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FIXED POINT THEOREMS FOR FUZZY CONTRACTIONS MAPPINGS IN A DISLOCATED B-METRIC SPACES WITH APPLICATIONS

MUHAMMED RAJI[1*] AND MUSA ADEKU IBRAHIM[2]

Abstract. The purpose of this article, is to establish some fixed point results for fuzzy mappings in a complete dislocated b-metric space. Examples are provided to support results and concepts presented herein. As an application of our results, multivalued mappings and fuzzy mappings are studied.

1. Introduction and preliminaries

Banach's fixed point theorems for contraction mappings is one of the pivotal results of mathematical analysis. Banach contraction principle [5] is one of the most notable results which has played a vital role in the development of a metric fixed point theory. This principle and its variants provide a useful apparatus in guaranteeing the existence and uniqueness of solutions of various nonlinear problems: differential equations, integral equations, optimization problems, variational inequalities. A host of this principle has been made modified and extended by several mathematicians in different perspectives, some of them are as follows: Czerwik [10] had an excellent idea to introduced the concepts of b-metric spaces by weakening the coefficient of the triangle inequality and generalized Banach's Contraction Principle. Sub- sequently, Boriceanu [7] obtained some concrete examples of b-metric spaces and studied the fixed-point properties of set-valued operators in b-metric spaces. Hussain et al. [13] introduced the new concept of dislocated b-metric space as a generalization of metric space and established to prove some common fixed point results for four mappings satisfying the generalized weak contractive conditions in a partially ordered dislocated b-metric space. Zadeh [23] introduced the idea of fuzzy sets. Later on, Weiss [22] gave the idea of a fuzzy mapping and obtained many fixed point results. Afterward, Heilpern [11] initiated the idea of fuzzy contraction mappings and proved a fixed point theorem for fuzzy contraction mappings. Shahzad et al. [19] introduced the concept of F-contraction to obtain some common fixed point results for fuzzy mappings satisfying a new Ciric type rational F-contraction in the context of complete dislocated metric spaces.

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* Corresponding author.
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In view of the above considerations, we prove some fixed point results for fuzzy mappings in a complete dislocated b-metric space. We also give illustrative examples to furnish the highlighted realized improvements. As an application of our results, multivalued mappings and fuzzy mappings are studied.
Throughout the article, we denoted by R is the set of all real numbers, by R^+ is the set of all positive real numbers and by N is the set of natural numbers.

In the sequel, we shall present the notion of some relevant definitions.

Definition 1.1. [10] Let X be non-empty set and $s \geq 1$ be a given real number. If a function $d : X \times X \to R^+$ satisfies the following condition
(b_1) $d(x, y) = 0$ iff x=y
(b_2) $d(x, y) = d(y, x)$
(b_3) $d(x, z) \leq s[d(x, y) + d(y, z)]$ for all $x, y \in X$,
then the pair (X, d) is called a b-metric space.

Definition 1.2. [18] Let X be a nonempty set. A function $d : X \times X \to R^+$. A pair (X, d) is called a distance space. If d satisfies the following conditions:
i. $d(x, y) = 0$ if $x = y$;
ii. $d(x, y) = d(y, x)$;
iii. $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.
Then, a function $d : X \times X \to R^+$ is called a dislocated metric on X. If d is a dislocated metric on X, then the pair (X, d) is said to be a dislocated metric space.

Definition 1.3. [11, 17] A fuzzy set in X is a function with domain X and value in $[0, 1]$, $F(X)$ is the collection of all fuzzy sets in X. If A is a fuzzy set and $x \in X$, then the function value $A(x)$ is called the grade of membership of x in A. The α-level set of fuzzy set A, is denoted by $[A]_\alpha$, and defined as:

$$[A]_\alpha = \{x : A(x) \geq \alpha\} \ \ where\ \alpha \in (0, 1], [A]_0 = \overline{\{x : A(x) > 0\}}.$$

Definition 1.4. [8, 4] Let X be any nonempty set and Y be a metric space. A mapping T is called a fuzzy mapping, if T is a mapping from X into $F(Y)$. A fuzzy mapping T is a fuzzy subset on $X \times Y$ with membership function $T(x)(y)$. The function $T(x)(y)$ is the grade of membership of y in $T(x)$. For convenience, we denote the α-level set of $T(x)$ by $[Tx]_\alpha$ instead of $[T(x)]_\alpha$.

Definition 1.5. [20, 15] A point $x \in X$ is called a fuzzy fixed point of a fuzzy mapping $T : X \to F(X)$ if there exists $\alpha \in (0, 1]$ such that $x \in [Tx]_\alpha$.

Definition 1.6. [12, 6] Let X be a nonempty set. A function $d_{lb} : X \times X \to [0, \infty)$ is called dislocated b-metric (or simply d_{lb}-metric) if, for any $x, y, z \in X$ and $b \geq 1$, the following conditions hold:
i. If $d_{lb}(x, y) = 0$, then $x = y$;
ii. $d_{lb}(x, y) = d_{lb}(y, x)$;
iii. $d_{lb}(x, y) \leq b[d_{lb}(x, z) + d_{lb}(z, y)]$.
The pair (X, d_{lb}) is called a dislocated b-metric space. It should be noted that the class of d_{lb} metric spaces is effectively larger than that of d_l metric spaces, since d_{lb} is a d_l metric when $b = 1$.

It is clear that if $d_{lb}(x,y)=0$, then from (i), $x=y$. But if $x=y$, (X,d_{lb}) may not be 0.

Example 1.7. [21] If $X=R^{+}\cup\{0\}$, then $d_{lb}(x,y)=(x+y)^2$ defines a dislocated b-metric d_{lb} on X.

