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ANALYZING THE EXPRESSIONS OF T_1-CORONA COMPOSITE GRAPHS VIA ZAGREB INDICES

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Abstract. To enrich the field of transformation graphs, we put forward four T_1-Corona composite graphs. The Zagreb indices plays vital role in chemical graph theory. In this paper, we obtain the explicit expressions for first and second Zagreb indices of T_1-Corona composite graphs.

1. Introduction and preliminaries

A structural representation of a chemical compound is a molecular graph. The atoms of chemical compound represent the vertices and chemical bonds represent the edges of the molecular graphs. Topological index is a unique number that dogmatise some physico-chemical properties of a chemical compound. The applications of topological indices are usually associated with quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR) [9, 10].

All graphs in this paper are finite, simple and connected. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of a graph G. The degree of a vertex u in G is the number of vertices adjacent to u and denoted by $d_G(u)$. The degree of an edge $e = uv$ is defined as $d_G(e) = d_G(u) + d_G(v) - 2$. The number of vertices and edges of G are denoted by n_G and m_G respectively. For undefined terminologies refer [5].

The *first and second Zagreb indices* [4] are introduced by Gutman and Trinajstić in the year 1972, and are respectively defined as

$$M_1(G) = \sum_{u \in V(G)} d_G(u)^2 = \sum_{uv \in E(G)} d_G(u) + d_G(v) \tag{1.1}$$

and

$$M_2(G) = \sum_{uv \in E(G)} d_G(u)\, d_G(v). \tag{1.2}$$

The *F-index* [3] is another vertex degree based molecular descriptor defined as

$$F(G) = \sum_{u \in V(G)} d_G(u)^3 = \sum_{uv \in E(G)} d_G(u)^2 + d_G(v)^2. \tag{1.3}$$

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Milićević et al. [7] in 2004 reformulated the Zagreb indices in terms of edge-degrees instead of vertex-degrees.

The *first and second reformulated Zagreb indices* are defined respectively as

$$EM_1(G) = \sum_{e\in E(G)} d_G(e)^2 = \sum_{e\sim f}[d_G(e) + d_G(f)].$$

and

$$EM_2(G) = \sum_{e\sim f} d_G(e)\ d_G(f).$$

For more study on Zagreb indices of transformation graphs, one can refer [1, 2, 6].

The *corona* [5] $G \circ H$ of two graphs G and H is the graph obtained by taking one copy of graph G and n_G copies of H, and then joining the ith vertex of G to every vertex in ith copy of H.

The order and size of $G\circ H$ are $n_G(1+n_H)$ and $m_G+n_Gm_H+n_Gn_H$, respectively. The degree of a vertex $x \in V(G \circ H)$ is given by

$$d_{G\circ H}(x) = \begin{cases} d_G(x) + n_H & if\ x \in V(G), \\ d_H(x) + 1 & if\ x \in V(H). \end{cases} \tag{1.4}$$

Sampathkumar and Chikkodimath introduced the concept of semitotal line graph. The semitotal line graph is also known as edge semitotal graph or middle graph or Q-graph.

The *semitotal line graph* [8] $T_1(G)$ of G is the graph whose vertex set is $V(G)\cup E(G)$ whose two vertices are adjacent if and only if (i) they are adjacent lines of G or (ii) one is vertex of G and the another is an edge of G incident with it.

In section 2, we introduce T_1-Corona composite graphs. In section 3 and 4, we give expressions for first Zagreb index and second Zagreb index of T_1-Corona composite graphs.

2. T_1-Corona composite graphs

In this section, we introduce four T_1-Corona composite graphs namely, T_1-vertex corona, T_1-edge corona, T_1-vertex neighborhood corona and T_1-edge neighborhood corona involving semitotal line graph and are defined as follows:

Definition 2.1. *The T_1-vertex corona* of G and H, denoted by $T_1(G) \odot H$ is the graph obtained from one copy of $T_1(G)$ and n_G copies of H, and then joining a vertex of $V(G)$, that is on the ith position in $T_1(G)$ to every vertex in the ith copy of H.

