\left\{\frac{\mu}{u}\right\} - \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\frac{\partial^2 \phi}{\partial y^2} + \phi - \phi^2\right\}\right] \tag{4.10}$$

Since the approximation is independent of$t$, so all it derivatives with respect to $t$ are zero. Then, we obtain

$$\phi_{r+1}(y,t) = \mu t - \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\frac{\partial^2 \phi}{\partial y^2} + \phi - \phi^2\right\}\right] \tag{4.11}$$

Applying Homotopy Perturbation Method to equation (4.11). Assume equation can be written as

$$\phi(y,t) = \phi_0(y,t) + p\phi_1(y,t) + p^2\phi_2(y,t) + p^3\phi(y,t) + \ldots \tag{4.12}$$

$$\phi(y,t) = \sum_{m=0}^{\infty} p^m \phi_m(y,t) \tag{4.13}$$

Taking into consideration the nonlinear term, it can be simplified as

$$N\phi(y,t) = \sum_{m=0}^{\infty} p^m H_m(\phi) \tag{4.14}$$

Where He is the polynomial $\phi$ that denotes the nonlinear terms, and $H_m(\phi)$. The initial terms are given by

$$\begin{aligned} H_0(\phi) &= \phi_0\phi_0 \\ H_1(\phi) &= \phi_0\phi_1 + \phi_1\phi_0 \\ H_2(\phi) &= \phi_0\phi_2 + \phi_1\phi_1 + \phi_2\phi_0 \\ H_3(\phi) &= \phi_0\phi_3 + \phi_1\phi_2 + \phi_2\phi_1 + \phi_3\phi_3 \end{aligned}$$

Then, by substituting (4.13) and (4.14) into (4.11) we have

$$\sum_{m=0}^{\infty} p^m \phi_{m+1}(y,t) = \mu t + p\left(\mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\sum_{m=0}^{\infty}\frac{\partial^2\phi_m}{\partial y^2} + \sum_{m=0}^{\infty}\phi_m - \sum_{m=0}^{\infty}H_m(\phi)\right\}\right]\right) \tag{4.15}$$

$$\begin{aligned} \phi_0 + p^1\phi_1 + p^2\phi_2 + \cdots = \mu - p^0\mathcal{A}^{-1}\Bigg[\frac{1}{u}\mathcal{A}\Bigg\{&\left(\sum_{m=0}^{\infty}\frac{\partial^2\phi_0}{\partial y^2} + \sum_{m=0}^{\infty}\phi_0 - \sum_{m=0}^{\infty}H_0^2(\phi)\right) \\ &+p^1\left(\sum_{m=0}^{\infty}\frac{\partial^2\phi_1}{\partial y^2} + \sum_{m=0}^{\infty}\phi_1 - \sum_{m=0}^{\infty}H_1^2(\phi)\right) \\ &+p^2\left(\sum_{m=0}^{\infty}\frac{\partial^2\phi_2}{\partial y^2} + \sum_{m=0}^{\infty}\phi_2 - \sum_{m=0}^{\infty}H_2^2(\phi)\right) + \ldots\Bigg\}\Bigg] \end{aligned} \tag{4.16}$$

The desired approximation is obtained by equating the corresponding exponents of $p$ in equation (4.16) such that $\phi_0(y,t)$ is defined as $\phi_0(y,t) = \alpha + t\beta$, where $\alpha$ and $\beta$ denotes the initial conditions.

$$p^0 : \phi_0(y,t) = \mu \tag{4.17}$$

$$\begin{aligned} p^1 : \phi_1(y,t) &= \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\left(\frac{\partial^2\phi_0}{\partial y^2} + \phi_0 - H0^2(\phi)\right)\right\}\right] \\ &= \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{(\mu - \mu^2)\right\}\right] \\ &= \mathcal{A}^{-1}\left[\frac{1}{u}.\left[\frac{\mu}{u^2} - \frac{\mu^2}{u^2}\right]\right] \\ &= \mathcal{A}^{-1}\left[\frac{\mu}{u^3} - \frac{\mu^2}{u^3}\right] \\ &= (\mu - \mu^2)\, t \end{aligned} \tag{4.18}$$