Definition 1.8. [2] Let (X,d_{lb}) be a dislocated b-metric space.
i. A sequence $\{x_n\}$ in (X,d_{lb}) is called Cauchy sequence if, given $\epsilon>0$, there corresponds $n_0\in N$ such that, for all $n,m\geq n_0$, we have $d_{lb}(x_m,x_n)<\epsilon$ or $\lim_{n,m\to\infty} d_{lb}(x_m,x_n)$
ii. A sequence $\{x_n\}$ dislocated b-converges (for short d_{lb} -converges) to x if $\lim_{n\to\infty} d_{lb}(x_n,n)=0$. In this case x is called a d_{lb} limit of $\{x_n\}$.

Definition 1.9. [21] Let K be a nonempty subset of dislocated b-metric space X, and let $x\in X$. An element $y_0\in K$ is called a best approximation in K if

$$d_{lb}(x,K)=d_{lb}(x,y_0),$$

where

$$d_{lb}(x,K)=\inf_{y\in k} y\in K\ d_{lb}(x,y)$$

. If each $x\in X$ has at least one best approximation in K, then K is called a proximinal set. Denote by $P(X)$ the set of all proximinal subsets of X.

Definition 1.10. [21] The function $H_{d_{lb}}:P(X)\times P(X)\to R^{+}$, defined by

$$H_{d_{lb}}(A,B)=max\{\sup_{a\in A} d_{lb}(a,B),\sup_{b\in B} d_{lb}(A,b)\},$$

is called dislocated Hausdorff b-metric on $P(X)$.

Lemma 1.11. [21] *Let A and B be nonempty proximal subsets of a dislocated b-metric space (X,d_{lb}). If $a\in A$, then*

$$d(a,B)\leq H(A,B).$$

Lemma 1.12. [21] *Let (X,d_{lb}) be a dislocated metric space. Let $(P(X),H_{d_{lb}})$ be a dislocated Hausdorff b-metric space. Then, for all $A,B\in P(X)$ and for each $a\in A$, there exists $b_a\in B$ satisfying*

$$d_{lb}(a,B)=d_{lb}(a,b_a),$$

then

$$H_{d_{lb}}(A,B)\geq d_{lb}(a,b_a).$$

Definition 1.13. [16, 14] Let (X,d) be a complete metric space and $S,T:X\to X$ be two given mappings. Then we say that $x\in X$ is a common fixed point of S and T if $x=Sx=Tx$.

2. Main results

In this section, we begin with the following Theorem.

Theorem 2.1. *Let (X, d_{lb}) be a complete dislocated b-metric space with constant $b \geq 1$. Let $T : X \to F(x)$ be a fuzzy mapping. Suppose there exists $\alpha(x) \in (0, 1]$ satisfying the following conditions:*

$$\begin{aligned} H_{d_{lb}}([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) &\leq a_1 d_{lb}(x, y) + a_2 d_{lb}(x, [Tx]_{\alpha(x)}) + a_3 d_{lb}(y, [Ty]_{\alpha(y)}) + \\ a_4 d_{lb}(x, [Ty]_{\alpha(y)}) + a_5 d_{lb}(y, [Tx]_{\alpha(x)}) &+ a_6 \frac{d_{lb}(x, [Tx]_{\alpha(x)}) d_{lb}(y, [Ty]_{\alpha(y)})}{d_{lb}(x, y) + d_{lb}(x, [Ty]_{\alpha(y)}) + d_{lb}(y, [Tx]_{\alpha(x)})} \\ &+ a_7 \frac{d_{lb}(x, [Tx]_{\alpha(x)}) d_{lb}(x, [Ty]_{\alpha(y)}) + d_{lb}(y, [Tx]_{\alpha(x)}) d_{lb}(y, [Ty]_{\alpha(y)})}{d_{lb}(x, [Ty]_{\alpha(y)}) + d_{lb}(y, [Tx]_{\alpha(x)})} \end{aligned} \tag{2.1}$$

for all $x, y \in X$,$x \neq y$ and $a_1, a_2, a_3, a_4, a_5, a_6, a_7 \geq 0$ with $b(a_1 + a_2) + a_3 + b(1 + b)a_4 + b(a_6 + a_7) < 1$ and $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 < 1$.Then there exists $x^* \in X$ such that $x^* \in [Tx^*]_{\alpha(x^*)}$.

Proof. Let $x_0 \in X$ be any arbitary point in X. Let $x_1 \in [Tx_0]_{\alpha(x_0)}$ be an element such that $d_{lb}(x_0, [Tx_0]_{\alpha(x_0)}) = d_{lb}(x_0, x_1)$ Again let $x_2 \in [Tx_1]_{\alpha(x_1)}$ be an element such that $d_{lb}(x_1, [Tx_1]_{\alpha(x_1)}) = d_{lb}(x_1, x_2)$. Continuing in this process,we construct a sequence x_n of points in X such that $x_n \in [Tx_{n-1}]_{\alpha(x_{n-1})}$, for all $n \in N$. Now by using (2.1) and Lemma 1.12,we get

$$\begin{aligned} d_{lb}(x_n, x_{n+1}) &\leq H_{d_{lb}}([Tx_{n-1}]_{\alpha(x_{n-1})}, [Tx_n]_{\alpha(x_n)}) \leq a_1\ d_{lb}(x_{n-1}, x_n) + \\ a_2 d_{lb}(x_{n-1}, [Tx_{n-1}]_{\alpha(x_{n-1})}) &+ a_3 d_{lb}(x_n, [Tx_n]_{\alpha(x_n)}) + a_4 d_{lb}(x_{n-1}, [Tx_n]_{\alpha(x_n)}) + \\ a_5 d_{lb}(x_n, [Tx_{n-1}]_{\alpha(x_{n-1})}) &+ a_6 \frac{d_{lb}(x_{n-1}, [Tx_{n-1}]_{\alpha(x_{n-1})}) d_{lb}(x_n, [Tx_n]_{\alpha(x_n)})}{d_{lb}(x_{n-1}, x_n) + d_{lb}(x_{n-1}, [Tx_n]_{\alpha(x_n)}) + d_{lb}(x_n, [Tx_{n-1}]_{\alpha(x_{n-1})})} \\ &+ a_7 \frac{d_{lb}(x_{n-1}, [Tx_{n-1}]_{\alpha(x_{n-1})}) d_{lb}(x_{n-1}, [Tx_n]_{\alpha(x_n)}) + d_{lb}(x_n, [Tx_{n-1}]_{\alpha(x_{n-1})}) d_{lb}(x_n, [Tx_n]_{\alpha(x_n)})}{d_{lb}(x_{n-1}, [Tx_n]_{\alpha(x_n)}) + d_{lb}(x_n, [Tx_{n-1}]_{\alpha(x_{n-1})})} \end{aligned}$$

