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CONNECTIONS OF GREEN'S RELATIONS OF A Γ-SEMIGROUP WITH OPERATOR SEMIGROUPS

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ABSTRACT. The theory of Γ-semigroups is an extension of the semigroup theory. In this paper, we examine the left operator semigroup L and the right operator semigroup R via modified definition of Γ-semigroup and deduce some results of operator semigroups acting on a Γ-semigroup. Further, we study some relationships between Green's equivalence relations of a Γ-semigroup and its left (right) operator semigroup.

1. INTRODUCTION

Nobusawa's groundbreaking work [15] introduced the innovative concept of the Γ-ring, representing a significant extension of the classical ring theory. This seminal idea has catalysed a surge of research activity among numerous scholars, resulting in a substantial body of new findings, as evidenced by the notable contributions cited in [2, 3, 13, 14]. Inspired by the profound implications of this ring generalisation, a parallel exploration unfolded within the domain of semigroup theory. Sen [18] spearheaded this endeavour by pioneering the theory of Γ-semigroups. Subsequently, Sen collaborated with Saha [19], undertook a refinement of the defining conditions, slightly adjusting them to redefine Γ-semigroups and further advanced the theoretical framework.

The evolution of Γ-semigroups hinges on the fact that subsets of a semigroup inherently exhibit associativity but not necessarily closed. This fundamental insight has paved way for numerous generalisations stemming from the modified definition as seen in [10, 17, 20, 21, 22]. Building upon on this concept, Dutta and Chattopadhyay [8] elucidated the concept of operator semigroups within the framework of Γ-semigroups. The authors delved into the intricate relationships between the set of Γ-ideals and operator semigroups, echoing the groundwork laid by Dutta and Adhikari [9]. Additionally, Heidari and Amooshashi [12] linked transformation semigroups to Γ-semigroups, exploring interconnectedness with Γ-ideals and Green's relations based on the idea that a transformation semigroup τ is a function from τ to itself (see [1]).

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The foundational work of Green [11] in establishing Green's equivalence relations for semigroups has been instrumental in unravelling the structural intricacies of semigroups (for further elaboration, see [6]). Researchers have extended this line of inquiry to characterise Green's relations within the framework of Γ-semigroups, as evidenced by the contributions in [4, 5, 7, 16]. This body of research, coupled with the insights gleaned from [12], serves as the impetus behind our paper. Within this context, we present, among other findings, an elucidation of the relationships between the Green's equivalence relations of a Γ-semigroup and operator semigroups.

2. PRELIMINARIES

In this section, we revisit some fundamental definitions and results pertinent to our discourse.

Definition 2.1. Let S and Γ be two non-empty sets. Then S is called a Γ-semigroup if there exist mappings $S \times \Gamma \times S \longrightarrow S \mid (a, \alpha, b) \longrightarrow a\alpha b \in S$ and $\Gamma \times S \times \Gamma \longrightarrow \Gamma \mid (\alpha, a, \beta) \longrightarrow \alpha a\beta \in \Gamma$ satisfying the identities $a\alpha(b\beta c) = a(\alpha b\beta)c = (a\alpha b)\beta c$ for all $a, b, c \in S$ and $\alpha, \beta \in \Gamma$.

The modified definition of Γ-semigroup by Sen and Saha [19] may be regarded as one-sided Γ-semigroup.

Definition 2.2. Let $S = \{a, b, c...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two non-empty sets. Then S is called a Γ-semigroup if there exists a mapping $S \times \Gamma \times S \longrightarrow S \mid (a, \alpha, b) \longrightarrow a\alpha b \in S$ satisfying the property $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in S$ and $\alpha, \beta \in \Gamma$.

Example 2.3. Let S be a semigroup and Γ be any any non-empty set. Define a mapping $S \times \Gamma \times S \longrightarrow S$ by $a\gamma b = ab$ for all $a, b \in S$ and $\gamma \in \Gamma$. Then S is a Γ-semigroup.

Example 2.4. Let S be a set of all negative rational numbers. Obviously S is not a semigroup under usual product of rational numbers. Let $\Gamma = \{-\frac{1}{p} \mid p \text{ is prime}\}$. Let $a, b, c \in S$ and $\alpha, \beta \in \Gamma$. Now if $a\alpha b$ is equal to the usual product of rational numbers a, α, b; then $a\alpha b \in S$ and $(a\alpha b)\beta c = a\alpha(b\beta c)$. Hence S is a Γ-semigroup.