$$\begin{aligned} p^2 : \phi_2(y,t) &= \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\left(\frac{\partial^2\phi_1}{\partial y^2} + \phi_1 - H_1^2(\phi)\right)\right\}\right] \\ &= \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{(\mu t - \mu^2 t - \mu(\mu t - \mu^2 t) - \mu(\mu t - \mu^2 t)\right\}\right] \\ &= \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{(\mu t - \mu^2 t - \mu^2 t - \mu^2 t)\right\}\right] \end{aligned} \tag{4.19}$$

$$= (\mu - 3\mu^2 + 2\mu^3)\frac{t^2}{2!} \tag{4.20}$$

The approximate solution to equation (4.1) is given as

$$\sum_{r=0}^{\infty}\phi(y,t) = \mu + \left(\mu - \mu^2\right)t + \left(\mu - 3\mu^2 + 2\mu^3\right)\frac{t^2}{2!} + \ldots \tag{4.21}$$

The exact solution of equation (4.21) is written as

$$\phi(y,t) = \frac{\mu e^t}{1-\mu+\mu e^t} \tag{4.22}$$

Equation (4.34) is the same as the solution obtained in [2]

**Example 4.2.** Let solve a nonlinear Fisher's equation of the form [25]:

$$\frac{\partial \phi(y,t)}{\partial t} = \frac{\partial^2 \phi(y,t)}{\partial y^2} + 6\left(1-\phi(y,t)\right)\phi(y,t) \tag{4.23}$$

With

$$\phi(y,0) = e^y \tag{4.24}$$

**Solution** Taking the Aboodh transform of equation of both sides of (4.23)

$$u\mathcal{A}\left\{\phi(y,t)\right\} - \frac{\phi\left(y,0\right)}{u} = \mathcal{A}\left\{\frac{\partial^2 \phi(y,t)}{\partial y^2} + 6\left(1-\phi\right)\phi\right\} \tag{4.25}$$

$$u\mathcal{A}\left\{\phi(y,t)\right\} = \frac{\phi\left(y,0\right)}{u} + \mathcal{A}\left\{\frac{\partial^2 \phi(y,t)}{\partial y^2} + 6\left(1-\phi\right)\phi\right\} \tag{4.26}$$

Multiplying equation (4.26) by $\frac{1}{u}$. Then we have,

$$\mathcal{A}\left\{\phi(y,t)\right\} = \frac{\phi\left(y,0\right)}{u^2} + \frac{1}{u}\mathcal{A}\left\{\frac{\partial^2 \phi(y,t)}{\partial y^2} + 6\left(1-\phi\right)\phi\right\} \tag{4.27}$$

Taking the Aboodh inverse transform of (4.27)

$$\mathcal{A}\left\{\phi(y,t)\right\} = \mathcal{A}^{-1}\left[\frac{e^y}{u^2}\right] + \mathcal{A}^{-1}\left[\frac{1}{u}\mathcal{A}\left\{\frac{\partial^2 \phi(y,t)}{\partial y^2} + 6\left(1-\phi\right)\phi\right\}\right] \tag{4.28}$$

Applying HPM technique to equation (4.28). Thus, we get

$$\sum_{m=0}^{\infty}\phi_{r+1}(y,t) = e^y + p\left(\mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}^{-1}\left[\frac{\partial^2}{\partial y^2}\sum_{m=0}^{\infty}p^m\phi_m(y,t) + 6\sum_{m=0}^{\infty}p^m\phi_m(y,t) - 6\sum_{m=0}^{\infty}p^m\phi_m^2\right]\right\}\right) \tag{4.29}$$

Applying He's polynomial to simplify the nonlinear terms,

$$\sum_{m=0}^{\infty}p^m\phi_m^2(\phi) = \sum_{m=0}^{\infty}p^m H_m(\phi) \tag{4.30}$$

$$\sum_{m=0}^{\infty}\phi_m(y,t) = \phi_0(y,t) + p\phi_1(y,t) + p^2\phi_2(y,t) + \ldots \tag{4.31}$$

Substituting (4.30) and (4.31) into equation (4.29), we have

$$\sum_{m=0}^{\infty}\phi_{m+1}(y,t) = e^y + p\left(\mathcal{A}^{-1}\left\{u^2\mathcal{A}\left[\frac{\partial^2}{\partial y^2}\sum_{m=0}^{\infty}p^m\phi_m(y,t) + 6\sum_{m=0}^{\infty}p^m\phi_m(y,t) - 6\sum_{m=0}^{\infty}p^m H_m(\phi)\right]\right\}\right) \tag{4.32}$$

