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APPLICATION OF MOHAND TRANSFORM COUPLED WITH HOMOTOPY PERTURBATION METHOD TO SOLVE NEWEL-WHITE-SEGEL-EQUATION

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Abstract. To solve the Newell-White-Segel equations (NWSE), this paper presents a novel analytical method that combines the Mohand Transform with the HPM in a unique way. Due to its intrinsic nonlinearity, NWSE, which is essential for simulating intricate events in mathematical biology and physics, poses analytical challenges. An efficient analytical framework for dealing with nonlinear systems is provided by extending the Mohand Transform in combination with the HPM. Comparative evaluations between the results of this study and those of previous research, together with proven analytical solutions, are carried out in the framework of three NWSE examples.

1. Introduction

Pattern formation in reaction-diffusion systems is a phenomenon of great significance across various scientific disciplines, encompassing fields such as biology, chemistry, physics, and materials science [35, 13, 22]. It involves the spontaneous emergence of organized structures or patterns in spatially extended systems, often governed by complex nonlinear partial differential equations. One equation that plays a crucial role in modeling pattern formation is the Newell-White-Segel equation [17, 35].

The Newell-White-Segel equation, proposed in the realm of reaction-diffusion dynamics, serves as a mathematical representation of the intricate processes underlying pattern formation [35]. This nonlinear partial differential equation captures the interplay between chemical reactions and diffusion, offering insights into the emergence of dynamic patterns observed in biological and physical systems [11, 36, 22]. The equation's form, which often involves multiple variables and intricate coupling terms, presents a formidable challenge for obtaining accurate and efficient solutions using traditional methods [11].

The pursuit of effective solution strategies for the Newell-White-Segel equation is motivated by the limitations of conventional methods in capturing the complexity

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inherent in real-world systems [6, 33]. Traditional approaches, such as analytical methods or straightforward numerical techniques, often fall short in providing comprehensive solutions, particularly when faced with the nonlinearities and intricate dynamics inherent in the equation [9, 4].

In response to these challenges, recent advancements in mathematical techniques have paved the way for innovative approaches to solving complex partial differential equations [20, 14]. The Mohand transform, a relatively novel mathematical tool, has shown promise in simplifying the solution process for such equations [2, 1]. When coupled with the HPM, a powerful technique for solving nonlinear problems, the hybrid methodology emerges as a potential breakthrough in addressing the Newell-White-Segel equation [29, 18, 37].

This study aim to bridge the gap between theoretical modeling and real-world observations of pattern formation [35]. By developing and applying a hybrid methodology combining the Mohand transform and the HPM, the research aims to provide a robust and efficient solution strategy for the Newell-White-Segel equation. The outcomes of this study are anticipated to contribute not only to the understanding of the mathematical aspects of pattern formation but also to offer practical insights with applications in diverse scientific and engineering domains [38].

As we delve into this research, the complexities of pattern formation and the challenges posed by the Newell-White-Segel equation underscore the necessity for advanced mathematical methods. This study represents a crucial step towards unraveling the mysteries of pattern formation in reaction-diffusion systems and advancing our ability to model and predict dynamic behaviors in complex, spatially extended systems.

The dynamic nature of pattern formation is evident in various natural phenomena, ranging from the intricate patterns on animal coats to the formation of spatial structures in chemical reactions [16]. Understanding and predicting these patterns are not only intellectually stimulating but also carry profound implications for fields such as biology, ecology, and materials science [10, 12].

Historically, researchers have grappled with the Newell-White-Segel equation and its counterparts, seeking to articulate the underlying dynamics that give rise to pattern formation [3, 26]. This endeavor has been met with challenges due to the inherent nonlinearities and coupling effects present in the equation. Consequently, the development of advanced mathematical methods becomes imperative to provide a nuanced understanding of the complex behaviors exhibited by reaction-diffusion systems [19].

The Mohand transform, as a relatively recent addition to the mathematician's toolkit, has exhibited promise in simplifying the solutions to partial differential equations [2]. Its unique approach to transforming complex equations into more manageable forms aligns well with the demands posed by the Newell-White-Segel equation [27, 7]. Additionally, the integration of the homotopy perturbation method, known for its efficacy in handling nonlinear problems, augments the potential for achieving accurate and efficient solutions.

The outcomes of this research could extend beyond the confines of reaction-diffusion systems, potentially influencing how we approach and model nonlinear phenomena in diverse scientific disciplines. The insights gained from this study may pave the way for the development of more robust solution strategies applicable to a broader class of nonlinear PDE.

The NWSE is a nonlinear hyperbolic PDE that is often used to explain physical processes such as fluid movement and chemical reactions. Despite its popularity, efforts to solve this equation analytically have been unsuccessful, necessitating the use of approximation computer approaches. Due to its importance in models of invasive species, chemical reactions, and population dynamics, the NWSE is of significant interest in mathematical physics [36, 5, 32]. This problem has just lately attracted attention because of its ability to be solved using methods other than numerical approaches. The NWSE is expressed as [24, 34]:

$$\frac{\partial\varphi(x,t)}{\partial t} = \ell^2\frac{\partial^2\varphi(x,t)}{\partial x^2} + \lambda_1\varphi(x,t) - \lambda_2\varphi^w(x,t) \tag{1.1}$$

Obtaining the result of a nonlinear PDE could be quite challenging, Varieties of approaches have been used. Researchers have used approximate analytic methods such as the HPM [15, 30], ADM [8, 25], and VIM [8, 21]. This work considers a new integral transformation known as Mohand transform in conjunction with HPM to solve the nonlinear models.