The graph $T_1(G) \odot H$ has a number of $n_G + m_G + n_Gn_H$ vertices and $m_G + n_Gm_H + n_Gn_H$ edges. The degree of a vertex $x \in V(T_1(G) \odot H)$ is given by

$$d_{T_1(G)\odot H}(x) = \begin{cases} d_G(x) + n_H & if\ x \in V(G), \\ d_G(x) + 2 & if\ x \in E(G), \\ d_H(x) + 1 & if\ x \in V(H). \end{cases} \tag{2.1}$$

Definition 2.2. The T_1-*edge corona* of G and H, denoted by $T_1(G) \ominus H$ is the graph obtained from one copy of $T_1(G)$ and m_G copies of H and joining a vertex of $E(G)$, that is on ith position in $T_1(G)$ to every vertex in the ith copy of H.

The graph $T_1(G) \ominus H$ has $n_G + m_G + m_G n_H$ number of vertices and $m_G + \frac{1}{2}M_1(G) + m_G m_H + m_G n_H$ number of edges. The degree of a vertex $x \in V(T_1(G) \ominus H)$ is given by

$$d_{T_1(G)\ominus H}(x) = \begin{cases} d_G(x) \;\; if\ x \in V(G), \\ d_G(x) + 2 + n_H \;\; if\ x \in E(G) \\ d_H(x) + 1 \;\; if\ x \in V(H). \end{cases} \tag{2.2}$$

Definition 2.3. The T_1-*vertex neighborhood corona* of G and H, denoted by $T_1(G) \boxdot H$, is the graph obtained from one copy of $T_1(G)$ and n_G copies of H and joining the neighbors of a vertex in $T_1(G)$ corresponding to a vertex of G, that is on the ith position in $T_1(G)$ to every vertex in the ith copy of H.

The graph $T_1(G) \boxdot H$ has $n_G + m_G + n_G n_H$ vertices and $m_G + \frac{1}{2}M_1(G) + n_G m_H + 2m_G n_H$ edges. The degree of vertices of $T_1(G) \boxdot H$ is given by :

$$\begin{aligned} d_{T_1(G)\boxdot H}(u) &= d_G(u) \;\; if\, u \in V(G), \\ d_{T_1(G)\boxdot H}(e) &= d_G(e) + 2 + 2n_H \;\; if\, e \in E(G). \\ d_{T_1(G)\boxdot H}(u') &= d_H(u') + d_G(u) \;\; if\, u' \in V(H), u \in V(G). \end{aligned} \tag{2.3}$$

In the last expression, $u' \in V(H)$ is the vertex in ith copy of H corresponding to ith vertex $u \in V(G)$ in $T_1(G)$.

Definition 2.4. The T_1-*edge neighborhood corona* of G and H, denoted by $T_1(G) \boxminus H$, is the graph obtained from one copy of $T_1(G)$ and m_G copies of H and joining the neighbors of a vertex in $T_1(G)$ corresponding to an edge of G, that is on the ith position in $T_1(G)$ to every vertex in the ith copy of H.

The graph $T_1(G) \boxminus H$ has $n_G + m_G + m_G n_H$ vertices and $m_G + M_1(G)[n_H + \frac{1}{2}] + m_G m_H$ edges. The degree of vertices of $T_1(G) \boxminus H$ is given by:

$$\begin{aligned} &d_{T_1(G)\boxminus H}(u) = d_G(u)(1 + n_H) \;\; if\, u \in V(G), \\ &d_{T_1(G)\boxminus H}(e) = d_G(e)(1 + n_H) + 2 \;\; if\, e \in E(G) \\ &d_{T_1(G)\boxminus H}(v) = d_H(v) + d_G(e') + 2 \;\; if\, v \in V(H), e' \in E(G). \end{aligned} \tag{2.4}$$

3. First Zagreb index of T_1-Corona composite graphs

Theorem 3.1. *Let G and H be two connected simple graphs. Then*

$$M_1(T_1(G) \odot H) \;\; = \;\; 5M_1(G) + EM_1(G) + n_G M_1(H) + n_G(n_H^2 + n_H + 4m_H) + 4m_G(n_H - 1).$$