$$\begin{aligned} &\leq a_1 d_{lb}(x_{n-1}, x_n) + a_2 d_{lb}(x_{n-1}, x_n) + a_3 d_l b(x_n, x_{n+1}) + a_4 d_{lb}(x_{n-1}, x_{n+1}) + a_5 d_{lb}(x_n, x_n) \\ &+ a_6 \frac{d_{lb}\ (x_{n-1}, x_n) d_{lb}(x_n, x_{n+1}))}{d_l b(x_{n-1}, x_n) + d_l b(x_{n-1}, x_{n+1}) + d_l b(x_n, x_n)} \\ &+ a_7 \frac{d_l b\ (x_{n-1}, x_n) d_{lb}(x_{n-1}, x_{n+1}) + d_{lb}(x_n, x_n) d_{lb}(x_n, x_{n+1}))}{d_{lb}(x_{n-1}, x_{n+1}) + d_l b(x_n, x_n)} \\ &\leq a_1 d_{lb}(x_{n-1}, x_n) + a_2 d_{lb}(x_{n-1}, x_n) + a_3 d_{lb}(x_n, x_{n+1}) + b a_4 [d_{lb}(x_{n-1}, x_n) + d_{lb}(x_n, x_{n+1})] \\ &+ a_6 d_{lb}(x_{n-1}, x_n) + a_7 d_{lb}(x_{n-1}, x_n) \end{aligned}$$

$$d_{lb}(x_n, x_{n+1}) \leq \frac{a_1 + a_2 + b a_4 + a_6 + a_7}{1 - (a_3 + b a_4)} d_{lb}(x_{n-1}, x_n). \tag{2.2}$$

Let

$$\lambda = \frac{a_1 + a_2 + b a_4 + a_6 + a_7}{1 - (a_3 + b a_4)}.$$

Since

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 < 1$$

implies that

$$\frac{a_1 + a_2 + ba_4 + a_6 + a_7}{1 - (a_3 + ba_4)} < 1.$$

Hence,

$$d_{lb}(x_n, x_{n+1}) \leq \lambda\, d_{lb}(x_{n-1}, x_n) \;\; for \;\; all \;\; n \in N. \tag{2.3}$$

Then, by repeated application of (2.3), we have

$$d_{lb}(x_n, x_{n+1}) \leq \lambda\, d_{lb}\,(x_{n-1}, x_n) \leq \lambda^2 d_{lb}(x_n, x_{n+1}) \leq \cdots \leq \lambda^{n+1} d_{lb}(x_0, x_1) \tag{2.4}$$

for all $n \in N$ and for any $n > m$ where $n, m \in N$, we have, by using condition (iii) of definition 1.6

$$\begin{aligned} d_{lb}(x_m, x_n) &\leq b(d_{lb}(x_m, x_{m+1})) + b^2(d_{lb}(x_{m+1}, x_{m+2})) + \cdots + b^{n-m}(d_{lb}(x_{n-1}, x_n)) \\ &\leq b\lambda^m d_{lb}(x_0, x_1) + b^2 \lambda^{m+1} d_{lb}(x_0, x_1) + \cdots + b^{n-m} \lambda^{n-1} d_{lb}(x_0, x_1) \; by \; (2.4) \\ &\leq b\lambda^m [1 + b\lambda + \cdots + b^{n-m-1} \lambda^{n-m-1}] d_{lb}(x_0, x_1) \\ &\leq \frac{b\lambda^m}{1 - b\lambda} d_{lb}(x_0, x_1) \to 0 \; as \; m \to \infty. \end{aligned} \tag{2.5}$$

This proves that $\{x_n\}$ is Cauchy sequence in X.
Since X is complete, there exists $x^* \in X$ such that $x_n \to x^*$ as $n \to \infty$. Now, by Lemma 1.11 and (2.1), we have

$$\begin{aligned} d_{lb}(x^*, [Tx^*]_\alpha(x^*)) &\leq b[d_{lb}(x^*, x_{n+1}) + d_{lb}(x_{n+1}, [Tx^*]_{\alpha(x^*)})] \\ &\leq b[d_{lb}(x^*, x_{n+1}) + H d_{lb}([Tx_n]_{\alpha(x_n)}, [Tx^*]_{\alpha(x^*)})] \\ &\leq b[d_{lb}(x^*, x_{n+1}) + a_1 d_{lb}(x_n, x^*) + a_2 d_{lb}(x_n, [Tx_n]_{\alpha(x_n)}) + \\ &\quad a_3 d_{lb}(x^*, [Tx^*]_{\alpha(x^*)}) + a_4 d_{lb}(x_n, [Tx^*]_{\alpha(x^*)}) \\ &+ a_5 d_{lb}(x^*, [Tx_n]_{\alpha(x_n)}) + a_6 \frac{d_{lb}(x_n, [Tx_n]_{\alpha(x_n)}) d_{lb}(x^*, [Tx^*]_{\alpha(x^*)})}{d_{lb}(x_n, x^*) + d_{lb}(x_n, [Tx^*]_{\alpha(x^*)}) + d_{lb}(x^*, [Tx_n]_{\alpha(x_n)})} \\ &+ a_7 \frac{d_{lb}(x_n, [Tx_n]_{\alpha(x_n)}) d_{lb}(x_n, [Tx^*]_{\alpha(x^*)}) + d_{lb}(x^*, [Tx_n]_{\alpha(x_n)}) d_{lb}(x^*, [Tx^*]_{\alpha(x^*)})}{d_{lb}(x_n, [Tx^*]_\alpha(x^*)) + d_{lb}(x^*, [Tx_n]_{\alpha(x_n)})} \end{aligned} \tag{2.6}$$