This study's contribution extends beyond its specific application, offering advancements in mathematical methods tailored for nonlinear partial differential equations. The outcomes are expected to benefit not only the understanding of the Newell-White-Segel equation but also hold the potential for addressing similar challenges in a broader scientific and engineering context. In summary, the research is justified by the imperative need for efficient solution methods, the promise of the Mohand transform, and the broader implications for unraveling the intricacies of pattern formation in dynamic systems.

2. Materials and Method

2.1. **Mohand Transform.** The Mohand Transform was created by Mohand Mahgoub, based on the Classical Fourier integral [28]. It is used to solve ODE and PDE in the time domain, due to its basic features and ease of calculation. The Mohand transform and its basic features are essential tools for solving differential equations. [2, 1].

Definition 2.1 ([28, 2, 1].)**.** We examine functions in set A that are defined using the Mohand transform, which applies to exponential order functions.

$$A = \left\{ f(t) : \exists\, M,\; k_1, k_2 > 0.\;\; |f(t)| < Me^{\frac{|t|}{k_j}},\;\; \text{if}\;\; i \in (-1)^j \times [0, \infty) \right\} \tag{2.1}$$

k_1 and k_2 may be infinite or finite given a function in the set A, but the constant M must have a finite value. The operator $\mathcal{M}$ represents the Mohand transform, which is defined by the integral equation.

$$\mathcal{M}\{[f(t)]\} = R(v) = \nu^2 \int_0^\infty f(t)e^{-vt}dt. \qquad t \geq 0, \quad k_1 \leq v \leq k_2 \tag{2.2}$$

Table 1. Properties of Mohand Transform

S/N	Property	Mathematical Expression
1.	Linearity	$\mathcal{M}\{a\varphi_1(t) + b\varphi_2(t)\} = a\mathcal{M}\{\varphi_1(t)\} + b\mathcal{M}\{\varphi_2(t)\}$
2.	Change of Scale	$\mathcal{M}\{F(at)\} = af\left(\frac{\nu}{a}\right)$
3.	$Shifting$	$\mathcal{M}e^{at}F(t) = \left(\frac{\nu}{\nu-1}\right)^2 f(\nu - a)$
4.	First Derivative	$\mathcal{M}\{G'(t)\} = vf(\nu - \nu^2 f(0)$
5.	Second Derivative	$\mathcal{M}F'(t) = \nu^2 f(v) - \nu^3 f(0) - \nu^2 g'(0)$
6.	n^{th}Derivative	$\mathcal{M}\{G^{(n)}(t)\} = \nu^n f(v) - \nu^{n+1} f(0) - \nu^n g'(0) - ... - \nu^2 G^{(n-1)}(0)$
7.	Convolution	$\mathcal{M}\{F_1(t) * F_2(t)\} = \frac{1}{\nu^2}\mathcal{M}\{F_1(t)\}\mathcal{M}\{F_2(t)\}$

2.2. **Mohand Transform of Some functions.** It is assumed that the integral equation (2.2) can be applied to any function $f(t)$, provided that the Mohand transform is both piecewise continuous and of exponential order. Otherwise, the equation might not exist at all. It is essential that the function $f(t)$ meets these requirements for $t \geq 0$. The Mohand Transform of basic functions can be determined as:

(1) Let $f(t) = 1$, then, By the definition of simple functions.

$$\mathcal{M}\{1\} = R(v) = \nu^2 \int_0^\infty e^{-vt}dt = \nu^2 \left[\frac{-1}{\nu^2}e^{-vt}\right]_0^\infty = v \tag{2.3}$$

(2) Let $f(t) = t$, then:

$$\mathcal{M}\{t\} = \nu^2 \int_0^\infty te^{-vt}dt = 1 \tag{2.4}$$

$$\mathcal{M}\{t\} = 1 \tag{2.5}$$

In general case if $n > 0$ is integer number, then.

$$\mathcal{M}[t^n] = \frac{n!}{\nu^{n-1}} \tag{2.6}$$

This results are used useful to find the Mohand transforms of the function in table 2

TABLE 2. Mohand Transform of some common functions [2, 1]

S/N	$f(t)$	$\mathcal{M}\{f(t)\} = R(v)$
1.	1	ν
2.	k	$k\nu$
3.	t	1
4.	$t^n, \quad n \in N$	$\frac{n!}{\nu^{n-1}}$
5.	e^{at}	$\frac{\nu^2}{\nu-a}$
6.	$t^n e^{at}$	$\frac{n\nu^2}{(\nu-a)^{n+1}}$
7.	$\sin at$	$\frac{a\nu^2}{\nu^2+a^2}$
8.	$\cos at$	$\frac{\nu^3}{\nu^2+a^2}$
9.	$\sinh at$	$\frac{a\nu^2}{\nu^2-a^2}$
10.	$\cosh at$	$\frac{\nu^3}{\nu^2-a^2}$

TABLE 3. Inverse Mohand Transform of Frequently encountered functions [2, 1]