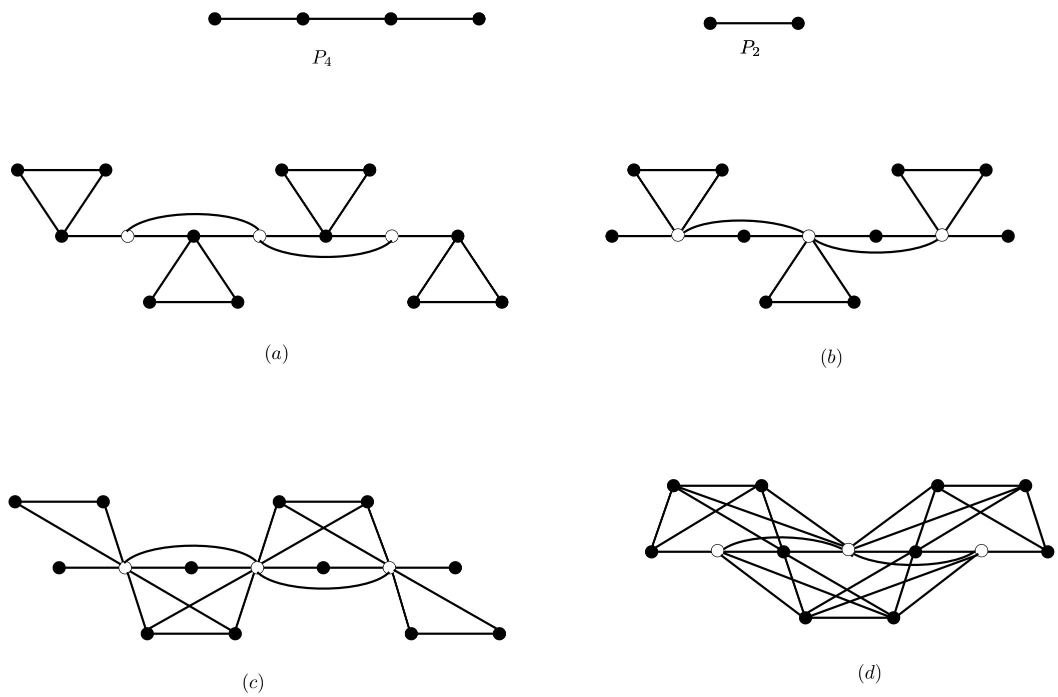

FIGURE 1. (a) $T_1(P_4) \odot P_2$, (b) $T_1(P_4) \ominus P_2$, (c) $T_1(P_4) \boxdot P_2$, (d) $T_1(P_4) \boxminus P_2$

Proof. Using (2.1) in equation (1.1), we get

$$
\begin{aligned}
M_1(T_1(G) \odot H) &= \sum_{u \in V(G)} (d_G(u) + n_H)^2 + \sum_{e \in E(G)} (d_G(e) + 2)^2 + n_G \sum_{u' \in V(H)} (d_H(u') + 1)^2 \\
&= \sum_{u \in V(G)} \{d_G(u)^2 + n_H^2 + 2n_H d_G(u)\} + \sum_{e \in E(G)} \{d_G(e)^2 + 4 + 4d_G(e)\} \\
&+ n_G \sum_{u' \in V(H)} \{d_H(u')^2 + 1 + 2d_H(u')\} \\
&= \left(M_1(G) + n_H^2 n_G + 4n_H m_G\right) + EM_1(G) + 4m_G + 8\left(-m_G + \frac{1}{2}\sum_{u \in V(G)} d_G(u)^2\right) \\
&+ n_G\{M_1(H) + n_H + 4m_H\} \\
&= M_1(G) + n_H^2 n_G + 4n_H m_G + EM_1(G) + 4m_G - 8m_G + 4M_1(G) \\
&+ n_G\{M_1(H) + n_H + 4m_H\} \\
&= 5M_1(G) + EM_1(G) + n_G M_1(H) + n_G(n_H^2 + n_H + 4m_H) + 4m_G(n_H - 1).
\end{aligned}
$$

□

Theorem 3.2. *Let G and H be two connected simple graphs. Then*

$$
\begin{aligned}
M_1(T_1(G) \ominus H) &= (2n_H + 5)M_1(G) + EM_1(G) + m_G M_1(H) + m_G(n_H - 2)(n_H + 2) \\
&+ m_G(n_H + 4m_H).
\end{aligned}
$$