Using HPM, the series becomes

$$\sum_{m=0}^{\infty}\phi_{m+1}(y,t)=e^{y}+p\left(\mathcal{A}^{-1}\left\{u^{2}\mathcal{A}\left[\left(\frac{\partial^{2}}{\partial y^{2}}\sum_{m=0}^{\infty}p^{m}\phi_{m}(y,t)+6\sum_{r=0}^{\infty}p^{m}\phi_{m}(y,t)-6\sum_{r=0}^{\infty}H_{m}(\phi)\right)\right.\right.$$
$$\left.\left.+p\left(\frac{\partial^{2}}{\partial y^{2}}\sum_{m=0}^{\infty}p^{m}\phi_{m}(y,t)+6\sum_{m=0}^{\infty}p^{m}\phi_{m}(y,t)-6\sum_{m=0}^{\infty}H_{m}(\phi)\right)+\ldots\right]\right\}\right) \quad (4.33)$$

Comparing the respective powers of $p$ of equation (4.33), the following approximations are obtained.

$$\phi_{0}(y,t)=\phi\left(y,0\right)=e^{y} \quad (4.34)$$
$$H_{0}(\phi)=\phi_{0}\phi_{1} \quad (4.35)$$
$$\phi_{1}(y,t)=\mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\left(\frac{\partial^{2}\phi_{0}}{\partial y^{2}}+6\phi_{0}-6H_{0}(\phi)\right)\right\} \quad (4.36)$$
$$=\mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\left(e^{y}+6e^{y}-e^{2y}\right)\right\}$$
$$=\mathcal{A}^{-1}\left\{\frac{1}{u}\left(e^{y}+6e^{y}-6ve^{2y}\right)\right\}$$
$$=\mathcal{A}^{-1}\left\{\frac{1}{u}\left(7e^{y}-6e^{2y}\right)\right\}$$
$$=7te^{y}-6te^{2y}$$
$$=\left(7-6e^{y}\right)te^{y} \quad (4.37)$$

The approximate solution to equation (4.23) is given as

$$\sum_{r=0}^{\infty}\phi(y,t)=e^{y}+\left(7+6e^{y}\right)te^{y}+\ldots \quad (4.38)$$

Equation (4.38) is the same as the solution obtained in [25]

**Example 4.3.** Considering the one dimensional Fisher's equation of the form [25]

$$\frac{\partial\phi}{\partial t}(y,t)-\frac{\partial^{2}\phi}{\partial y^{2}}-6\phi(y,t)\left[1-\phi(y,t)\right]=0 \quad (4.39)$$

With the initial condition

$$\phi(y,0)=\frac{1}{\left(1+e^{x}\right)^{2}} \quad (4.40)$$

**Solution**

Equation (4.39) can also be written as

$$\frac{\partial\phi}{\partial t}(y,t)=\frac{\partial^{2}\phi}{\partial y^{2}}+6\phi(y,t)[1-\phi(y,t)] \quad (4.41)$$

Taking the Aboodh transform of equation (4.41), we get

$$v\mathcal{A}\left\{\phi(y,t)\right\}-\frac{\phi\left(y,0\right)}{u}=\mathcal{A}\left\{\frac{\partial^{2}\phi}{\partial y^{2}}+6\phi(y,t)[1-\phi(y,t)]\right\} \quad (4.42)$$

Multiplying equation (4.42) by $\frac{1}{u}$. Then, the equation becomes

$$u\mathcal{A}\left\{\phi(y,t)\right\} = \frac{\phi\left(y,0\right)}{u^2} + \frac{1}{u}\mathcal{A}\left\{\frac{\partial^2\phi}{\partial y^2} + 6\phi(y,t)[1-\phi(y,t)]\right\} \tag{4.43}$$

Taking the Aboodh inverse transform of both sides of equation (4.43). Thus,

$$\phi(y,t) = \mathcal{A}\left\{\frac{\phi\left(y,0\right)}{u^2}\right\} + \mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\left[\frac{\partial^2\phi}{\partial y^2} + 6\phi(y,t)[1-\phi(y,t)]\right]\right\} \tag{4.44}$$