S/N	$R(v)$	$f(t)(= \mathcal{M}^{-1}\{R(v)\}$
1.	ν	1
2.	1	t
3.	$\frac{1}{\nu}$	$\frac{t^2}{2!}$
4.	$\frac{1}{\nu^{n-1}}, \quad n \in N$	$\frac{t^n}{n!}$
5.	$\frac{\nu^2}{\nu-a}$	e^{at}
6.	$\frac{\nu^2}{(\nu-a)^{n+1}}$	$\frac{t^n}{n} e^{at}$
7.	$\frac{\nu^2}{\nu^2+a^2}$	$\frac{\sin at}{a}$
8.	$\frac{\nu^3}{\nu^2+a^2}$	$\cos at$
9.	$\frac{\nu^2}{\nu^2-a^2}$	$\sinh at$
10.	$\frac{\nu^3}{\nu^2-a^2}$	$\cosh at$

2.3. **Mohand Transform Homotopy Perturbation Method (MTHPM).** Consider a general NWSE problem of the form [24, 34]

$$\frac{\partial\varphi(x,t)}{\partial t} = \ell^2\frac{\partial^2\varphi(x,t)}{\partial x^2} + \lambda_1\varphi(x,t) - \lambda_2\varphi^w(x,t) \tag{2.7}$$

with

$$\varphi(x,0) = h(x) \tag{2.8}$$

with $\lambda_1,\ \lambda_2$ are constants and ℓ, w are non-negative

Procedure of the Method

Taking Mohand transform of both side of equation (2.7)

$$\mathcal{M}\left[\frac{\partial\varphi(x,t)}{\partial t}\right] = \mathcal{M}\left[\ell^2\frac{\partial^2\varphi(x,t)}{\partial x^2}\right] + \mathcal{M}\left[\lambda_1\varphi(x,t)\right] - \mathcal{M}\left[\lambda_2\varphi^w(x,t)\right] \tag{2.9}$$

$$v\mathcal{M}[\varphi(x,t)] - \nu^2[\varphi(x,0)] = \mathcal{M}\left[\ell^2\frac{\partial^2\varphi(x,t)}{\partial x^2}\right] + \lambda_1\mathcal{M}[\varphi(x,t)] - \lambda_2\mathcal{M}[\varphi^w(x,t)] \tag{2.10}$$

$$v\mathcal{M}[\varphi(x,t)] - \lambda_1\mathcal{M}[\varphi(x,t)] = \nu^2[\varphi(x,0)] + \mathcal{M}\left[\ell^2\frac{\partial^2\varphi(x,t)}{\partial x^2} - \lambda_2\varphi^w(x,t)\right] \tag{2.11}$$

$$(\nu-\lambda_1)\mathcal{M}[\varphi(x,t)] = \nu^2[\varphi(x,0)] + \mathcal{M}\left[\ell^2\frac{\partial^2\varphi(x,t)}{\partial x^2} - \lambda_2\varphi^w(x,t)\right] \tag{2.12}$$

multiplying equation (2.12) by $\dfrac{1}{\nu-\lambda_1}$

$$\mathcal{M}[\varphi(x,t)] = \frac{\nu^2}{\nu-\lambda_1}\varphi(x,0) + \frac{1}{\nu-\lambda_1}\mathcal{M}\left[\ell^2\frac{\partial^2\varphi(x,t)}{\partial x^2} - \lambda_2\varphi^w(x,t)\right] \tag{2.13}$$

$$\mathcal{M}\left[\varphi(x,t)\right] = \frac{\nu^2}{\nu-\lambda_1}\varphi(x,0) + \frac{1}{\nu-\lambda_1}\left[\mathcal{M}\left\{\ell^2\frac{\partial^2}{\partial x^2}\varphi(x,t) - \lambda_2\varphi^w(x,t)\right\}\right] \tag{2.14}$$

Applying Homotopy assumptions, the solutions can be expressed in power series. Let

$$\varphi(x,t) = \varphi_r(x,t) = \varphi_{r+1}, \tag{2.15}$$

$$\varphi^w(x,t) = H_r(u) \tag{2.16}$$

where H_r is He's Polynomial Coefficients and defined as

$$H_r = \frac{1}{r!}\frac{d^r}{d\eta^r}\left[N\sum_{i=0}^{r}\eta^i\varphi_i\right]_{\eta=0} \tag{2.17}$$

Substituting (2.15) and (2.16) into (2.14). Thus, equation (2.14) becomes

$$\mathcal{M}\left[\varphi_{r+1}(x,t)\right] = \frac{\nu^2}{\nu-\lambda_1}\varphi_r(x,0) + \frac{1}{\nu-\lambda_1}\left[\mathcal{M}\left\{\ell^2\frac{\partial^2}{\partial x^2}\varphi_r(x,t) - \lambda_2 H_r(u)\right\}\right] \tag{2.18}$$

Substituting equation (2.8) into equation (2.18). Then we have

$$\mathcal{M}\left[\varphi_{r+1}(x,t)\right] = \frac{\nu^2 h(x)}{\nu-\lambda_1} + \frac{1}{\nu-\lambda_1}\left[\mathcal{M}\left\{\ell^2\frac{\partial^2}{\partial x^2}\varphi_r(x,t) - \lambda_2 H_r(u)\right\}\right] \tag{2.19}$$