On taking $n \to \infty$ in (2.6), we get

$$(1 - b(a_3 + a_4)) d_{lb}(x^*, [Tx^*]_{\alpha(x^*)}) \leq 0. \tag{2.7}$$

From (2.7), we get

$$x^* \in [Tx^*]_{\alpha(x^*)}.$$

Hence, $x^* \in X$ is the fixed point. □

Example 2.2. Consider $X = Q^+ \cup \{0\}$ and $d_{lb}(x, y) = (x + y)^2$, whenever $x, y \in X$, then (X, d_{lb}) is a complete dislocated b-metric space with constant $b \geq 1$. Let $T : X \to F(X)$ be a fuzzy mapping define by

$$T(x)(t) = \begin{cases} 1 & 0 \leq t \leq \frac{x}{4} \\ \frac{1}{2} & \frac{x}{4} < t \leq \frac{x}{3} \\ \frac{1}{4} & \frac{x}{3} < t \leq \frac{x}{2} \\ 0 & \frac{x}{2} < t \leq 1 \end{cases}$$

For all $x \in X$, there exists $\alpha(x) = 1$ such that

$$[Tx]_{\alpha(x)} = [0, \frac{x}{4}].$$

Let $a_1 = \frac{1}{50}, a_2 = \frac{1}{10}, a_3 = \frac{1}{20}, a_4 = \frac{1}{30}, a_5 = \frac{1}{40}, a_6 = \frac{1}{30}, a_7 = \frac{1}{30}$. Then

$$H_{(d_{lb})}([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq \frac{1}{50}(x-y)^2 + \frac{1}{10}(x+\frac{x}{4})^2 + \frac{1}{20}(y+\frac{y}{4})^2 + \frac{1}{30}(x+\frac{y}{4})^2$$
$$+\frac{1}{40}(y+\frac{x}{4})^2 + \frac{1}{30}.\frac{\frac{1}{200}(x+\frac{x}{4})^2(y+\frac{y}{4})^2}{\frac{1}{50}(x-y)^2 + \frac{1}{30}(x+\frac{y}{4})^2 + \frac{1}{40}(y+\frac{x}{4})^2} +$$
$$\frac{1}{30}.\frac{\frac{1}{300}(x+\frac{x}{4})^2(x+\frac{y}{4})^2 + \frac{1}{800}(y+\frac{x}{4})^2(y+\frac{y}{4})^2}{\frac{1}{30}(x+\frac{y}{4})^2 + \frac{1}{40}(y+\frac{x}{4})^2}$$

Since all the conditions of Theorem 2.1 are satisfied, there exists $0 \in X$ which is the fixed point of T.

Theorem 2.3. *Let (X, d_{lb}) be a complete dislocated b-metric space with constant $b \geq 1$. Let $S, T : X \to F(X)$ be a pair of fuzzy mapping. Suppose there exist $\alpha_{S(x)}, \alpha_{T(x)} \in (0, 1]$ satisfying the following conditions:*

$$H_{d_{lb}}([TX]_{\alpha_T(x)}), [Sy]_{\alpha_s(y)}) \leq a_1 d_{lb}(x, y) + a_2 d_{lb}(x, [Tx]_{\alpha_T(x)}) + a_3 d_{lb}(y, [Sy]_{\alpha_s(y)}) +$$
$$a_4 d_{lb}(x, [Sy]_{\alpha_s(y)}) + a_5 d_{lb}(y, [Tx]_{\alpha_T(x)}) + a_6 \frac{d_{lb}(x, [Tx]_{\alpha_T(x)}) d_{lb}(y, [Sy]_{\alpha_S(y)})}{d_{lb}(x, y) + d_{lb}(x, [Sy]_{\alpha_S(y)}) + d_{lb}(y, [Tx]_{\alpha_T(x)})}$$
$$+ a_7 \frac{d_{lb}(x, [Tx]_{\alpha_T(x)}) d_{lb}(x, [Sy]_{\alpha_S(y)}) + d_{lb}(y, [Tx]_{\alpha_T(x)}) d_{lb}(y, [Sy]_{\alpha_S(y)})}{d_{lb}(x, [Sy]_{\alpha_S(y)}) + d_{lb}(y, [Tx]_{\alpha_T(x)})} \tag{2.8}$$

for all $x, y \in X$, $x \neq y$ and $a_1, a_2, a_3, a_4, a_5, a_6, a_7 \geq 0$ with $2ba_1 + (a_2 + a_3)(b + 1) + b(a_4 + a_5)(b + 1) + 2b(a_6 + a_7) < 2$ and $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 < 1$. Then there exists $x^* \in X$ such that x^* is a common fixed point of S and T.

Proof. Let $x_0 \in X$ be any arbitrary point in X. Let $x_1 \in [Tx_0]_{\alpha_T(x_0)}$ be an element such that $d_{lb}(x_0, [Tx_0]_{\alpha_T(x_0)}) = d_{lb}(x_0, x_1)$. Again let $x_2 \in [Sx_1]_{\alpha_S(x_1)}$ be an element such that $d_{lb}(x_1, [Tx_1]_{\alpha_S(x_1)}) = d_{lb}(x_1, x_2)$. Continuing this process, we construct a sequence x_n of points in X such that $x_{2n+1} \in [Tx_{2n}]_{\alpha_T(x_{2n})}$ and $x_{2n+2} \in [Sx_{2n+1}]_{\alpha_S(x_{2n+1})}$, for all $n \in N$. Now, by using (2.1) and Lemma 1.12, we get