Applying the HPM technique to assume the series of solution of the function $\phi(y,t)$. Then, we obtain

$$\phi(y,t) = \sum_{m=0}^{\infty}\phi_m(y,t) \tag{4.45}$$

Using equation (4.45) in equation (4.44), we get

$$\sum_{m=0}^{\infty}\phi_{m+1}(y,t) = \frac{1}{(1+e^y)^2} + p\left(\mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\left[\frac{\partial^2}{\partial y^2}\sum_{m=0}^{\infty}p^m\phi_m(y,t) + 6\sum_{m=0}^{\infty}p^m\phi_m(y,t) - 6\sum_{m=0}^{\infty}p^m\phi_m^2\right]\right\}\right) \tag{4.46}$$

Applying He's polynomial to simplify the nonlinear terms in equation (4.46),

$$\sum_{m=0}^{\infty}p^m\phi_m^2 = \sum_{m=0}^{\infty}p^m H_m^2(\phi) \tag{4.47}$$

$$\sum_{m=0}^{\infty}\phi_m(y,t) = \phi_0(y,t) + p\phi_1(y,t) + p^2\phi_2(y,t) + \ldots \tag{4.48}$$

Substituting (4.47) and (4.48) into equation (4.46), we have

$$\sum_{m=0}^{\infty}\phi_{m+1}(y,t) = \frac{1}{(1+e^y)^2} + p\left(\mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\left[\frac{\partial^2}{\partial y^2}\sum_{m=0}^{\infty}p^m\phi_m(y,t) + 6\sum_{m=0}^{\infty}p^m\phi_m(y,t) - 6\sum_{m=0}^{\infty}p^m H_m(\phi)\right]\right\}\right) \tag{4.49}$$

$$\begin{aligned}\sum_{m=0}^{\infty}\phi_{m+1}(y,t) = \frac{1}{(1+e^y)^2} + p\Bigg(\mathcal{A}^{-1}\Bigg\{\frac{1}{u}\mathcal{A}\Bigg[&\left(\frac{\partial^2}{\partial y^2}\sum_{m=0}^{\infty}\phi_0(y,t) + 6\sum_{m=0}^{\infty}\phi_0(y,t) - 6\sum_{m=0}^{\infty}H_m(\phi)\right)\\ &+p\left(\frac{\partial^2}{\partial y^2}\sum_{m=0}^{\infty}\phi_1(y,t) + 6\sum_{m=0}^{\infty}\phi_1(y,t) - 6\sum_{m=0}^{\infty}H_m(\phi)\right) + \ldots\Bigg]\Bigg\}\Bigg)\end{aligned} \tag{4.50}$$

Using HPM, the series becomes
Comparing the respective powers of $p$ of equation (4.50), the following approximations are obtained.

$$\phi_0(y,t) = \phi \tag{4.51}$$

$$(y,0) = \frac{1}{(1+e^y)^2}$$

$$H_0(\phi) = \phi_0\phi_1$$

$$\phi_1(y,t) = \mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\left(\frac{\partial^2\phi_0}{\partial y^2} + 6\phi_0 - 6H_0(\phi)\right)\right\} \tag{4.52}$$

$$= \frac{10e^y t}{(1+e^y)^3}$$

$$\phi_2(y,t) = \mathcal{A}^{-1}\left\{\frac{1}{u}\mathcal{A}\left(\frac{\partial^2\phi}{\partial y^2} + 6\phi_1 - 6H_1(\phi)\right)\right\}$$

$$= \frac{25e^y\left(-1+2e^y\right)t^2}{(1+e^y)^4} \tag{4.53}$$

The approximate solution to equation (4.39) is given as

$$\sum_{m=0}^{\infty}\phi_{m+1}(y,t) = \frac{1}{(1+e^y)^2} + \frac{10e^y t}{(1+e^y)^3} + \frac{25e^y\left(-1+2e^y\right)t^2}{(1+e^y)^4} + \ldots \tag{4.54}$$

Equation (4.54) is the same as the exact solution obtained in [25]

## 5. Conclusion

In this work, one dimensional Fisher's problem was solved by using the combination of Aboodh transform and homotopy perturbation method (HPM). The outcomes demonstrate the effectiveness and potency of the AHPM approach in locating both precise and approximate solutions to nonlinear differential equations. When compared to previous methods, AHPM provides very accurate numerical solutions for nonlinear problems because it yields an efficient recurring relation for solving Fishers Type equations. According to this approach, the answer is found by adding an infinite series that quickly converges to the precise answers.

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