Example 2.5. Let $S = \{-i, i, 0\}$ and $\Gamma = S$. Then S is a Γ-semigroup with respect to multiplication of complex numbers whereas S does not reduce to semigroup with respect to multiplication of complex numbers.

The examples highlighted above illustrate that every semigroup inherently falls within the category of Γ-semigroups. Hence, Γ-semigroups are generalization of semigroups.

The following definitions except Definitions 2.7 and 2.8 can be found in [19].

Definition 2.6. Let S be a Γ-semigroup. A non-empty subset A of S is said to be a Γ-subsemigroup of S if $A\Gamma A \subseteq A$, that is, if $a\alpha b \in A$ for every $a, b \in A$ and every $\alpha \in \Gamma$.

Definition 2.7. A left ideal of a Γ-semigroup S is a non-empty subset A of S such that $S\Gamma A \subseteq A$, that is, if $s \in S$, $\alpha \in \Gamma$ and $a \in A$, implies $s\alpha a \in A$. It is a right idea of S if $A\Gamma S \subseteq A$, that is, if $a \in A$, $\alpha \in \Gamma$ and $s \in S$, implies $a\alpha s \in A$.

Definition 2.8. A Γ-semigroup S is called commutative if $a\alpha b = b\alpha a$ for every $a, b \in S$ and $\alpha \in \Gamma$.

Definition 2.9. Let S be a Γ-semigroup, $a \in S$ and $\alpha \in \Gamma$. An element $a \in S$ is said to be α-idempotent if $a\alpha a = a$. The set of of all α-idempotents is denoted by $E_\alpha(S)$. Also, a is said to be (α, β)-regular if there exists $b \in S$ such that $a = a\alpha b\beta a$. We denote the set of all (α, β)-regular elements of S by $Reg_{\alpha,\beta}(S)$.

Definition 2.10. [8] Let S be a Γ-semigroup. We define a relation ρ on $S \times \Gamma$ as follows:

$$(x, \alpha)\rho(y, \beta) \Longleftrightarrow x\alpha s = y\beta s, \ \forall \ s \in S.$$

Obviously ρ is an equivalence relation. Let $[x, \alpha]$ denote the equivalence class containing (x, α). Let $L = S \times \Gamma/\rho = \{[x, \alpha] : x \in S, \ \alpha \in \Gamma\}$. Then L is a semigroup with respect to multiplication defined by $[x, \alpha][y, \beta] = [x\alpha y, \beta]$. The semigroup L is called the left operator semigroup of S. Similarly, the right operator semigroup R of a Γ-semigroup S is defined as $R = \Gamma \times S/\rho = \{[\alpha, x] : \alpha \in \Gamma, \ x \in S\}$, where $[\alpha, x][\beta, y] = [\alpha, x\beta y]$, for all $x, y \in S$ and $\alpha, \beta \in \Gamma$.

Definition 2.11. [4] Let S be a Γ-semigroup and $a, b \in S$. Then Green's equivalence relations $\mathcal{L}$ and $\mathcal{R}$ on a Γ-semigroup S are defined by the following rules:

(*i*) $(a, b) \in \mathcal{L}$ if and only if $\langle a\rangle_l = \langle b\rangle_l$;
(*ii*) $(a, b) \in \mathcal{R}$ if and only if $\langle a\rangle_r = \langle b\rangle_r$.

Green's relations of a Γ-semigroup are defined similarly. Let $a \in S$. Then

$$\langle a\rangle_l = S\Gamma a \cup \{a\} \text{ and } \langle a\rangle_r = a\Gamma S \cup \{a\}.$$

Thus, $(a, b) \in \mathcal{L}$ if and only if $a = b$ or there exist $x, y \in S$ and $\alpha, \beta \in \Gamma$ such that $a = x\alpha b$ and $b = y\beta a$. Similarly, $(a, b) \in \mathcal{R}$ if and only if $a = b$ or there exist $x, y \in S$ and $\alpha, \beta \in \Gamma$ such that $a = b\alpha x$ and $b = a\beta y$.

Theorem 2.12. [4] *Let S be a Γ-semigroup, $\alpha \in \Gamma$ and e be an α-idempotent. Then*

(i) *$a\alpha e = a$ for all $a \in L_e$.*
(ii) *$e\alpha a = a$ for all $a \in R_e$.*

3. Main results

Throughout S stands for one-sided Γ-semigroup unless otherwise mentioned.

