ANALYZING THE EXPRESSIONS OF $T_1$-CORONA COMPOSITE GRAPHS VIA ZAGREB INDICES

MANJUNATHA GALI$^1$ D. G. PRAKASHA$^2$ AND CHETANA GALI$^3$,∗

ABSTRACT. To enrich the field of transformation graphs, we put forward four $T_1$-Corona composite graphs. The Zagreb indices plays vital role in chemical graph theory. In this paper, we obtain the explicit expressions for first and second Zagreb indices of $T_1$-Corona composite graphs.

1. INTRODUCTION AND PRELIMINARIES

A structural representation of a chemical compound is a molecular graph. The atoms of chemical compound represent the vertices and chemical bonds represent the edges of the molecular graphs. Topological index is a unique number that dogmatise some physico-chemical properties of a chemical compound. The applications of topological indices are usually associated with quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR) [9, 10].

All graphs in this paper are finite, simple and connected. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of a graph $G$. The degree of a vertex $u$ in $G$ is the number of vertices adjacent to $u$ and denoted by $d_G(u)$. The degree of an edge $e = uv$ is defined as $d_G(e) = d_G(u) + d_G(v) - 2$. The number of vertices and edges of $G$ are denoted by $n_G$ and $m_G$ respectively. For undefined terminologies refer [5].

The first and second Zagreb indices [4] are introduced by Gutman and Trinajstić in the year 1972, and are respectively defined as

$$M_1(G) = \sum_{u \in V(G)} d_G(u)^2 = \sum_{uv \in E(G)} d_G(u) + d_G(v)$$

(1.1)

and

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v).$$

(1.2)

The $F$-index [3] is another vertex degree based molecular descriptor defined as

$$F(G) = \sum_{u \in V(G)} d_G(u)^3 = \sum_{uv \in E(G)} d_G(u)^2 + d_G(v)^2.$$  

(1.3)
Milićević et al. [7] in 2004 reformulated the Zagreb indices in terms of edge-degrees instead of vertex-degrees. The first and second reformulated Zagreb indices are defined respectively as

\[ EM_1(G) = \sum_{e \in E(G)} d_G(e)^2 = \sum_{e \sim f}[d_G(e) + d_G(f)]. \]

and

\[ EM_2(G) = \sum_{e \sim f} d_G(e) d_G(f). \]

For more study on Zagreb indices of transformation graphs, one can refer [1, 2, 6].

The corona [5] \( G \circ H \) of two graphs \( G \) and \( H \) is the graph obtained by taking one copy of graph \( G \) and \( n_G \) copies of \( H \), and then joining the \( i \)th vertex of \( G \) to every vertex in \( i \)th copy of \( H \).

The order and size of \( G \circ H \) are \( n_G(1+n_H) \) and \( m_G+n_Gm_H+n_Gn_H \), respectively. The degree of a vertex \( x \in V(G \circ H) \) is given by

\[
d_{G\circ H}(x) = \begin{cases} 
d_G(x) + n_H & \text{if } x \in V(G), \\
d_G(x) + 2 & \text{if } x \in E(G), \\
d_H(x) + 1 & \text{if } x \in V(H). 
\end{cases} \tag{1.4}
\]

Sampathkumar and Chikkodimath introduced the concept of semitotal line graph. The semitotal line graph is also known as edge semitotal graph or middle graph or Q-graph.

The semitotal line graph [8] \( T_1(G) \) of \( G \) is the graph whose vertex set is \( V(G) \cup E(G) \) whose two vertices are adjacent if and only if (i) they are adjacent lines of \( G \) or (ii) one is vertex of \( G \) and the another is an edge of \( G \) incident with it.

In section 2, we introduce \( T_1 \)-Corona composite graphs. In section 3 and 4, we give expressions for first Zagreb index and second Zagreb index of \( T_1 \)-Corona composite graphs.

2. \( T_1 \)-Corona composite graphs

In this section, we introduce four \( T_1 \)-Corona composite graphs namely, \( T_1 \)-vertex corona, \( T_1 \)-edge corona, \( T_1 \)-vertex neighborhood corona and \( T_1 \)-edge neighborhood corona involving semitotal line graph and are defined as follows:

**Definition 2.1.** The \( T_1 \)-vertex corona of \( G \) and \( H \), denoted by \( T_1(G) \circ H \) is the graph obtained from one copy of \( T_1(G) \) and \( n_G \) copies of \( H \), and then joining a vertex of \( V(G) \), that is on the \( i \)th position in \( T_1(G) \) to every vertex in the \( i \)th copy of \( H \).