Using He's Polynomial to simplify the nonlinear Term

$$H_0(u) = \varphi_0\varphi_{0x} \tag{2.20}$$

$$H_1(u) = \varphi_0\varphi_{1x} + \varphi_1\varphi_{0x} \tag{2.21}$$

$$H_2(u) = \varphi_0\varphi_{2x} + \varphi_1\varphi_{1x} + \varphi_2\varphi_{0x} \tag{2.22}$$

$$H_3(u) = \varphi_0\varphi_{3x} + \varphi_1\varphi_{2x} + \varphi_2\varphi_{1x} + \varphi_3\varphi_{3x} \tag{2.23}$$

Taking the inverse Mohand Transform of equation (2.18)

$$\varphi_{r+1}(x,t) = \mathcal{M}^{-1}\left[\frac{\nu^2 h(x)}{\nu-\lambda_1}\right] + p\left(\mathcal{M}^{-1}\left\{\frac{1}{\nu-\lambda_1}\left[\mathcal{M}\left\{\ell^2\frac{\partial^2}{\partial x^2}\varphi_r(x,t) - \lambda_2 H_r(u)\right\}\right]\right\}\right) \tag{2.24}$$

Comparing the terms of the He's Polynomial coefficient of equation (2.24). The following approximations are gotten

$$p^0: \qquad \varphi_0(x,t) = \mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-\lambda_1}h(x)\right\} \tag{2.25}$$

$$p^1: \qquad \varphi_1(x,t) = \mathcal{M}^{-1}\left\{\frac{1}{\nu-\lambda_1}\left[\mathcal{M}\left\{\ell^2\frac{\partial^2}{\partial x^2}\varphi_0(x,t) - \lambda_2 H_0(u)\right\}\right]\right\} \tag{2.26}$$

$$p^2: \qquad \varphi_2(x,t) = \mathcal{M}^{-1}\left\{\frac{1}{\nu-\lambda_1}\left[\mathcal{M}\left\{\ell^2\frac{\partial^2}{\partial x^2}\varphi_1(x,t) - \lambda_2 H_1(u)\right\}\right]\right\} \tag{2.27}$$

$$p^3: \qquad \varphi_3(x,t) = \mathcal{M}^{-1}\left\{\frac{1}{\nu-\lambda_1}\left[\mathcal{M}\left\{\ell^2\frac{\partial^2}{\partial x^2}\varphi_2(x,t) - \lambda_2 H_2(u)\right\}\right]\right\} \tag{2.28}$$

3. Main Results

Example 3.1. Consider a linear homogeneous NWSE Problem of the form [24, 34]

$$\frac{\partial\varphi(x,t)}{\partial t} - \frac{\partial^2\varphi(x,t)}{\partial x^2} + 3\varphi(x,t) = 0 \tag{3.1}$$

Subject to an initial condition

$$\varphi(x,0) = e^{2x} \tag{3.2}$$

Solution

To begin the procedure, Let's take the Mohand transform (3.1)

$$\nu^2\mathcal{M}[\varphi(x,t)] - \nu^2\varphi(x,0) = \mathcal{M}\left[\frac{\partial^2\varphi(x,t)}{\partial x^2} - 3\varphi(x,t)\right] \tag{3.3}$$

Using the Linearity property of Mohand transform, equation (3.3) becomes

$$\nu^2\mathcal{M}[\varphi(x,t)] - \nu^2\varphi(x,0) = \mathcal{M}\left[\frac{\partial^2\varphi(x,t)}{\partial x^2}\right] - 3\mathcal{M}[\varphi(x,t)] \tag{3.4}$$

$$\nu^2\mathcal{M}[\varphi(x,t)] + 3\mathcal{M}[\varphi(x,t)] = \nu^2\varphi(x,0) + \mathcal{M}\left[\frac{\partial^2\varphi(x,t)}{\partial x^2}\right] \tag{3.5}$$

Factorizing the LHS of equation (3.5). Then it becomes

$$(v+3)\mathcal{M}[\varphi(x,t)] = \nu^2\varphi(x,0) + \mathcal{M}\left[\frac{\partial^2\varphi(x,t)}{\partial x^2}\right] \tag{3.6}$$

Multiplying both sides of equation (3.6) by $\dfrac{1}{v+3}$. The equation becomes

$$\mathcal{M}[\varphi(x,t)] = \frac{\nu^2}{v+3}\varphi(x,0) + \frac{1}{v+3}\left[\mathcal{M}\left[\frac{\partial^2\varphi(x,t)}{\partial x^2}\right]\right] \tag{3.7}$$

Taking the inverse Mohand of transform of equation (3.7)

$$\varphi(x,t) = \mathcal{M}^{-1}\left\{\frac{\nu^2}{v+3}\varphi(x,0)\right\} + \mathcal{M}^{-1}\left\{\frac{1}{v+3}\left[\mathcal{M}\left[\frac{\partial^2\varphi(x,t)}{\partial x^2}\right]\right]\right\} \tag{3.8}$$

Let

$$\varphi(x,t) = \sum_{r=0}^{\infty}\varphi_r(x,t) = \varphi_{r+1}(x,t) \quad \forall \; r \geq 0 \tag{3.9}$$

$$H_r = \frac{1}{r!}\frac{d^r}{d\eta^r}\left[N\sum_{i=0}^{r}\eta^i\varphi_i\right]_{\eta=0} \tag{3.10}$$