Definition 3.1. Let S be a Γ-semigroup and its left and right operator semigroup are respectively L and R. Then for every $a \in S$ and $\alpha \in \Gamma$, we define $L \times S \longrightarrow S$ and $S \times R \longrightarrow S$ respectively as follow:

$$[a, \alpha]s := a\alpha s \text{ and } s[\alpha, a] := s\alpha a.$$

Remark 3.2. If S is a commutative Γ-semigroup, then $L \times S = S \times R$.

Example 3.3. Using Example 2.3, it is evident that S is commutative and we obtain the action of the operator semigroups on S as follow:
The equivalence class $[x, \alpha] = \{(y, \beta) \in \rho \mid (y, \beta)\rho(x, \alpha)\}$, where ρ is an equivalence relation on $S \times \Gamma$. Let $L = \{[-i, -i], [i, i], [-i, i], [i, -i]\}$. Then it is not difficult to see that $L \times S = S \times R$ for every $s \in S$ and $\alpha \in \Gamma$.

Proposition 3.4. *If $A \subseteq S$, then $L \times A$ and $A \times R$ are Γ-subsemigroups of $L \times S$ and $S \times R$ respectively.*

Proof. Suppose that $[a, \alpha], [b, \beta] \in L$ such that $a, b \in S$ and $\alpha, \beta \in \Gamma$. Then, for every $t \in A$, we have

$$\begin{aligned} [a, \alpha][b, \beta]t &= [a, \alpha](b\beta t) \\ &= a\alpha(b\beta t) \\ &= (a\alpha b)\beta t \\ &= [a\alpha b, \beta]t \end{aligned}$$

Thus, $[a\alpha b, \beta]t \in L \times A$. So, $L \times A$ is closed under multiplication. Therefore, $L \times A$ is a Γ-subsemigroup of $L \times S$. Similarly, we can prove that $A \times R$ is a Γ-subsemigroup of $R \times S$. □

Proposition 3.5. *Let A and B be two subsets of $L \times S$. Then*

(*i*) $(A \cap B) \times S \subseteq (A \times S) \cap (B \times S)$,
(*ii*) $(A \cup B) \times S = (A \times S) \cup (B \times S)$.

Proof. The proofs of (i) and (ii) are straightforward. □

Proposition 3.6. *There exist a surjective $f : S \times \Gamma \longrightarrow L$.*

Proof. Let S be Γ-semigroup and L the left operator semigroup of S. Define a mapping $f : S \times \Gamma \longrightarrow L$ by $f((a, \alpha)) = [a, \alpha]$.

Now, we prove that the mapping f is well-defined. If $(a, \alpha) = (b, \beta)$ for some $a, b \in S$ and $\alpha, \beta \in \Gamma$, then for every $s \in S$ we have $a\alpha s = [a, \alpha]s = [b, \beta]s = b\beta s$ and so $[a, \alpha] = [b, \beta]$. By the definition it is clear that f is surjective, since for any $[a, \alpha] \in L$, there exists $(a, \alpha) \in S \times \Gamma$ such that $f((a, \alpha)) = [a, \alpha]$. □

Proposition 3.7. *If A is an ideal of S, then $L \times A$ is an ideal of $L \times S$.*

Proof. Suppose that A is an ideal of S and $[a, \alpha]t \in L \times A$. Then for every $s, t \in S$ and $\beta \in \Gamma$, we have $a\alpha(s\beta t) \in A$. Thus, $[a, \alpha][s, \beta]t = [a\alpha s, \beta]t \in L \times A$. Similarly, we can prove that $[s, \beta][a, \alpha]t = [s\beta a, \alpha]t \in L \times A$. Hence, $L \times A$ is an ideal of $L \times S$. □

Theorem 3.8. *There exists an inclusion preserving bijection between the set of all right ideals of S and the set all right ideals of $L \times S$.*

Proof. Suppose that $\mathcal{I}(S)$ and $\mathcal{I}(L \times S)$ sre the sets of all right ideals of S and $L \times S$ respectively. Clearly, the mapping $f : \mathcal{I}(S) \longrightarrow \mathcal{I}(L \times S)$ by $f(I) = L \times I$ is well-defined. Since I is a right ideal of S, $I\Gamma S \subseteq I$. Thus, $I \subseteq L \times I$. On the other hand, since S has a left unity, $L \times I \subseteq (L \times I)\Gamma S \subseteq I$. Thus, $L \times I = I$. Hence, f is bijective. It remains to show that f is an inclusion preserving mapping.