Proof. Using (2.2) in equation (1.1), we get

$$\begin{aligned}
M_1(T_1(G) \ominus H) &= \sum_{u \in V(G)} d_G(u)^2 + \sum_{e \in E(G)} [d_G(e) + (n_H + 2)]^2 + m_G \sum_{u' \in V(H)} (d_H(u') + 1)^2 \\
&= M_1(G) + EM_1(G) + (n_H+2)^2 m_G + 2(n_H+2)\left[2\left(-m_G + \frac{1}{2}M_1(G)\right)\right] \\
&+ m_G\{M_1(H) + n_H + 4m_H\} \\
&= M_1(G) + EM_1(G) + (n_H+2)^2 m_G - 4(n_H+2)m_G + 2(n_H+2)M_1(G) \\
&+ m_G\{M_1(H) + n_H + 4m_H\} \\
&= (2n_H+5)M_1(G) + EM_1(G) + m_G M_1(H) + m_G(n_H-2)(n_H+2) \\
&+ m_G(n_H + 4m_H).
\end{aligned}$$

□

Theorem 3.3. *Let G and H be two connected simple graphs. Then*

$$\begin{aligned}
M_1(T_1(G) \boxdot H) &= 5(n_H+1)M_1(G) + EM_1(G) + n_G M_1(H) + 4m_G(n_H+1)^2 \\
&+ 8m_G(m_H - n_H - 1).
\end{aligned}$$

Proof. Using (2.3) in equation (1.1), we get

$$\begin{aligned}
M_1(T_1(G) \boxdot H) &= \sum_{u \in V(G)} d_G(u)^2 + \sum_{e \in E(G)} [d_G(e) + 2(n_H+1)]^2 + \sum_{u \in V(G); u' \in V(H)} [d_G(u) + d_H(u')]^2 \\
&= M_1(G) + EM_1(G) + 4m_G(n_H+1)^2 + 4(n_H+1)\left[2\left(-m_G + \frac{1}{2}M_1(G)\right)\right] \\
&+ n_H \sum_{u \in V(G)} d_G(u)^2 + n_G \sum_{u' \in V(H)} d_H(u')^2 + 2\sum_{u \in V(G)} d_G(u) \cdot \sum_{u' \in V(H)} d_H(u') \\
&= M_1(G) + EM_1(G) + 4m_G(n_H+1)^2 - 8m_G(n_H+1) + 4(n_H+1)M_1(G) \\
&+ n_H M_1(G) + n_G M_1(H) + 8m_G m_H \\
&= 5(n_H+1)M_1(G) + EM_1(G) + n_G M_1(H) + 4m_G(n_H+1)^2 \\
&+ 8m_G(m_H - n_H - 1).
\end{aligned}$$

□

Theorem 3.4. *Let G and H be two connected simple graphs. Then*

$$\begin{aligned}
M_1(T_1(G) \boxminus H) &= [(n_H+1)(n_H+5) + 4(n_H+m_H)]M_1(G) + [(n_H+1)^2 + n_H]EM_1(G) \\
&+ m_G M_1(H) - 4m_G(3n_H+1).
\end{aligned}$$

Proof. Using (2.4) in equation (1.1), we get

$$\begin{aligned}
M_1(T_1(G) \boxminus H) &= \sum_{u\in V(G)} [(n_H+1)d_G(u)]^2 + \sum_{e\in E(G)} [(n_H+1)d_G(e)+2]^2 \\
&+ \sum_{e\in E(G)\,;\,u'\in V(H)} [d_G(e)+d_H(u')+2]^2 \\
&= (n_H+1)^2 M_1(G) + (n_H+1)^2 EM_1(G) + 4m_G + 4(n_H+1)\left[2\left(-m_G+\frac{1}{2}M_1(G)\right)\right] \\
&+ m_G M_1(H) + n_H EM_1(G) + 8m_G m_H - 8m_G n_H + 4n_H M_1(G) \\
&- 8m_G m_H + 4m_H M_1(G) + 4n_H m_G \\
&= [(n_H+1)(n_H+5) + 4(n_H+m_H)]M_1(G) + [(n_H+1)^2 + n_H]EM_1(G) \\
&+ m_G M_1(H) - 4m_G(3n_H+1).
\end{aligned}$$