The graph \( T_1(G) \circ H \) has a number of \( n_G + m_G + n_Gn_H \) vertices and \( m_G + n_Gm_H + n_Gn_H \) edges. The degree of a vertex \( x \in V(T_1(G) \circ H) \) is given by

\[
d_{T_1(G)\circ H}(x) = \begin{cases} 
d_G(x) + n_H & \text{if } x \in V(G), \\
d_G(x) + 2 & \text{if } x \in E(G), \\
d_H(x) + 1 & \text{if } x \in V(H). \end{cases} \tag{2.1}
\]
Definition 2.2. The $T_1$-edge corona of $G$ and $H$, denoted by $T_1(G) \ominus H$ is the graph obtained from one copy of $T_1(G)$ and $m_G$ copies of $H$ and joining a vertex of $E(G)$, that is on $ith$ position in $T_1(G)$ to every vertex in the $ith$ copy of $H$.

The graph $T_1(G) \ominus H$ has $n_G + m_G + m_G n_H$ number of vertices and $m_G + \frac{1}{2} M_1(G) + m_G m_H + m_G n_H$ number of edges. The degree of a vertex $x \in V(T_1(G) \ominus H)$ is given by

$$dt_{T_1(G) \ominus H}(x) = \begin{cases} 
  d_G(x) & \text{if } x \in V(G), \\
  d_G(x) + 2 + n_H & \text{if } x \in E(G) \\
  d_H(x) + 1 & \text{if } x \in V(H). 
\end{cases} \tag{2.2}$$

Definition 2.3. The $T_1$-vertex neighborhood corona of $G$ and $H$, denoted by $T_1(G) \square H$, is the graph obtained from one copy of $T_1(G)$ and $n_G$ copies of $H$ and joining the neighbors of a vertex in $T_1(G)$ corresponding to a vertex of $G$, that is on the $ith$ position in $T_1(G)$ to every vertex in the $ith$ copy of $H$.

The graph $T_1(G) \square H$ has $n_G + m_G + n_G n_H$ vertices and $m_G + \frac{1}{2} M_1(G) + n_G m_H + 2 m_G n_H$ edges. The degree of vertices of $T_1(G) \square H$ is given by:

$$\begin{align*}
  dt_{T_1(G) \square H}(u) &= d_G(u) \text{ if } u \in V(G), \\
  dt_{T_1(G) \square H}(e) &= d_G(e) + 2 + 2n_H \text{ if } e \in E(G), \\
  dt_{T_1(G) \square H}(u') &= d_H(u') + d_G(u) \text{ if } u' \in V(H), u \in V(G). \tag{2.3}
\end{align*}$$

In the last expression, $u' \in V(H)$ is the vertex in $ith$ copy of $H$ corresponding to $ith$ vertex $u \in V(G)$ in $T_1(G)$.

Definition 2.4. The $T_1$-edge neighborhood corona of $G$ and $H$, denoted by $T_1(G) \triangleright H$, is the graph obtained from one copy of $T_1(G)$ and $m_G$ copies of $H$ and joining the neighbors of a vertex in $T_1(G)$ corresponding to an edge of $G$, that is on the $ith$ position in $T_1(G)$ to every vertex in the $ith$ copy of $H$.

The graph $T_1(G) \triangleright H$ has $n_G + m_G + m_G n_H$ vertices and $m_G + M_1(G)[n_H + \frac{1}{2}] + m_G m_H$ edges. The degree of vertices of $T_1(G) \triangleright H$ is given by:

$$\begin{align*}
  dt_{T_1(G) \triangleright H}(u) &= d_G(u)(1 + n_H) \text{ if } u \in V(G), \\
  dt_{T_1(G) \triangleright H}(e) &= d_G(e)(1 + n_H) + 2 \text{ if } e \in E(G) \\
  dt_{T_1(G) \triangleright H}(v) &= d_H(v) + d_G(e') + 2 \text{ if } v \in V(H), e' \in E(G). \tag{2.4}
\end{align*}$$