Thus, equation (3.8), becomes

$$\varphi_{r+1}(x,t) = \mathcal{M}^{-1}\left\{\frac{\nu^2}{v+3}\varphi_r(x,0)\right\} + \mathcal{M}^{-1}\left\{\frac{1}{v+3}\left[\mathcal{M}\left[\frac{\partial^2}{\partial x^2}\varphi_r(x,t)\right]\right]\right\} \tag{3.11}$$

Substituting equation (3.2) into (3.11), The equation becomes

$$\varphi_{r+1}(x,t) = \mathcal{M}^{-1}\left\{\frac{\nu^2 e^{2x}}{v+3}\right\} + p\left(\mathcal{M}^{-1}\left\{\frac{1}{v+3}\left[\mathcal{M}\left[\frac{\partial^2}{\partial x^2}\varphi_r(x,t)\right]\right]\right\}\right) \tag{3.12}$$

Comparing the coefficient of p

$$\begin{aligned}
\varphi_0(x,t) &= \mathcal{M}^{-1}\left[\frac{\nu^2}{v+3}\right] \\
&= \mathcal{M}^{-1}\left[\frac{ke^{2x}}{v+3}\right] \\
&= e^{2x}\left[\mathcal{M}^{-1}\left\{\frac{\nu^2}{v+3}\right\}\right] \\
&= e^{2x}[e^{-3t}] \\
&= e^{2x-3t}
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
\varphi_1(x,t) &= \mathcal{M}^{-1}\left\{\frac{1}{v+3}\mathcal{M}\left[\frac{\partial^2\varphi_0(x,t)}{\partial x^2}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{1}{v+3}\mathcal{M}\left[\frac{\partial^2}{\partial x^2}e^{2x-3t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{1}{v+3}\mathcal{M}\left[4e^{2x-3t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{4e^{2x}}{v+3}\mathcal{M}\left[e^{-3t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{4\nu^2e^{2x}}{(v+3)^2}\right\} \\
&= 4e^{2x}\mathcal{M}^{-1}\left\{\frac{\nu^2}{(v+3)^2}\right\} \\
&= 4te^{2x-3t}
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
\varphi_2(x,t) &= \mathcal{M}^{-1}\left\{\frac{1}{v+3}\mathcal{M}\left[\frac{\partial^2\varphi_1(x,t)}{\partial x^2}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{1}{v+3}\mathcal{M}\left[\frac{\partial^2}{\partial x^2}4te^{2x-3t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{1}{v+3}\mathcal{M}\left[16te^{2x-3t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{16e^{2x}}{v+3}\mathcal{M}\left[te^{-3t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{16\nu^2e^{2x}}{(v+3)^3}\right\} \\
&= 16e^{2x}\mathcal{M}^{-1}\left\{\frac{\nu^2}{(v+3)^3}\right\} \\
&= \frac{16t^2e^{2x-3t}}{2} \\
&= 8t^2e^{2x-3t}
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
\varphi_3(x,t) &= \mathcal{M}^{-1}\left\{\frac{1}{v+3}\mathcal{M}\left[\frac{\partial^2\varphi_2(x,t)}{\partial x^2}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{1}{v+3}\mathcal{M}\left[\frac{\partial^2}{\partial x^2}8t^2e^{2x-3t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{1}{v+3}\mathcal{M}\left[32te^{2x-3t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{32e^{2x}}{v+3}\mathcal{M}\left[t^2e^{-3t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{16\nu^2e^{2x}}{(v+3)^4}\right\} \\
&= 32e^{2x}\mathcal{M}^{-1}\left\{\frac{\nu^2}{(v+3)^3}\right\} \\
&= \frac{32t^3e^{2x-3t}}{3} \\
&= \frac{32}{3}t^3e^{2x-3t}
\end{aligned} \tag{3.16}$$

The series solution is expressed as

$$\varphi(x,t) = e^{2x-3t}\left(1+4t+8t^2+\frac{32}{3}t^3+...\right) \tag{3.17}$$

Equation (3.17) can be written in closed form as

$$= e^{2x+t} \tag{3.18}$$

Equation (3.18) agrees with the solution obtained in [24, 34]

Example 3.2. Consider a nonlinear Newell-White-Segel Equation of the form [24, 34]

$$\frac{\partial(x,t)}{\partial t} = 5\frac{\partial^2\varphi(x,t)}{\partial x^2} + 2\varphi(x,t) + \varphi^2(x,t) \tag{3.19}$$

Subjected to

$$\varphi(x,t) = \xi$$

Solution

To begin the procedure, Let's take the Mohand transform (3.19)

$$\nu^2\mathcal{M}[\varphi(x,t)] - \nu^2\varphi(x,0) = \mathcal{M}\left[5\frac{\partial^2\varphi(x,t)}{\partial x^2} + 2\varphi(x,t) - \varphi^2(x,t)\right] \tag{3.20}$$

Using the Linearity property of Mohand transform, equation (3.20) becomes

$$\nu^2\mathcal{M}[\varphi(x,t)] - \nu^2\varphi(x,0) = \mathcal{M}\left[5\frac{\partial^2\varphi(x,t)}{\partial x^2}\right] + 2\mathcal{M}[\varphi(x,t)] + \mathcal{M}[\varphi^2(x,t)] \tag{3.21}$$

$$\nu^2\mathcal{M}[\varphi(x,t)] - 2\mathcal{M}[\varphi(x,t)] = \nu^2\varphi(x,0) + \mathcal{M}\left[5\frac{\partial^2\varphi(x,t)}{\partial x^2} + \varphi^2(x,t)\right] \tag{3.22}$$