□

4. Second Zagreb index of T_1-Corona composite graphs

Theorem 4.1. *Let G and H be two connected simple graphs. Then*

$$\begin{aligned}
M_2(T_1(G) \odot H) &= 2(n_H+1)M_1(G) + 2M_2(G) + 2EM_1(G) + EM_2(G) + F(G) \\
&+ n_G\{M_1(H) + M_2(H)\} + n_G n_H(n_H+2m_H) + m_H(n_G+4m_G) + m_G(2n_H-4).
\end{aligned}$$

Proof. Using (2.1) in equation (1.2), we get

$$\begin{aligned}
M_2(T_1(G) \odot H) &= \sum_{a\sim b\,;\,a,b\in E(G)} [(d_G(a)+2)(d_G(b)+2)] + n_G \sum_{u'v'\in E(H)} [(d_H(u')+1)(d_H(v')+1)] \\
&+ \sum_{u\sim e}[(d_G(u)+n_H)(d_G(e)+2)] + \sum_{u\sim u'}[(d_G(u)+n_H)(d_H(u')+1)] \\
&= EM_2(G) + 2EM_1(G) + 4\left(-m_G+\frac{1}{2}M_1(G)\right) + n_G[M_2(H)+M_1(H)+m_H] \\
&+ \sum_{uv\in E(G)} [d_G(u)+d_G(v)+2n_H][d_G(u)+d_G(v)] \\
&+ \sum_{u\in V(G)} (d_G(u)+n_H)\cdot \sum_{u'\in V(H)} (d_H(u')+1) \\
&= EM_2(G) + 2EM_1(G) - 4m_G + 2M_1(G) + n_G[M_2(H)+M_1(H)+m_H] \\
&+ F(G) + 2M_2(G) - 2M_1(G) - 4n_H m_G + 2n_H M_1(G) + 2M_1(G) + 4n_H m_G \\
&+ 4m_G m_H + 2n_H m_G + 2n_G n_H m_H + n_H^2 n_G \\
&= 2(n_H+1)M_1(G) + 2M_2(G) + 2EM_1(G) + EM_2(G) + F(G) \\
&+ n_G\{M_1(H)+M_2(H)\} + n_G n_H(n_H+2m_H) + m_H(n_G+4m_G) + m_G(2n_H-4).
\end{aligned}$$

□

Theorem 4.2. *Let G and H be two connected simple graphs. Then*

$$\begin{aligned} M_2(T_1(G) \ominus H) &= \left[\frac{(n_H+2)^2}{2} + 2(n_H+m_H)\right] M_1(G) + 2M_2(G) + F(G) \\ &+ m_G\{M_1(H) + M_2(H)\} + (n_H+2)EM_1(G) + EM_2(G) \\ &+ m_G[n_H^2 - (n_H+2)^2] + m_G m_H(2n_H+1). \end{aligned}$$

Proof. Using (2.2) in equation (1.2), we get

$$\begin{aligned} M_2(T_1(G) \ominus H) = &\sum_{a\sim b\,;\,a,b\in E(G)} [(d_G(a)+n_H+2)(d_G(b)+n_H+2)] \\ &+ m_G \sum_{u'v'\in E(H)} [(d_H(u')+1)(d_H(v')+1)] \\ &+ \sum_{u\sim e}[(d_G(u))(d_G(e)+n_H+2)] + \sum_{u'\sim e}[(d_H(u')+1)(d_G(e)+n_H+2)] \\ &= EM_2(G) + (n_H+2)EM_1(G) + (n_H+2)^2\left(-m_G + \frac{1}{2}M_1(G)\right) \\ &+ m_G\{M_1(H)+M_2(H)+m_H\} + \sum_{uv\in E(G)} [d_G(u)+d_G(v)][d_G(u)+d_G(v)+n_H] \\ &+ \sum_{u'\in V(H)} (d_H(u')+1) \cdot \sum_{e\in E(G)} (d_G(e)+n_H+2) \\ &= EM_2(G) + (n_H+2)EM_1(G) - (n_H+2)^2 m_G + \frac{(n_H+2)^2}{2} M_1(G) \\ &+ m_G\{M_1(H)+M_2(H)+m_H\} + F(G) + 2M_2(G) + n_H M_1(G) \\ &+ (2m_H+n_H)\left[2\left(-m_G+\frac{1}{2}M_1(G)\right) + (n_H+2)m_G\right] \\ &= \left[\frac{(n_H+2)^2}{2} + 2(n_H+m_H)\right] M_1(G) + 2M_2(G) + F(G) + m_G\{M_1(H)+M_2(H)\} \\ &+ (2+n_H)EM_1(G) + EM_2(G) + m_G[n_H^2 - (n_H+2)^2] + m_G m_H(2n_H+1). \end{aligned}$$