Let I and J be two right ideals of S such that $I \subseteq J$. We have to show that $L \times I \subseteq L \times J$. Let $[a, \alpha] \in L$. Then for every $s \in S$ we have $s \in I$ and so $[a, \alpha]s \in L \times I$. Since $I \subseteq J$, we have $s \in J$. Hence, $[a, \alpha]s \in L \times J$. Therefore, $L \times I \subseteq L \times J$. Hence the proof. □

In the following, we study some relationships between Green's relations of a Γ-semigroup and left (right) operator semigroup. The notations $\mathcal{L}_S$ and $\mathcal{R}_S$ are used for Green's relation of a Γ-semigroup S. Similarly, $\mathcal{L}_{O_l}$ and $\mathcal{R}_{O_l}$ are used for Green's relations of the left operator semigroup of S. Also, $\mathcal{L}_{O_r}$ and $\mathcal{R}_{O_r}$ are used for Green's relations of the right operation semigroup of S.

Proposition 3.9. *Let S be a Γ-semigroup, $a \in S$ and $\alpha, \beta \in \Gamma$.*

(i) If $a \in E_\alpha(S)$, then L is idempotent.
(ii) If $a \in Reg_{\alpha,\beta}(S)$, then L is regular.
(iii) If $a \in E_\alpha(S)$, then R is idempotent.
(iv) If $a \in Reg_{\alpha,\beta}(S)$, then R is regular.

Proof. (i) Since $a \in E_\alpha(S)$, we have $[a, \alpha][a, \alpha] = [a\alpha a, \alpha] = [a, \alpha]$. Thus, L is idempotent.
(ii) Since $a \in Reg_{\alpha,\beta}(S)$, then there exists $b \in S$ such that

$a = a\alpha b\beta a$. So, $[a, \alpha][b, \beta][a, \alpha] = [a\alpha b\beta a, \alpha] = [a, \alpha]$ and thus L is regular.

The proofs of (iii) and (iv) are similar to the proofs of (i) and (ii) respectively. □

Theorem 3.10. *Let S be a Γ-semigroup and $a, b \in S$.*

(i) If $(a, b) \in \mathcal{L}_S$, then $([a, \alpha], [b, \alpha]) \in \mathcal{L}_{O_l}$ for every $\alpha \in \Gamma$.
(ii) If $(a, b) \in \mathcal{R}_S$, then $([\alpha, a], [\alpha, b]) \in \mathcal{R}_{O_r}$ for every $\alpha \in \Gamma$.
(iii) If $a \in E_\alpha(S)$ and $b \in E_\beta(S)$ such that $([a, \alpha], [b, \beta]) \in \mathcal{R}_{O_l}$, then $(a, b) \in \mathcal{R}_S$.
(iv) If S is commutative, then $\mathcal{L}_{O_l} = \mathcal{R}_{O_l}$ ($\mathcal{L}_{O_r} = \mathcal{R}_{O_r}$)

Proof. (i) Suppose that $(a, b) \in \mathcal{L}_S$. Then there exist $x, y \in S$ and $\gamma, \delta \in \Gamma$ such that $a = x\gamma b$ and $b = y\delta a$. Thus for every $s \in S$, we have $a\alpha s = x\gamma b\alpha s$ and $b\alpha s = y\delta a\alpha s$ which implies that $[a, \alpha] = [x, \gamma][b, \alpha]$ and $[b, \alpha] = [y, \delta][a, \alpha]$. Therefore, $([a, \alpha], [b, \alpha]) \in \mathcal{L}_{O_l}$.
(ii) If $(a, b) \in \mathcal{R}_S$, then there exist $x, y \in S$ and $\gamma, \delta \in \Gamma$ such that $a = b\gamma x$ and $b = a\delta y$. Thus for every $s \in S$, we have $s\alpha a = s\alpha b\gamma x$ and $s\alpha b = s\alpha a\delta y$ which implies that $[\alpha, a] = [\alpha, b][\gamma, x]$ and $[\alpha, b] = [\alpha, a][\delta, y]$. Therefore, $([\alpha, a], [\alpha, b]) \in \mathcal{R}_{O_r}$.
(iii) Suppose that $([a, \alpha], [b, \beta]) \in \mathcal{R}_{O_l}$. Then there exist $x, y \in S$ and $\gamma, \delta \in \Gamma$ such that $[a, \alpha] = [b, \beta][x, \gamma] = [b\beta x, \gamma]$ and $[b, \beta] = [a, \alpha][y, \delta] = [a\alpha y, \delta]$. Thus, $a = a\alpha a = [a, \alpha]a = b\beta x\gamma a = b\beta(x\gamma a) \in \langle b\rangle_r$ and $b\beta b = a\alpha y\delta b = a\alpha(y\delta b) \in \langle a\rangle_r$. Therefore, $(a, b) \in \mathcal{R}_S$.
(iv) Let S be commutative. Then we have