$$d_{lb}(x_{2n+1}, x_{2n+2}) \leq H_{d_{lb}}([Tx_{2n}]_{\alpha_{T(x_{2n})}}), [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}})$$
$$\leq a_1 d_{lb}(x_{2n}, x_{2n+1}) + a_2 d_{lb}(x_{2n}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) + a_3 d_{lb}(x_{2n+1}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}}) +$$
$$a_4 d_{lb}(x_{2n}, [Sx_{2n+1})]_{\alpha_{S(x_{2n+1})}}) + a_5 d_{lb}(x_{2n+1}, [Tx_{2n}]_{\alpha_T(x_{2n})})$$
$$+a_6 \frac{d_{lb}(x_{2n}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) d_{lb}(x_{2n+1}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}})}{d_{lb}(x_{2n}, x_{2n+1}) + d_{lb}(x_{2n}, [Sx_{2n+1}]_{(\alpha_{S(x_{2n+1})})}) + d_{lb}](x_{2n+1}, [Tx_{2n}]\alpha_{T(x_{2n})})}$$
$$+a_7 [\frac{d_{lb}(x_{2n}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) d_{lb}(x_{2n}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}})}{d_{lb}(x_{2n}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}}) + d_{lb}(x_{2n+1}, [Tx]_{\alpha_T(x)})} +$$
$$\frac{d_{lb}(x_{2n+1}, [Tx_{2n}]_{\alpha_T(x_{2n})}) d_{lb}(x_{2n+1}, [Sx_{2n+1}]_{\alpha_{s(x_{2n+1})}})}{d_{lb}(x_{2n}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}}) + d_{lb}(x_{2n+1}, [Tx]_{\alpha_T(x)})}]$$

$$\leq a_1 d_{lb}(x_{2n}, x_{2n+1}) + a_2 d_{lb}(x_{2n}, x_{2n+1}) + a_3 d_{lb}(x_{2n+1}, x_{2n+2}) + a_4 d_{lb}(x_{2n}, x_{2n+2}) +$$
$$a_5 d_{lb}(x_{2n+1}, x_{2n+1}) + a_6 \frac{d_{lb}(x_{2n}, x_{2n+1}) d_{lb}(x_{2n+1}, x_{2n+2})}{d_{lb}(x_{2n}, x_{2n+1}) + d_{lb}(x_{2n}, x_{2n+2}) + d_{lb}(x_{2n+1}, x_{2n+1})}$$
$$+a_7 \frac{d_{lb}(x_{2n}, x_{2n+1}) d_{lb}(x_{2n}, x_{2n+2}) + d_{lb}(x_{2n+1}, x_{2n+1}) d_{lb}(x_{2n+1}, x_{2n+2})}{d_{lb}(x_{2n}, x_{2n+2}) + d_{lb}(x_{2n+1}, x_{2n+1})}$$
$$\leq a_1 d_{lb}(x_{2n}, x_{2n+1}) + a_2 d_{lb}(x_{2n}, x_{2n+1}) + a_3 d_{lb}(x_{2n+1}, x_{2n+2}) +$$
$$a_4 b[d_{lb}(x_{2n}, x_{2n+1}) + d_{lb}(x_{2n+1}, x_{2n+2})] + a_6 d_{lb}(x_{2n}, x_{2n+1}) + a_7 d_{lb}(x_{2n}, x_{2n+1})$$
$$d_{lb}(x_{2n+} x_{2n+2}) \leq \frac{a_1 + a_2 + ba_4 + a_6 + a_7}{1 - (a_3 + ba_4)} d_{lb}(x_{2n}, x_{2n+1}) \tag{2.9}$$

Similarly, by using condition (ii) of definition 1.6, we have

$$d_{lb}(x_{2n+2}, x_{2n+1}) \leq H_{d_{lb}})([Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}}), [Tx_{2n}]_{\alpha_{T(x_{2n})}})$$
$$\leq a_1 d_{lb}(x_{2n+1}, x_{2n}) + a_2 d_{lb}(x_{2n+1}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}}) + a_3 d_{lb}(x_{2n}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) +$$
$$a_4 d_{lb}(x_{2n+1}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) + a_5 d_{lb}(x_{2n}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}})$$
$$+a_6 \frac{d_{lb}(x_{2n+1}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}}) d_{lb}(x_{2n}, [Tx_{2n}]_{\alpha_{T(x_{2n})}})}{d_{lb}(x_{2n+1}, x_{2n}) + d_{lb}(x_{2n+1}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) + d_{lb}(x_{2n}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}})}$$
$$+a_7 [\frac{d_{lb}(x_{2n+1}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}}) d_{lb}(x_{2n+1}, [Tx_{2n}]_{\alpha_{T(x_{2n})}})}{d_{lb}(x_{2n+1}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) + d_{lb}(x_{2n}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}})} +$$
$$\frac{d_{lb}(x_{2n}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}}) d_{lb}(x_{2n}, [Tx_{2n}]_{\alpha_{T(x_{2n})}})}{d_{lb}(x_{2n+1}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) + d_{lb}(x_{2n}, [Sx_{2n+1}]_{\alpha_{S(x_{2n+1})}})}]$$
$$\leq a_1 d_{lb}(x_{2n+1}, x_{2n}) + a_2 d_{lb}(x_{2n+1}, x_{2n+2}) + a_3 d_{lb}(x_{2n}, x_{2n+1}) + a_4 d_{lb}(x_{2n+1}, x_{2n+1}) +$$
$$a_5 d_{lb}(x_{2n}, x_{2n+2}) + a_6 \frac{d_{lb}(x_{2n+1}, x_{2n+2}) d_{lb}(x_{2n}, x_{2n+1})}{d_{lb}(x_{2n+1}, x_{2n}) + d_{lb}(x_{2n+1}, x_{2n+1}) + d_{lb}(x_{2n}, x_{2n+2})}$$
$$+a_7 \frac{d_{lb}(x_{2n+1}, x_{2n+2}) d_{lb}(x_{2n+1}, x_{2n+1}) + d_{lb}(x_{2n}, x_{2n+2}) d_{lb}(x_{2n}, x_{2n+1})}{d_{lb}(x_{2n+1}, x_{2n+1}) + d_{lb}(x_{2n}, x_{2n+2})}$$
$$\leq a_1 d_{lb}(x_{2n+1}, x_{2n}) + a_2 d_{lb}(x_{2n+1}, x_{2n+2}) + a_3 d_{lb}(x_{2n}, x_{2n+1}) +$$
$$a_5 b[d_{lb}(x_{2n}, x_{2n+1}) + d_{lb}(x_{2n+1}, x_{2n+2})] + a_6 d_{lb}(x_{2n}, x_{2n+1}) + a_7 d_{lb}(x_{2n}, x_{2n+1})$$
$$d_{lb}(x_{2n+2}, x_{2n+1}) \leq \frac{a_1 + a_3 + ba_5 + a_6 + a_7}{1 - (a_2 + ba_5)} d_{lb}(x_{2n}, x_{2n+1}) \tag{2.10}$$