□

Theorem 4.3. *Let G and H be two connected simple graphs. Then*

$$\begin{aligned} M_2(T_1(G) \boxdot H) &= [2(n_H+1)^2 + 6n_H + 5m_H]M_1(G) + 6M_2(G) + 3F(G) + 2(n_H+1)EM_1(G) \\ &+ EM_2(G) + 2m_G M_1(H) + n_G M_2(H) + 4m_G\{2n_H m_H - (n_H+1)^2\}. \end{aligned}$$

Proof. Using (2.3) in equation (1.2), we get

$$\begin{aligned} M_2(T_1(G) \boxdot H) = &\sum_{a\sim b\,;\,a,b\in E(G)} [d_G(a)+2(n_H+1)][d_G(b)+2(n_H+1)] \\ &+ \sum_{u'\sim e}[(d_H(u')+d_G(u))(d_G(e)+2+2n_H)] + \sum_{u\sim e}[(d_G(u))(d_G(e)+2+2n_H)] \\ &+ \sum_{u\in V(G)} \sum_{u'v'\in V(H)} [d_H(u')+d_G(u)][d_H(v')+d_G(u)] \end{aligned}$$

u' and v' are the vertices of H corresponding to a vertex u of G.

$$= EM_2(G) + 2(n_H+1)EM_1(G) + 4(n_H+1)^2\left(-m_G + \frac{1}{2}M_1(G)\right)$$
$$+ 2\sum_{u'\in V(H)} d_H(u') \sum_{e\in E(G)} [d_G(e) + 2(n_H+1)] + 3\sum_{uv\in E(G)} [d_G(u)+d_G(v)][d_G(u)+d_G(v)+2n_H]$$
$$+ m_H \sum_{u\in V(G)} d_G(u)^2 + \sum_{u\in V(G)} d_G(u) \sum_{u'v'\in E(H)} [d_H(u')+d_H(v')] + n_G \sum_{u'v'\in E(H)} [d_H(u')\cdot d_H(v')]$$
$$= EM_2(G) + 2(n_H+1)EM_1(G) - 4m_G(n_H+1)^2 + 2(n_H+1)^2 M_1(G)$$
$$+ 4m_H\{M_1(G) + 2n_H m_G\} + 3\{F(G) + 2M_2(G) + 2n_H M_1(G)\}$$
$$+ m_H M_1(G) + 2m_G M_1(H) + n_G M_2(H)$$
$$= [2(n_H+1)^2 + 6n_H + 5m_H]M_1(G) + 6M_2(G) + 3F(G) + 2(n_H+1)EM_1(G)$$
$$+ EM_2(G) + 2m_G M_1(H) + n_G M_2(H) + 4m_G\{2n_H m_H - (n_H+1)^2\}.$$

□

Theorem 4.4. *Let G and H be two connected simple graphs. Then*

$$\begin{aligned} M_2(T_1(G)\boxminus H) &= 2[n_H(m_H - n_H + 1) + 3(m_H+1)]M_1(G) + 2(n_H+1)(2n_H+1)M_2(G) \\ &+ (n_H+1)(2n_H+1)F(G) + [2(n_H+1)(n_H+m_H+1) + (2n_H+1)]EM_1(G) \\ &+ (n_H+1)(3n_H+1)EM_2(G) + M_1(G)M_1(H) + m_G M_2(H) \\ &- 8m_G(n_H+m_H+1). \end{aligned}$$