3. First Zagreb index of $T_1$-corona composite graphs

Theorem 3.1. Let $G$ and $H$ be two connected simple graphs. Then

$$M_1(T_1(G) \odot H) = 5M_1(G) + EM_1(G) + n_G M_1(H) + n_G (n_H^2 + n_H + 4m_H) + 4m_G (n_H - 1).$$
Theorem 3.2. Let $G$ and $H$ be two connected simple graphs. Then

$$M_1(T_1(G) ⊙ H) = (2n_H + 5)M_1(G) + EM_1(G) + m_G M_1(H) + m_G(n_H - 2)(n_H + 2) + m_G(n_H + 4m_H).$$
Proof. Using (2.2) in equation (1.1), we get

\[
M_1(T_1(G) \ominus H) = \sum_{u \in V(G)} d_G(u)^2 + \sum_{e \in E(G)} [d_G(e) + (n_H + 2)]^2 + m_G \sum_{u' \in V(H)} (d_H(u') + 1)^2
\]

\[
= M_1(G) + EM_1(G) + (n_H + 2)^2m_G + 2(n_H + 2) \left[ 2 \left( -m_G + \frac{1}{2} M_1(G) \right) \right]
+ m_G \{ M_1(H) + n_H + 4m_H \}
\]

\[
= M_1(G) + EM_1(G) + (n_H + 2)^2m_G - 4(n_H + 2)m_G + 2(n_H + 2)M_1(G)
+ m_G \{ M_1(H) + n_H + 4m_H \}
\]

\[
= (2n_H + 5)M_1(G) + EM_1(G) + m_GM_1(H) + m_G(n_H - 2)(n_H + 2)
+ m_G(n_H + 4m_H).
\]

\[ \square \]

Theorem 3.3. Let G and H be two connected simple graphs. Then

\[
M_1(T_1(G) \Box H) = 5(n_H + 1)M_1(G) + EM_1(G) + n_GM_1(H) + 4m_G(n_H + 1)^2
+ 8m_G(m_H - n_H - 1).
\]

Proof. Using (2.3) in equation (1.1), we get

\[
M_1(T_1(G) \Box H) = \sum_{u \in V(G)} d_G(u)^2 + \sum_{e \in E(G)} [d_G(e) + 2(n_H + 1)]^2 + \sum_{u \in V(G), u' \in V(H)} [d_G(u) + d_H(u')]^2
\]

\[
= M_1(G) + EM_1(G) + 4m_G(n_H + 1)^2 + 4(n_H + 1) \left[ 2 \left( -m_G + \frac{1}{2} M_1(G) \right) \right]
+ n_H \sum_{u \in V(G)} d_G(u)^2 + n_G \sum_{u' \in V(H)} d_H(u')^2 + 2 \sum_{u \in V(G)} d_G(u) \cdot \sum_{u' \in V(H)} d_H(u')
\]

\[
= M_1(G) + EM_1(G) + 4m_G(n_H + 1)^2 - 8m_G(n_H + 1) + 4(n_H + 1)M_1(G)
+ n_HM_1(G) + n_GM_1(H) + 8m_Gm_H
\]

\[
= 5(n_H + 1)M_1(G) + EM_1(G) + n_GM_1(H) + 4m_G(n_H + 1)^2
+ 8m_G(m_H - n_H - 1).
\]

\[ \square \]

Theorem 3.4. Let G and H be two connected simple graphs. Then

\[
M_1(T_1(G) \oplus H) = [(n_H + 1)(n_H + 5) + 4(n_H + m_H)]M_1(G) + [(n_H + 1)^2 + n_H]EM_1(G)
+ m_GM_1(H) - 4m_G(3n_H + 1).
\]
Proof. Using (2.4) in equation (1.1), we get

\[ M_1(T_1(G) \Box H) = \sum_{u \in V(G)} [(n_H + 1)d_G(u)]^2 + \sum_{e \in E(G)} [(n_H + 1)d_G(e) + 2]^2 \]
\[ + \sum_{e \in E(G); u' \in V(H)} [d_G(e) + d_H(u') + 2]^2 \]
\[ = (n_H + 1)^2 M_1(G) + (n_H + 1)^2 EM_1(G) + 4m_G + 4(n_H + 1) \left[ 2 \left( -m_G + \frac{1}{2} M_1(G) \right) \right] \]
\[ + m_G M_1(H) + n_H EM_1(G) + 8m_G m_H - 8m_G n_H + 4n_H M_1(G) \]
\[ - 8m_G m_H + 4m_H M_1(G) + 4n_H m_G \]
\[ = [(n_H + 1)(n_H + 5) + 4(n_H + m_H)] M_1(G) + [(n_H + 1)^2 + n_H] EM_1(G) \]
\[ + m_G M_1(H) - 4m_G (3n_H + 1). \]

\[ \square \]