Adding (2.9) and (2.10), we get

$$d_{lb}(x_{2n+1}, x_{2n+2}) \leq \frac{2a_1 + a_2 + a_3 + ba_4 + ba_5 + 2a_6 + 2a_7}{2 - (a_2 + a_3 + ba_4 + ba_5)} d_{lb}(x_{2n}, x_{2n+}) \tag{2.11}$$

Let

$$\mu = \frac{2a_1 + a_2 + a_3 + ba_4 + ba_5 + 2a_6 + 2a_7}{2 - (a_2 + a_3 + ba_4 + ba_5)}.$$

Since

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 < 1, \frac{2a_1 + a_2 + a_3 + ba_4 + ba_5 + 2a_6 + 2a_7}{2 - (a_2 + a_3 + ba_4 + ba_5)} < 1.$$

Then, by (2.11), we get

$$d_{lb}(x_{2n+1}, x_{2n+2}) \leq \mu d_{lb}(x_{2n}, x_{2n+1}) \tag{2.12}$$

for all $n \in N$. Repeated application of (2.12), we have

$$d_{lb}(x_{2n+1}, x_{2n+2}) \leq \mu d_{lb}(x_{2n}, x_{2n+1}) \leq \mu^2 d_{lb}(x_{2n-1}, x_{2n}) \leq \cdots \leq \mu^{n+1} d_{lb}(x_0, x_1) \tag{2.13}$$

for all $n \in N$. Again, for any $n > m$ where $n, m \in N$, we have, by using condition (iii) of definition 1.6

$$\begin{aligned} d_{lb}(x_m, x_n) &\leq b(d_{lb}(x_m, x_{m+1})) + b^2(d_{lb}(x_{m+1}, x_{m+2)})) + \cdots + b^{n-m}(d_{lb}(x_{n-1}, x_n)) \\ &\leq b\mu^m d_{lb}(x_0, x_1) + b^2\mu^{m+1} d_{lb}(x_0, x_1) + \cdots + b^{n-m}\mu^{n-1} d_{lb}(x_0, x_1) \ \ by \ \ (2.13) \\ &\leq b\mu^m[1 + b\mu + \cdots + b^{n-m-1}\mu^{n-m-1}]d_{lb}(x_0, x_1) \\ &\leq \frac{b\mu^m}{1 - b\mu} d_{lb}(x_0, x_1) \to 0 \ as \ m \to \infty. \end{aligned} \tag{2.14}$$

This proves that $\{x_n\}$ is Cauchy sequence in X.
Since X is complete, there exists $x^* \in X$ such that $x_n \to x^* \ asn \to \infty$. Now, by Lemma 1.11 and (2.8), we now prove that $x^* \in X$ have a common fixed point of S and T.

$$\begin{aligned} d_{lb}(x^*, [Sx^*]_{\alpha_{S(x^*)}}) &\leq b[d_{lb}(x^*, x_{2n+1}) + d_{lb}(x_{2n+1}, [Sx^*]_{\alpha_{S(x^*)}})] \\ &\leq b[d_{lb}(x^*, x_{2n+1}) + H_{d_{lb}}([Tx_{2n}]_{\alpha_{T(x_{2n}}}, [Sx^*]_{\alpha_{S(x^*)}})] \\ &\leq b[d_{lb}(x^*, x_{2n+1}) + a_1 d_{lb}(x_{2n}, x^*) + a_2 d_{lb}(x_{2n}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) + a_3 d_{lb}(x^*, [Sx^*]_{\alpha_{S(x^*)}}) + \\ &\quad a_4 d_{lb}(x_{2n}, [Sx^*]_{\alpha_{S(x^*)}}) + a_5 d_{lb}(x^*, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) \\ &\quad + a_6 \frac{d_{lb}(x_{2n}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) d_{lb}(x^*, [Sx^*]_{\alpha_{S(x^*)}})}{d_{lb}(x_n, x^*) + d_{lb}(x_{2n}, [Sx^*]_{\alpha_{S(x^*)}}) + d_{lb}(x^*, [Tx_{2n}]_{\alpha_{T(x_{2n})}})} \\ &\quad + a_7 \frac{d_{lb}(x_{2n}, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) d_{lb}(x_{2n}, [Sx^*]_{\alpha_{S(x^*)}}) + d_{lb}(x^*, [Tx_{2n}]_{\alpha_{T(x_{2n})}}) d_{lb}(x^*, [Sx^*]_{\alpha_{S(x^*)}})}{d_{lb}(x_{2n}, [Sx^*]_{\alpha_{S(x^*)}}) + d_{lb}(x^*, [Tx_{2n}]_{\alpha_{T(x_{2n})}})}] \end{aligned} \tag{2.15}$$

On taking $n \to \infty$ in (2.15), we get

$$(1 - b(a_3 + a_4)) d_{lb}(x^*, [Sx^*]_{\alpha_{S(x^*)}}) \leq 0. \tag{2.16}$$

From (2.16), we get

$$x^* \in [Sx^*]_{\alpha_{S(x^*)}}.$$

Hence, $x^* \in X$ is the fixed point of S.
Similarly, we can prove that $x^* \in X$ is the fixed point of T. Hence, $x^* \in X$ have a common fixed point of S and T. □