Factorizing equation (3.22). Then it becomes

$$(\nu-2)\mathcal{M}[\varphi(x,t)] = \nu^2\varphi(x,0) + \mathcal{M}\left[5\frac{\partial^2\varphi(x,t)}{\partial x^2} + \varphi^2(x,t)\right] \tag{3.23}$$

Multiplying both sides of equation (3.23) by $\frac{1}{\nu-2}$. The equation becomes

$$\mathcal{M}[\varphi(x,t)] = \frac{\nu^2}{\nu-2}\varphi(x,0) + \frac{1}{\nu-2}\left[\mathcal{M}\left[5\frac{\partial^2\varphi(x,t)}{\partial x^2} + \varphi^2(x,t)\right]\right] \tag{3.24}$$

Taking the inverse Mohand of transform of equation (3.24)

$$\varphi(x,t) = \mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-2}\varphi(x,0)\right\} + \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\left[\mathcal{M}\left[5\frac{\partial^2\varphi(x,t)}{\partial x^2} + \varphi^2(x,t)\right]\right]\right\} \tag{3.25}$$

According to ADM[..],

$$\varphi(x,t) = \sum_{r=0}^{\infty}\varphi_r(x,t) \tag{3.26}$$

$$H_r = \frac{1}{r!}\frac{d^r}{d\eta^r}\left[N\sum_{i=0}^{r}\eta^i\varphi_i\right]_{\eta=0} \tag{3.27}$$

Using The He's Polynomial to simply the nonlinear Term. Then, equation (3.25), becomes

$$H_0(u) = \varphi_0\varphi_{0x} \tag{3.28}$$

$$H_1(u) = \varphi_0\varphi_{1x} + \varphi_1\varphi_{0x} \tag{3.29}$$

$$H_2(u) = \varphi_0\varphi_{2x} + \varphi_1\varphi_{1x} + \varphi_2\varphi_{0x} \tag{3.30}$$

$$H_3(u) = \varphi_0\varphi_{3x} + \varphi_1\varphi_{2x} + \varphi_2\varphi_{1x} + \varphi_3\varphi_{3x} \tag{3.31}$$

Thus, we obtain

$$\varphi_{r+1}(x,t) = \mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-2}\varphi_r(x,0)\right\} + p\left(\mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\left[\mathcal{M}\left[5\frac{\partial^2}{\partial x^2}\varphi_r(x,t) + H_r(u)\right]\right]\right\}\right) \tag{3.32}$$

Comparing the coefficient of p

$$\begin{aligned}\varphi_0(x,t) &= \mathcal{M}^{-1}\left[\frac{\nu^2}{\nu-2}\varphi_0(x,0)\right] \\ &= \mathcal{M}^{-1}\left[\frac{\nu^2\xi}{\nu-2}\right] \\ &= \xi\left[\mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-2}\right\}\right] \\ &= \xi e^{2t}\end{aligned} \tag{3.33}$$

$$\begin{aligned}
H_0(u) &= \varphi_0^2 \\
&= [\xi e^{2t}]^2 \\
&= \xi^2 e^{4t} \\
\varphi_1(x,t) &= \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[5\frac{\partial^2\varphi_0(x,t)}{\partial x^2} + H_0(u)\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[\xi^2 e^{4t}\right]\right\} \\
&= \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\times\frac{\nu^2\xi^2}{\nu-4}\right\} \\
&= \xi^2\mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\times\frac{\nu^2}{\nu-4}\right\} \\
&= \xi^2\mathcal{M}^{-1}\left\{\frac{\nu^2}{(\nu-2)(\nu-4)}\right\} \\
&= \xi^2\mathcal{M}^{-1}\left\{\xi^2\nu^2\left(\frac{A}{\nu-2}+\frac{B}{\nu-4}\right)\right\}
\end{aligned}$$

Solving the partial fraction in equation (3.34). Then, $A = -\frac{1}{2}$ and $B = \frac{1}{2}$becomes

$$\begin{aligned}
&= \xi^2\mathcal{M}^{-1}\left\{\nu^2\left[\frac{1}{2(\nu-4)} - \frac{1}{2(\nu-2)}\right]\right\} && (3.34) \\
&= \xi^2\mathcal{M}^{-1}\left\{\frac{\nu^2}{2}\left[\frac{1}{\nu-4} - \frac{1}{\nu-2}\right]\right\} && (3.35) \\
&= \frac{\xi^2}{2}\mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-4} - \frac{\nu^2}{\nu-2}\right\} \\
&= \frac{\xi^2}{2}(e^{4t} - e^{2t}) \\
&= \frac{\xi^2}{2}e^{2t}(e^{2t} - 1) \\
H_1 &= 2\varphi_0\varphi_1 && (3.36) \\
&= 2\xi e^{2t}\times\frac{\xi^2}{2}e^{2t}(e^{2t}-1) \\
&= \xi^3 e^{4t}(e^{2t}-1) && (3.37) \\
\varphi_2(x,t) &= \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[5\frac{\partial^2\varphi_1(x,t)}{\partial x^2} + H_1(u)\right]\right\} && (3.38) \\
&= \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[\xi^3 e^{4t}(e^{2t}-1)\right]\right\} && (3.39) \\
&= \xi^3\mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[e^{6t} - e^{4t}\right]\right\} && (3.40) \\
&= \xi^3\mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-2}\left[\frac{1}{(\nu-6)} - \frac{1}{(\nu-4)}\right]\right\}
\end{aligned}$$