4. Second Zagreb index of \( T_1 \)-Corona composite graphs

Theorem 4.1. Let \( G \) and \( H \) be two connected simple graphs. Then

\[ M_2(T_1(G) \circ H) = 2(n_H + 1) M_1(G) + 2M_2(G) + 2EM_1(G) + EM_2(G) + F(G) \]
\[ + n_G \{ M_1(H) + M_2(H) \} + n_G n_H (n_H + 2m_H) + m_H (n_G + 4m_G) + m_G (2n_H - 4). \]

Proof. Using (2.1) in equation (1.2), we get

\[ M_2(T_1(G) \circ H) = \sum_{a \sim b; a, b \in E(G)} [(d_G(a) + 2)(d_G(b) + 2)] + n_G \sum_{u' \in V(H)} [(d_H(u') + 1)(d_H(v') + 1)] \]
\[ + \sum_{u \sim e} [(d_G(u) + n_H)(d_G(e) + 2)] + \sum_{u \sim u'} [(d_G(u) + n_H)(d_H(u') + 1)] \]
\[ = EM_2(G) + 2EM_1(G) + 4 \left( -m_G + \frac{1}{2} M_1(G) \right) + n_G [M_2(H) + M_1(H) + m_H] \]
\[ + \sum_{uv \in E(G)} [d_G(u) + d_G(v) + 2n_H][d_G(u) + d_G(v)] \]
\[ + \sum_{u \in V(G)} (d_G(u) + n_H) \cdot \sum_{u' \in V(H)} (d_H(u') + 1) \]
\[ = EM_2(G) + 2EM_1(G) - 4m_G + 2M_1(G) + n_G [M_2(H) + M_1(H) + m_H] \]
\[ + F(G) + 2M_2(G) - 2M_1(G) - 4n_H m_G + 2n_H M_1(G) + 2M_1(G) + 4n_H m_G \]
\[ + 4m_G m_H + 2n_H m_G + 2n_G n_H m_H + n_H^2 m_G \]
\[ = 2(n_H + 1) M_1(G) + 2M_2(G) + 2EM_1(G) + EM_2(G) + F(G) \]
\[ + n_G \{ M_1(H) + M_2(H) \} + n_G n_H (n_H + 2m_H) + m_H (n_G + 4m_G) + m_G (2n_H - 4). \]

\[ \square \]
Theorem 4.2. Let $G$ and $H$ be two connected simple graphs. Then

\[
M_2(T_1(G) \oplus H) = \left[ \frac{(n_H + 2)^2}{2} + 2(n_H + m_H) \right] M_1(G) + 2M_2(G) + F(G) \\
+ m_G \left\{ M_1(H) + M_2(H) \right\} + (n_H + 2)EM_1(G) + EM_2(G) \\
+ m_G [n_H^2 - (n_H + 2)^2] + m_G m_H (2n_H + 1).
\]

Proof. Using (2.2) in equation (1.2), we get

\[
M_2(T_1(G) \oplus H) = \sum_{a \sim b : a, b \in E(G)} [(d_G(a) + n_H + 2)(d_G(b) + n_H + 2)] \\
+ m_G \sum_{u'v' \in E(H)} [(d_H(u') + 1)(d_H(v') + 1)] \\
+ \sum_{u \sim e} [(d_G(u))(d_G(e) + n_H + 2)] + \sum_{u' \sim e} [(d_H(u') + 1)(d_H(e) + n_H + 2)] \\
= EM_2(G) + (n_H + 2)EM_1(G) + (n_H + 2)^2 \left( -m_G + \frac{1}{2} M_1(G) \right) \\
+ m_G \left\{ M_1(H) + M_2(H) + m_H \right\} + \sum_{u \in E(G)} [d_G(u) + d_G(v)][d_G(u) + d_G(v) + n_H] \\
+ \sum_{u' \in V(H)} (d_H(u') + 1) \cdot \sum_{e \in E(G)} (d_G(e) + n_H + 2) \\
= EM_2(G) + (n_H + 2)EM_1(G) - (n_H + 2)^2 m_G + \frac{(n_H + 2)^2}{2} M_1(G) \\
+ m_G \left\{ M_1(H) + M_2(H) + m_H \right\} + F(G) + 2M_2(G) + n_H M_1(G) \\
+ (2m_H + n_H) \left[ 2 \left( -m_G + \frac{1}{2} M_1(G) \right) + (n_H + 2)m_G \right] \\
= \left[ \frac{(n_H + 2)^2}{2} + 2(n_H + m_H) \right] M_1(G) + 2M_2(G) + F(G) + m_G \left\{ M_1(H) + M_2(H) \right\} \\
+ (2 + n_H)EM_1(G) + EM_2(G) + m_G [n_H^2 - (n_H + 2)^2] + m_G m_H (2n_H + 1).
\]

\[\square\]