Example 2.4. Let $X = \{0,1,2\}$ and $d_{lb} : X \times X \to [0,\infty)$ be a mapping defined by

$$d_{lb}(x,y) = \begin{cases} 0 & x = y \text{ and } x,y \in \{0,1\}, \\ \frac{2}{5} & x = y \text{ and } x,y \in \{2\}, \\ \frac{1}{2} & x \neq y \text{ amd } x,y \in \{0,2\}, \\ 1 & x \neq y \text{ and } x,y \in \{0,1\}, \\ \frac{1}{4} & x \neq y \text{ and } x,y \in [1,2\}. \end{cases}$$

It is clear that d is a complete dislocated b-metric space with constant $b = \frac{4}{3}$. Define the fuzzy mapping $S, T : X \to F(X)$ by

$$T(0)(t) = \begin{cases} \frac{3}{4} & t = 0 \\ 0 & t = 1,2 \end{cases}$$

$$T(1)(t) = \begin{cases} 0 & t = 0,1 \\ \frac{3}{4} & t = 2 \end{cases}$$

$$T(2)(t) = \begin{cases} 0 & t = 0,2 \\ \frac{3}{4} & t = 1 \end{cases}$$

and

$$S(0)(t) = S(1)(t) = S(2)(t) = \begin{cases} \frac{3}{4} & t = 0, \\ 0 & t = 1,2 \end{cases}$$

Define $\alpha_{S(x)} = \alpha_{T(x)} = \alpha$, where $\alpha \in (0, \frac{3}{4}]$. We now get

$$[Tx]_{\alpha_{T(x)}} = \begin{cases} \{0\} & x = 0, \\ \{2\} & x = 1, \\ \{1\} & x = 2, \end{cases}$$

and $[Sx]_{\alpha_{S(x)}} = \{0\}$ for all $x \in X$. *For* $x, y \in X$, *we get*

$$Hd_{lb}([Sx]_{\alpha_{S(x)}}), [Ty]_{\alpha_{T(y)}}) = \begin{cases} H(\{0\},\{0\}) = 0, & y = 0, \\ H(\{0\},\{2\}) = \frac{1}{2}, & y = 1, \\ H(\{0\},\{1\}) = 1, & y = 2, \end{cases}$$

let $a_1 = 0, a_2 = \frac{1}{4}, a_3 = \frac{1}{4}, a_4 = 0, a_5 = \frac{1}{4}, a_6 = 0, a_7 = 0$. Then, we can see that condition (2.8) in Theorem 2.3 are satisfied. Then, there exists $0 \in [Sx]_{\alpha_{S(x)}} \cap [Ty]_{\alpha_{T(y)}}$.

Corollary 2.5. *Let* (X, d_{lb}) *be a complete dislocated b-metric space with constant* $b \geq 1$. *Let* $T : X \to F(X)$ *be a fuzzy mapping. Suppose there exists* $\alpha(x) \in (0,1]$ *satisfying the following conditions:*

$$H_{d_{lb}}([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq a_1 d_{lb}(x,y) + a_2 d_{lb}(x, [Tx]_{\alpha(x)}) + a_3 d_{lb}(y, [Ty]_{\alpha(y)}) + a_4 d_{lb}(x, [Ty]_{\alpha(y)}) + a_5 d_{lb}(y, [Tx]_{\alpha(x)}) \tag{2.17}$$

for all $x, y \in X$,$x \neq y$ *and* $a_1, a_2, a_3, a_4, a_5 \geq 0$ *with* $b(a_1 + a_2) + a_3 + b(1 + b)a_4 < 1$ *and* $a_1 + a_2 + a_3 + a_4 + a_5 < 1$. *Then there exists* $x^* \in X$ *such that* $x^* \in [Tx^*]_{\alpha(x^*)}$.

Proof. Let $a_6 = a_7 = 0$ in Theorem 2.1, we get the result immediately. □

Corollary 2.6. *Let (X, d_{lb}) be a complete dislocated b-metric space with constant $b \geq 1$. Let $S, T : X \to F(X)$ be a pair of fuzzy mapping. Suppose there exists $\alpha_{S(x)}, \alpha_{T(x)} \in (0, 1]$ satisfying the following conditions:*

$$H_{d_{lb}}([Tx]_{\alpha_{T(x)}}, [Sy]_{\alpha_{S(y)}}) \leq a_1 d_{lb}(x, y) + a_2 d_{lb}(x, [Tx]_{\alpha_{T(x)}}) + a_3 d_{lb}(y, [Sy]_{\alpha_{S(y)}}) + a_4 d_{lb}(x, [Sy]_{\alpha_{S(y)}}) + a_5 d_{lb}(y, [Tx]_{\alpha_{T(x)}}) \tag{2.18}$$

for all $x, y \in X$, $x \neq y$ and $a_1, a_2, a_3, a_4, a_5 \geq 0$ with $2ba_1 + (a_2 + a_3)(b + 1) + b(a_4 + a_5)(b + 1) < 2$ and $a_1 + a_2 + a_3 + a_4 + a_5 < 1$. Then there exists $x^* \in X$ such that x^* is a common fixed point of S and T.

Proof. Let $a_6 = a_7 = 0$ in Theorem 2.3, we get the result immediately. □

3. Application

In this section, as an application of our results, we establish that Theorem 2.1 and Theorem 2.3 can be utilized to derive the existence of common fixed point results for multivalued mappings in a dislocated b-metric space. In the sequel, we begin with the following Theorem.