$$= \xi^3 \mathcal{M}^{-1}\left\{\nu^2\left[\frac{1}{(\nu-2)(\nu-6)} - \frac{1}{(\nu-2)(\nu-4)}\right]\right\}$$
$$= \frac{\xi^3}{4} e^{2t}(e^{2t}-1)^2$$

$$H_2 = \varphi_1^2 + 2\varphi_0\varphi_2 \tag{3.41}$$

$$\varphi_3(x,t) = \mathcal{M}^{-1}\left\{\frac{1}{\upsilon+3}\mathcal{M}\left[5\frac{\partial^2\varphi_2(x,t)}{\partial x^2} + H_2(u)\right]\right\} \tag{3.42}$$
$$= \frac{\xi^4}{8} e^{2t}(e^{2t}-1)^3$$

The series solution is expressed as

$$\varphi(x,t) = e^{2t}\left(\xi + \frac{\xi^2}{2}(e^{2t}-1) + \frac{\xi^3}{4}(e^{2t}-1)^2 + \frac{\xi^4}{8}(e^{2t}-1)^3 ...\right) \tag{3.43}$$

Equation (3.43) can be written in closed form as

$$= \frac{2\xi e^{2t}}{2+\xi(1-e^{2t})} \tag{3.44}$$

Equation (3.44) agrees with the solution obtained in [24, 34]

Example 3.3. Let consider a nonlinear Newell-White-Segel Equation of the form [24, 31, 34]

$$\frac{\partial(x,t)}{\partial t} = \frac{\partial^2\varphi(x,t)}{\partial x^2} + 2\varphi(x,t) - 3\varphi^2(x,t) \tag{3.45}$$

Subjected to

$$\varphi(x,0) = 1$$

Solution

Taking the Mohand transform (3.45)

$$\nu^2\mathcal{M}[\varphi(x,t)] - \nu^2\varphi(x,0) = \mathcal{M}\left[\frac{\partial^2\varphi(x,t)}{\partial x^2} + 2\varphi(x,t) - 3\varphi^2(x,t)\right] \tag{3.46}$$

Using the Linearity property of Mohand transform, equation (3.46) becomes

$$\nu^2\mathcal{M}[\varphi(x,t)] - \nu^2\varphi(x,0) = \mathcal{M}\left[\frac{\partial^2\varphi(x,t)}{\partial x^2}\right] + 2\mathcal{M}[\varphi(x,t)] - 3\mathcal{M}[\varphi^2(x,t)] \tag{3.47}$$

$$\nu^2\mathcal{M}[\varphi(x,t)] - 2\mathcal{M}[\varphi(x,t)] = \nu^2\varphi(x,0) + \mathcal{M}\left[2\frac{\partial^2\varphi(x,t)}{\partial x^2} - 3\varphi^2(x,t)\right] \tag{3.48}$$

Factorizing the LHS of equation (3.48). Then it becomes

$$(\nu-2)\mathcal{M}[\varphi(x,t)] = \nu^2\varphi(x,0) + \mathcal{M}\left[2\frac{\partial^2\varphi(x,t)}{\partial x^2} - 3\varphi^2(x,t)\right] \tag{3.49}$$

Multiplying both sides of equation (3.49) by $\frac{1}{\nu-2}$. The equation becomes

$$\mathcal{M}[\varphi(x,t)] = \frac{\nu^2}{\nu-2}\varphi(x,0) + \frac{1}{\nu-2}\left[\mathcal{M}\left[2\frac{\partial^2\varphi(x,t)}{\partial x^2} - 3\varphi^2(x,t)\right]\right] \tag{3.50}$$

Taking the inverse Mohand of transform of equation (3.50)

$$\varphi(x,t) = \mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-2}\varphi(x,0)\right\} + \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\left[\mathcal{M}\left[2\frac{\partial^2\varphi(x,t)}{\partial x^2} - 3\varphi^2(x,t)\right]\right]\right\} \tag{3.51}$$

According to ADM[..],

$$\varphi(x,t) = \sum_{r=0}^{\infty}\varphi_r(x,t) \tag{3.52}$$

$$H_r = \frac{1}{r!}\frac{d^r}{d\eta^r}\left[N\sum_{i=0}^{r}\eta^i\varphi_i\right]_{\eta=0} \tag{3.53}$$

Using The He's Polynomial to simply the nonlinear Term. Then, equation (3.51), becomes

$$H_0(u) = \varphi_0\varphi_{0x} \tag{3.54}$$

$$H_1(u) = \varphi_0\varphi_{1x} + \varphi_1\varphi_{0x} \tag{3.55}$$

$$H_2(u) = \varphi_0\varphi_{2x} + \varphi_1\varphi_{1x} + \varphi_2\varphi_{0x} \tag{3.56}$$

$$H_3(u) = \varphi_0\varphi_{3x} + \varphi_1\varphi_{2x} + \varphi_2\varphi_{1x} + \varphi_3\varphi_{3x} \tag{3.57}$$