Theorem 4.3. Let $G$ and $H$ be two connected simple graphs. Then

\[
M_2(T_1(G) \boxplus H) = 2(n_H + 1)^2 + 6m_H + 5m_H [M_1(G) + 6M_2(G) + 3F(G) + 2(n_H + 1)EM_1(G) \\
+ EM_2(G) + 2m_G M_1(H) + n_G M_2(H) + 4m_G \{2n_H m_H - (n_H + 1)^2\}.
\]

Proof. Using (2.3) in equation (1.2), we get

\[
M_2(T_1(G) \boxplus H) = \sum_{a \sim b : a, b \in E(G)} [d_G(a) + 2(n_H + 1)][d_G(b) + 2(n_H + 1)] \\
+ \sum_{u' \sim e} [(d_H(u') + d_G(u))(d_G(e) + 2 + 2n_H)] + \sum_{u \sim e} [(d_G(u))(d_G(e) + 2 + 2n_H)] \\
+ \sum_{u \in V(G) \; u'v' \in V(H)} [d_H(u') + d_G(u)][d_H(v') + d_G(u)]
\]
Let $u'$ and $v'$ be the vertices of $H$ corresponding to a vertex $u$ of $G$.

$$
= EM_2(G) + 2(n_H + 1)EM_1(G) + 4(n_H + 1)^2 \left(-m_G + \frac{1}{2}M_1(G)\right)
$$

$$
+ 2 \sum_{u' \in V(H)} d_H(u') \sum_{e \in E(G)} [d_G(e) + 2(n_H + 1)] + 3 \sum_{u \in V(G)} [d_G(u) + d_G(v)][d_G(u) + d_G(v) + 2n_H]
$$

$$
+ m_H \sum_{u \in V(G)} d_G(u)^2 + \sum_{u \in V(G)} d_G(u) \sum_{u' \in V(H)} [d_H(u') + d_H(v')] + n_G \sum_{u' \in V(H)} [d_H(u') \cdot d_H(v')]
$$

$$
= EM_2(G) + 2(n_H + 1)EM_1(G) - 4m_G(n_H + 1)^2 + 2(n_H + 1)^2 M_1(G)
$$

$$
+ 4m_H \{M_1(G) + 2n_H M_1(G)\} + 3\{F(G) + 2M_2(G) + 2n_H M_1(G)\}
$$

$$
+ m_H M_1(G) + 2m_G M_1(H) + n_G M_2(H)
$$

$$
= [2(n_H + 1)^2 + 6n_H + 5m_H]M_1(G) + 6M_2(G) + 3F(G) + 2(n_H + 1)EM_1(G)
$$

$$
+ EM_2(G) + 2m_G M_1(H) + n_G M_2(H) + 4m_G \{2n_H m_H - (n_H + 1)^2\}.
$$

\[\Box\]

**Theorem 4.4.** Let $G$ and $H$ be two connected simple graphs. Then

$$
M_2(T_1(G) \boxdot H) = 2[n_H (m_H - n_H + 1) + 3(m_H + 1)M_1(G) + 2(n_H + 1)(2n_H + 1)M_2(G)
$$

$$
+ (n_H + 1)(2n_H + 1) F(G) + [2(n_H + 1)(n_H + m_H + 1) + (2n_H + 1)] EM_1(G)
$$

$$
+ (n_H + 1)(3n_H + 1) EM_2(G) + M_1(G) M_1(H) + m_G M_2(H)
$$

$$
- 8m_G (n_H + m_H + 1).
$$

**Proof.** Using (2.4) in equation (1.2), we get

$$
M_2(T_1(G) \boxdot H) = \sum_{a \sim b ; a, b \in E(G)} [d_G(a)(n_H + 1) + 2][d_G(b)(n_H + 1) + 2]
$$

$$
+ \sum_{u \sim e} [d_G(u)(n_H + 1)][d_G(e)(n_H + 1) + 2]
$$

$$
+ \sum_{e^2 \in (G)} \sum_{u' \in V(H)} [d_H(u') + d_G(e) + 2][d_H(v') + d_G(e) + 2]
$$