Proof. Using (2.4) in equation (1.2), we get

$$M_2(T_1(G)\boxminus H) = \sum_{a\sim b\ ;\ a,b\in E(G)} [d_G(a)(n_H+1)+2][d_G(b)(n_H+1)+2]$$
$$+ \sum_{u\sim e} [d_G(u)(n_H+1)][d_G(e)(n_H+1)+2]$$
$$+ \sum_{e\in E(G)} \sum_{u'v'\in V(H)} [d_H(u') + d_G(e) + 2][d_H(v') + d_G(e) + 2]$$
$$+ \sum_{e\in E(G)} \sum_{u\sim u'} [d_G(u)(n_H+1)][d_H(u') + d_G(e) + 2]$$
$$+ \sum_{e\sim u' \text{ and } u' \text{ is a vertex corresponding to copy } f\ ;\ e\sim f} [d_G(e)(n_H+1)+2][d_H(u') + d_G(f) + 2]$$
$$= (n_H+1)^2 EM_2(G) + 2(n_H+1)EM_1(G) + 4\left(-m_G + \frac{1}{2}M_1(G)\right)$$
$$+ (n_H+1)^2 \sum_{uv\in E(G)} [d_G(u)+d_G(v)]^2 - 2n_H(n_H+1)\sum_{uv\in E(G)} [d_G(u)+d_G(v)]$$
$$+ m_G \sum_{u'v'\in E(H)} d_H(u')d_H(v') + \sum_{u'v'\in E(H)} [d_H(u')+d_H(v')] \sum_{e\in E(G)} [d_G(e)+2] + \sum_{e\in E(G)} [d_G(e)+2]^2$$

$$
\begin{aligned}
&+ (n_H+1) \sum_{u' \in V(H)} d_H(u') \sum_{u \in V(G)} d_G(u)^2 + n_H(n_H+1) \sum_{uv \in E(G)} [d_G(u)+d_G(v)]^2 \\
&+ (n_H+1) \sum_{u' \in V(H)} d_H(u') \sum_{e \in E(G)} d_G(e)^2 + 2 \sum_{u' \in V(H)} d_H(u') \sum_{e \in E(G)} d_G(e) \\
&+ n_H(n_H+1) \left[2 \sum_{e \sim f} [d_G(e) d_G(f)] + 2 \sum_{e \in E(G)} d_G(e)^2 \right] + 2n_H \sum_{e \in E(G)} d_G(e)(d_G(e)+2) \\
&= (n_H+1)^2 EM_2(G) + 2(n_H+1) EM_1(G) + 2M_1(G) - 4m_G \\
&+ (n_H+1)\{(n_H+1)[F(G)+2M_2(G)] - 2n_H M_1(G)\} \\
&+ m_G M_2(H) + M_1(G)M_1(H) + EM_1(G) + 4m_G - 8m_G + 4M_1(G) \\
&+ 2m_H(n_H+1)M_1(G) + n_H(n_H+1)\{F(G)+2M_2(G)\} \\
&+ 2m_H(n_H+1)EM_1(G) + 4m_H \left[2 \left(-m_G + \frac{1}{2} M_1(G) \right) \right] \\
&+ n_H(n_H+1)[2EM_2(G) + 2EM_1(G)] + 2n_H \left[EM_1(G) + 4 \left(-m_G + \frac{1}{2} M_1(G) \right) \right] \\
&= (n_H+1)^2 EM_2(G) + 2(n_H+1) EM_1(G) + 2M_1(G) - 4m_G \\
&+ (n_H+1)^2 F(G) + 2(n_H+1)^2 M_2(G) - 2n_H(n_H+1) M_1(G) \\
&+ m_G M_2(H) + M_1(G)M_1(H) + EM_1(G) - 4m_G + 4M_1(G) \\
&+ 2(n_H+1)m_H M_1(G) + n_H(n_H+1)F(G) + 2n_H(n_H+1)M_2(G) + 4(n_H+m_H)M_1(G) \\
&+ [2m_H(n_H+1) + 2n_H(n_H+2)]EM_1(G) + 2n_H(n_H+1)EM_2(G) - 8m_G(n_H+m_H).
\end{aligned}
$$

□

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