$$\begin{aligned}
([a, \alpha], [b, \alpha]) \in \mathcal{L}_{O_l} &\iff [a, \alpha] = [x, \gamma][b, \alpha] = [x\gamma b, \alpha] \text{ and } [b, \alpha] = [y, \delta][a, \alpha] = [y\delta a, \alpha] \\
&\iff [a, \alpha] = [b\gamma x, \alpha] \text{ and } [b, \alpha] = [a\delta y, \alpha] \\
&\iff ([a, \alpha], [b, \alpha]) \in \mathcal{R}_{O_l}.
\end{aligned}$$

Similarly, we can prove that $\mathcal{L}_{O_r} = \mathcal{R}_{O_r}$. $\square$

Let σ be an equivalence relation on a set X and $a \in X$. Then the σ-class containing X is denoted by $[\sigma_X]_a$.

Theorem 3.11. *Let $\alpha \in \Gamma$ and $a \in E_\alpha(S)$. Then for every $b \in S$ the following statements are true.*

(i) *$b \in [\mathcal{L}_S]_a$ if and only if $[b, \alpha] \in [\mathcal{L}_{O_l}]_{[a,\alpha]}$;*
(ii) *$b \in [\mathcal{R}_S]_a$ if and only if $[\alpha, b] \in [\mathcal{L}_{O_r}]_{[\alpha,a]}$.*

Proof. (i)

$$\begin{aligned} b \in [\mathcal{L}_S]_a &\iff a = x\gamma b \text{ and } b = y\delta a \text{ for some } x, y \in S \text{ and } \gamma, \delta \in \Gamma \\ &\iff [a, \alpha]s = a\alpha s = (x\gamma b)\alpha s = [x, \gamma][b, \alpha]s \text{ and} \\ &\qquad [b, \alpha]s = b\alpha s = (y\delta a)\alpha s = [y, \delta][a, \alpha]s \ \forall \ s \in S \\ &\iff [a, \alpha] = [x, \gamma][b, \alpha] \in \langle [b, \alpha] \rangle_l \text{ and } [b, \alpha] = [y, \delta][a, \alpha] \in \langle [a, \alpha] \rangle_l \\ &\iff [b, \alpha] \in [\mathcal{L}_{O_l}]_{[a,\alpha]} \end{aligned}$$

(ii)

$$\begin{aligned} b \in [\mathcal{R}_S]_a &\iff a = b\gamma x \text{ and } b = a\delta y \text{ for some } x, y \in S \text{ and } \gamma, \delta \in \Gamma \\ &\iff s[\alpha, a] = s\alpha a = s\alpha(b\gamma x) = s[\alpha, b][\gamma, x] \text{ and} \\ &\qquad s[\alpha, b] = s\alpha b = s\alpha(a\delta y) = s[\alpha, a][\delta, y] \ \forall \ s \in S \\ &\iff [\alpha, a] = [\alpha, b][\gamma, x] \in \langle [\alpha, b] \rangle_l \text{ and } [\alpha, b] = [\alpha, a][\delta, y] \in \langle [\alpha, a] \rangle_l \\ &\iff [\alpha, b] \in [\mathcal{L}_{O_r}]_{[\alpha,a]} \end{aligned}$$

$\square$

Remark 3.12. Similar characterization can be proved for right operator semigroup of a Γ-semigroup in cases where left operator semigroup of a Γ-semigroup are mentioned.

4. Conclusion

Based on the modified definition of Γ-semigroup in this paper, we have studied some relationships between Green's equivalence relations of a Γ-semigroup and its left (right) operator semigroup. For further studies, the idea can be adopted to other Green's relations to see whether similar results will hold.

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