Thus, we obtain

$$\sum_{r=0}^{\infty}\varphi_{r+1}(x,t) = \mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-2}\varphi_r(x,0)\right\} + p\left(\mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\left[\mathcal{M}\left[2\frac{\partial^2}{\partial x^2}\varphi_r(x,t) - 3H_r(u)\right]\right]\right\}\right) \tag{3.58}$$

Comparing the coefficient of p in equation (3.58)

$$\begin{aligned}\varphi_0(x,t) &= \mathcal{M}^{-1}\left[\frac{\nu^2}{\nu-2}\varphi_0(x,0)\right] \\ &= \mathcal{M}^{-1}\left[\frac{\nu^2}{\nu-2}\right] \\ &= \mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-2}\right\} \\ &= e^{2t}\end{aligned} \tag{3.59}$$

$$\begin{aligned}H_0 &= \varphi_0^2 \\ &= (e^{2t})^2 \\ &= e^{4t}\end{aligned}$$

$$\begin{aligned} \varphi_1(x,t) &= \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[2\frac{\partial^2\varphi_0(x,t)}{\partial x^2} - 3H_0(u)\right]\right\} \quad (3.60)\\ &= -\mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[3e^{4t}\right]\right\}\\ &= -\mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\times\frac{3\nu^2}{\nu-4}\right\}\\ &= -3\mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\times\frac{\nu^2}{\nu-4}\right\} \quad (3.61) \end{aligned}$$

simplifying the partial fraction, equation (3.61) becomes

$$\begin{aligned} &= -3\mathcal{M}^{-1}\left\{\frac{\nu^2}{2}\left[\frac{1}{\nu-4} - \frac{1}{\nu-2}\right]\right\} \quad (3.62)\\ &= -\frac{3}{2}\mathcal{M}^{-1}\left\{\frac{\nu^2}{\nu-4} - \frac{\nu^2}{\nu-2}\right\}\\ &= -\frac{3}{2}(e^{4t} - e^{2t})\\ &= -\frac{3e^{2t}}{2}(e^{2t} - 1) \end{aligned}$$

$$\begin{aligned} H_1(u) &= 2\varphi_0\varphi_1\\ &= -2\times e^{2t}\times\frac{3e^{2t}}{2}(e^{2t} - 1)\\ &= 3e^{6t} - 3e^{4t}\\ &= -3(e^{6t} + e^{4t})\\ \varphi_2(x,t) &= \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[2\frac{\partial^2\varphi_1(x,t)}{\partial x^2}\right] - 3H_1(u)\right\} \quad (3.63)\\ &= \mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[-3(e^{6t} + e^{4t})\right]\right\}\\ &= -3\mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\mathcal{M}\left[e^{6t} + e^{4t}\right]\right\} \end{aligned}$$

(3.64)

$$
\begin{aligned}
&= -3\mathcal{M}^{-1}\left\{\frac{1}{\nu-2}\left(\frac{\nu^2}{\nu-6}+\frac{\nu^2}{\nu-4}\right)\right\}\\
&= -3\mathcal{M}^{-1}\left\{\nu^2\left(\frac{1}{(\nu-2)(\nu-4)}+\frac{1}{(\nu-2)(\nu-6)}\right)\right\}\\
&= -3\mathcal{M}^{-1}\left\{\nu^2\left(\frac{2}{4(\nu-4)}-\frac{3}{4(\nu-2)}+\frac{1}{4(\nu-6)}\right)\right\}\\
&= -3\mathcal{M}^{-1}\left\{\frac{2\nu^2}{4(\nu-4)}-\frac{3\nu^2}{4(\nu-2)}+\frac{\nu^2}{4(\nu-6)}\right\}\\
&= -\frac{3}{4}\left[2e^{4t}-3e^{2t}+e^{6t}\right]\\
&= -\frac{3}{4}e^{2t}\left[e^{4t}+2e^{2t}-3\right]
\end{aligned}
\tag{3.65}
$$

The series solution is expressed as

$$
\varphi(x,t) = e^{2t}\left(1-\frac{3}{2}(e^{2t}-1)-\frac{3}{4}(e^{4t}+2e^{2t}-3)...\right) \tag{3.66}
$$

Equation (3.45) can be written in closed form as

$$
= \frac{\frac{2}{3}e^{2t}}{e^{2t}-\frac{1}{3}} \tag{3.67}
$$

The solution obtained in (3.67) agrees with [24, 31, 34]

4. Conclusion

The use of the integral transform with the HPM to the Newell-White-Segel equation provides a potent and new tool to get analytical insights into the dynamics of pattern generation in reaction-diffusion systems. We have overcome the difficulties presented by the Newell-White-Segel equation's intrinsic nonlinearity by using integral transforms to linearize the equation and then applying the homotopy perturbation method's systematic perturbation methodology.

In addition to advancing our theoretical knowledge of the Newell-White-Segel equation, this work provides a useful approach to dealing with nonlinearities in other mathematical models. Integral transformations and the homotopy perturbation technique work together to provide scholars and practitioners in a variety of domains with a flexible toolset that strikes a compromise between analytical precision and real-world applications. With further research and development of this combined method, we might be able to apply it to a wider class of nonlinear equations and improve our understanding and forecasting of complicated events in a variety of scientific and technical fields.

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