$$
+ \sum_{e \in E(G)} \sum_{u \sim u'} [d_G(u)(n_H + 1)][d_H(u') + d_G(e) + 2]
$$

$$
+ \sum_{e \sim u' \text{ and } u' \text{ is a vertex corresponding to copy } f \sim f} [d_G(e)(n_H + 1) + 2][d_H(u') + d_G(f) + 2]
$$

$$
= (n_H + 1)^2 EM_2(G) + 2(n_H + 1) EM_1(G) + 4 \left(-m_G + \frac{1}{2}M_1(G)\right)
$$

$$
+ (n_H + 1)^2 \sum_{u \in E(G)} [d_G(u) + d_G(v)]^2 - 2n_H (n_H + 1) \sum_{u \in E(G)} [d_G(u) + d_G(v)]
$$

$$
+ m_G \sum_{u' \sim v' \in E(H)} d_H(u')d_H(v') + \sum_{u' \sim v' \in E(H)} [d_H(u') + d_H(v')] \sum_{e \in E(G)} [d_G(e) + 2] + \sum_{e \in E(G)} [d_G(e) + 2]^2
$$
\begin{align*}
  &+ (n_H + 1) \sum_{u' \in V(H)} d_H(u') \sum_{u \in V(G)} d_G(u)^2 + n_H(n_H + 1) \sum_{u \in V(G)} (d_G(u) + d_G(v))^2 \\
  &+ (n_H + 1) \sum_{u' \in V(H)} d_H(u') \sum_{e \in E(G)} d_G(e)^2 + 2 \sum_{u' \in V(H)} d_H(u') \sum_{e \in E(G)} d_G(e) \\
  &+ n_H(n_H + 1) \left[ 2 \sum_{e \sim f} [d_G(e)d_G(f)] + 2 \sum_{e \in E(G)} d_G(e)^2 \right] + 2n_H \sum_{e \in E(G)} d_G(e)(d_G(e) + 2) \\
  &= (n_H + 1)^2 EM_2(G) + 2(n_H + 1) EM_1(G) + 2M_1(G) - 4m_G \\
  &+ (n_H + 1) \{ (n_H + 1) [F(G) + 2M_2(G)] - 2n_H M_1(G) \} \\
  &+ M_2(M_2(H) + M_1(G)M_1(H) + EM_1(G) + 4m_G - 8m_G + 4M_1(G) \\
  &+ 2m_H(n_H + 1)M_1(G) + n_H(n_H + 1) \{ F(G) + 2M_2(G) \} \\
  &+ 2m_H(n_H + 1)EM_1(G) + 4m_H \left[ 2 \left( -m_G + \frac{1}{2} M_1(G) \right) \right] \\
  &+ n_H(n_H + 1) \{ 2EM_2(G) + 2EM_1(G) \} + 2n_H \left[ EM_1(G) + 4 \left( -m_G + \frac{1}{2} M_1(G) \right) \right] \\
  &= (n_H + 1)^2 EM_2(G) + 2(n_H + 1) EM_1(G) + 2M_1(G) - 4m_G \\
  &+ (n_H + 1)^2 F(G) + 2(n_H + 1)^2 M_2(G) - 2n_H(n_H + 1)M_1(G) \\
  &+ M_2(M_2(H) + M_1(G)M_1(H) + EM_1(G) - 4m_G + 4M_1(G) \\
  &+ 2(n_H + 1)m_H M_1(G) + n_H(n_H + 1) F(G) + 2n_H(n_H + 1)M_2(G) + 4(n_H + m_H)M_1(G) \\
  &+ [2m_H(n_H + 1) + 2n_H(n_H + 2)] EM_1(G) + 2n_H(n_H + 1)EM_2(G) - 8m_G(n_H + m_H). \\
\end{align*}

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\section*{References}


1 DEPARTMENT OF MATHEMATICS, SHRI GAVISIDDESHWAR ARTS, SCIENCE AND COMMERCE COLLEGE, KOPPAL-583231, INDIA.
   Email address: manjugalijack@gmail.com

2 DEPARTMENT OF MATHEMATICS, DAVANGERE UNIVERSITY, SHIVAGANGOTHRI., DAVANGERE-577007, INDIA.
   Email address: prakashadg@gmail.com

3 DEPARTMENT OF MATHEMATICS, DAVANGERE UNIVERSITY, SHIVAGANGOTHRI., DAVANGERE-577007, INDIA.
   Email address: chetanagali19@gmail.com