Theorem 3.1. *Let (X, d_{lb}) be a complete dislocated b-metric space with constant $b \geq 1$. Let $R : X \to P(X)$ be a multivalued mapping satisfying the following conditions:*

$$H_{d_{lb}}(Rx, Ry) \leq a_1 d_{lb}(x, y) + a_2 d_{lb}(x, Rx) + a_3 d_{lb}(y, Ry) + a_4 d_{lb}(x, Ry) + a_5 d_{lb}(y, Rx) + a_6 \frac{d_{lb}(x, Rx) d_{lb}(y, Ry)}{d_{lb}(x, y) + d_{lb}(x, Ry) + d_{lb}(y, Rx)} + a_7 \frac{d_{lb}(x, Rx) d_{lb}(x, Ry) + d_{lb}(y, Rx) d_{lb}(y, Ry)}{d_{lb}(x, Ry) + d_{lb}(y, Rx)} \tag{3.1}$$

for all $x, y \in X$,$x \neq y$ and $a_1, a_2, a_3, a_4, a_5, a_6, a_7 \geq 0$ with $b(a_1 + a_2) + a_3 + b(1 + b)a_4 + b(a_6 + a_7) < 1$ and $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 < 1$. Then there exists $x^* \in X$ such that $x^* \in Rx^*$.

Proof. Let $\alpha : X \to (0, 1]$ be an arbitrary mapping. Consider a fuzzy mapping $T : X \to F(X)$ defined by

$$(Tx)(t) = \begin{cases} \alpha(x), & t \in Rx, \\ 0, & t \notin Rx. \end{cases}$$

We have that

$$[Tx]_{\alpha(x)} = \{t : Tx(t) \geq \alpha(x)\} = Rx.$$

Hence, condition (3.1) becomes condition (2.1) in Theorem 2.1. It implies that there exists $x^* \in X$ such that $x^* \in [Tx^*]_{\alpha(x^*)} = Rx^*$. □

Theorem 3.2. *Let (X, d_{lb}) be a complete dislocated b-metric space with constant $b \geq 1$. Suppose $R, G : X \to P(X)$ are two multivalued mappings satisfying the following conditions:*

$$H_{d_{lb}}(Rx, Gy) \leq a_1 d_{lb}(x, y) + a_2 d_{lb}(x, Rx) + a_3 d_{lb}(y, Gy) + a_4 d_{lb}(x, Gy) + a_5 d_{lb}(y, Rx) + a_6 \frac{d_{lb}(x, Rx) d_{lb}(y, Gy)}{d_{lb}(x, y) + d_{lb}(x, Gy) + d_{lb}(y, Rx)} + a_7 \frac{d_{lb}(x, Rx) d_{lb}(x, Gy) + d_{lb}(y, Rx) d_{lb}(y, Gy)}{d_{lb}(x, Gy) + d_{lb}(y, Rx)} \tag{3.2}$$

for all $x, y \in X$, $x \neq y$ *and* $a_1, a_2, a_3, a_4, a_5, a_6, a_7 \geq 0$ *with* $2ba_1 + (a_2+a_3)(b+1) + b(a_4+a_5)(b+1) + 2b(a_6+a_7) < 2$ *and* $a_1+a_2+a_3+a_4+a_5+a_6+a_7 < 1$. *Then there exists* $x^* \in X$ *such that* x^* *is a common fixed point of* R *and* G.

Proof. Let $\alpha : X \to (0,1]$ be an arbitrary mapping. Consider a fuzzy mapping $S, T : X \to F(X)$ defined by

$$(Sx)(t) = \begin{cases} \alpha(x), & t \in Rx, \\ 0, & t \notin Rx, \end{cases}$$

$$(Tx)(t) = \begin{cases} \alpha(x), & t \in Gx, \\ 0, & t \notin Gx. \end{cases}$$

We have that

$$[Sx]_{\alpha(x)} = \{t : Sx(t) \geq \alpha(x)\} = Rx,$$

and

$$[Tx]_{\alpha(x)} = \{t : Tx(t) \geq \alpha(x)\} = Gx.$$

Hence, condition (3.2) becomes condition (2.8) in Theorem 2.3 It implies that there exists $x^* \in [Sx^*]_{\alpha(x^*)} \cap [Tx^*]_{\alpha(x^*)} = Rx^* \cap Gx^*$. □

If $a_6 = a_7 = 0$, we have Theorem 3.1 of [21] as corollary as follows.

Corollary 3.3. *Let* (X, d_{lb}) *be a complete dislocated b-metric space with constant* $b \geq 1$. *Let* $R : X \to P(X)$ *be a multivalued mapping satisfying the following conditions:*

$$H_{d_{lb}}(Rx, Ry) \leq a_1 d_{lb}(x,y) + a_2 d_{lb}(x, Rx) + a_3 d_{lb}(y, Ry) + a_4 d_{lb}(x, Ry) + a_5 d_{lb}(y, Rx) \tag{3.3}$$

for all $x, y \in X$, $x \neq y$ *and* $a_1, a_2, a_3, a_4, a_5 \geq 0$ *with* $b(a_1+a_2)+a_3+b(1+b)a_4 < 1$ *and* $a_1+a_2+a_3+a_4+a_5 < 1$.*Then there exists* $x^* \in X$ *such that* $x^* \in Rx^*$.

If $a_6 = a_7 = 0$, we have Theorem 3.2 of [21] as corollary as follows.

Corollary 3.4. *Let* (X, d_{lb}) *be a complete dislocated b-metric space with constant* $b \geq 1$. *Suppose* $R, G : X \to P(X)$ *are two multivalued mappings satisfying the following conditions:*

$$H_{d_{lb}}(Rx, Gy) \leq a_1 d_{lb}(x,y) + a_2 d_{lb}(x, Rx) + a_3 d_{lb}(y, Gy) + a_4 d_{lb}(x, Gy) + a_5 d_{lb}(y, Rx) \tag{3.4}$$

for all $x, y \in X$, $x \neq y$ *and* $a_1, a_2, a_3, a_4, a_5 \geq 0$ *with* $2ba_1 + (a_2+a_3)(b+1) + b(a_4+a_5)(b+1) < 2$ *and* $a_1+a_2+a_3+a_4+a_5 < 1$. *Then there exists* $x^* \in X$ *such that* x^* *is a common fixed point of* R *and* G.

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[1] Department of Mathematics, Confluence University of Science and Technology, Osara, Kogi State, Nigeria.
Email address: rajimuhammed11@gmail.com

[2] Department of Mathematics, Federal University Lokoja, Kogi State, Nigeria
Email address: adekukbash@gmail.com