Annals of Mathematics and Computer Science
ISSN: 2789-7206
Vol 3 (2021) 7-14

STARLIKENESS AND CLOSE-TO-CONVEXITY OF MEROMORPHIC FUNCTIONS

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ABSTRACT. In the present paper, we study a differential subordination involving a Ruscheweyh type operator. In particular, we obtain certain sufficient conditions for meromorphic starlike and meromorphic close-to-convex functions.

1. INTRODUCTION AND PRELIMINARIES

Let Σ_p denote the class of functions of the form

$$f(z) = \frac{1}{z^p} + \sum_{k=1}^{\infty} a_k z^{k-p} \ (p \in \mathbb{N} = \{1, 2, 3, \ldots\}),$$

which are analytic and p-valent in the punctured unit disc $\mathbb{E}_0 = \mathbb{E} \setminus \{0\}$, where $\mathbb{E} = \{z \in \mathbb{C} : |z| < 1\}$. A function $f \in \Sigma_p$ is said to be meromorphic p-valent starlike of order α if $f(z) \neq 0$ for $z \in \mathbb{E}_0$ and

$$-\Re\frac{1}{p}\left(\frac{zf'(z)}{f(z)}\right) > \alpha, \qquad (\alpha < p; z \in \mathbb{E}).$$

The class of all such functions is denoted by $\mathcal{MS}_p^*(\alpha)$.
A function $f \in \Sigma_p$ is called meromorphic p-valent close-to-convex of order α if $f'(z) \neq 0$ and

$$-\Re\left(\frac{f'(z)}{z^{-p-1}}\right) > \alpha, \qquad (\alpha < p; z \in \mathbb{E}).$$

The class of all meromorphic p-valent close-to-convex functions is denoted by $\mathcal{MC}_p(\alpha)$.

Let $\Sigma = \Sigma_1,\ \mathcal{MS}^*(\alpha) = \mathcal{MS}^*{}_1(\alpha)$ and $\mathcal{MC}(\alpha) = \mathcal{MC}_1(\alpha)$.
For two functions f and g analytic in $\mathbb{E}$, we say that the function $f(z)$ is subordinate to $g(z)$ in $\mathbb{E}$, and write $f(z) \prec g(z),\ z \in \mathbb{E}$, if there exists a Schwarz function $w(z)$, analytic in $\mathbb{E}$ with $w(0) = 0$ and $|w(z)| < 1,\ z \in \mathbb{E}$, such that $f(z) = g(w(z)),\ z \in \mathbb{E}$.

Date: Received: Aug 3, 2021; Accepted: Aug 23, 2021.

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2010 *Mathematics Subject Classification.* Primary 30C45, Secondary 30C80.

Key words and phrases. Meromorphic Multivalent Functions, Meromorphic Starlike Function, Meromorphic Close-to-Convex Function, Hadamard Product(or Convolutions), Ruscheweyh Derivative, Differential Subordination.

In particular, if the function g is univalent in $\mathbb{E}$, the above subordination is equivalent to $f(0) = g(0)$ and $f(\mathbb{E}) \subset g(\mathbb{E})$.
Let $\Phi : \mathbb{C}^2 \times \mathbb{E} \longrightarrow \mathbb{C}$ ($\mathbb{C}$ is the complex plane) and h be univalent in $\mathbb{E}$. If p is analytic in $\mathbb{E}$ and satisfies the differential subordination

$$\Phi(p(z), zp'(z); z) \prec h(z), \Phi(p(0), 0; 0) = h(0), \tag{1.1}$$

then p is called a solution of the differential subordination (1.1). The univalent function q is called a dominant of differential subordination (1.1) if $p \prec q$ for all p satisfying (1.1). A dominant $\tilde{q} \prec q$ for all dominants q of (1.1), is said to be the best dominant of (1.1).

For $f(z) \in \Sigma_p$ and $g(z) \in \Sigma_p$ given by

$$g(z) = \frac{1}{z^p} + \sum_{k=1}^{\infty} b_k z^{k-p} \qquad (p \in \mathbb{N}),$$

the Hadamard product (or convolution) of f and g is defined by

$$(f * g)(z) = \frac{1}{z^p} + \sum_{k=1}^{\infty} a_k b_k z^{k-p} = (f * g)(z).$$

To study the geometric properties of various classes in the field of complex analysis, Ruscheweyh operator has been studied by many authors. A linear operator $\mathcal{D}^{\lambda,p}$ was investigated by Raina and Srivastava [5] on the space of analytic and p-valent functions in $\mathbb{E}$. A more general convolution operator was studied by Liu and Srivastava [1].
In [2009], Goyal and Prajapat [6] studied the extended linear derivative operator of Ruscheweyh type for the functions belonging to class Σ_p,

$${\mathcal{D}_*}^{\lambda,p} : \Sigma_p \longrightarrow \Sigma_p,$$

and defined by the following convolution:

$${\mathcal{D}_*}^{\lambda,p} = \frac{1}{z^p(1-z)^{\lambda+1}} \qquad (\lambda > 1; f \in \Sigma_p).$$

In particular for $\lambda = n$ ($n \in \mathbb{N}$), it is easily observed that

$${\mathcal{D}_*}^{\lambda,p} = \frac{z^{-p}(z^{n+p} f(z))^{(n)}}{n!} \qquad (n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}).$$

By using the operator ${\mathcal{D}_*}^{\lambda,p}$ ($\lambda > -p; p \in \mathbb{N}$), they obtain some sufficient conditions for meromorphic functions. We refer to Liu and Srivastava [1], Mogra [2], Raina and Srivastava [4] and Xu and Yang [3] for the related work on the subject of meromorphic functions. The main objective of the present paper is to study differential subordination involving the operator ${\mathcal{D}_*}^{\lambda,p}$. Selecting different dominants to our main result, we obtain certain sufficient conditions for meromorphic starlike functions and meromorphic close-to-convex functions.
We shall use the following lemma of Miller and Mocanu [7] to prove our result.

Lemma 1.1. *Let q be univalent in $\mathbb{E}$ and let θ and ϕ be analytic in a domain $\mathbb{D}$ containing $q(\mathbb{E})$, with $\phi(w) \neq 0$, when $w \in q(\mathbb{E})$. Set $Q_1(z) = zq'(z)\phi[q(z)],\ h(z) = \theta[q(z)] + Q_1(z)$ and suppose that either*
(i) h is convex, or
(ii) Q_1 is starlike.
In addition, assume that
(iii) $\Re\left(\frac{zh'(z)}{Q_1(z)}\right) > 0$ for all z in $\mathbb{E}$.
If p is analytic in $\mathbb{E}$, with $p(0) = q(0),\ p(\mathbb{E}) \subset \mathbb{D}$ and

$$\theta[p(z)] + zp'(z)\phi[p(z)] \prec \theta[q(z)] + zq'(z)\phi[q(z)],\ z \in \mathbb{E},$$

then $p(z) \prec q(z)$ and q is the best dominant.

2. Main results

Theorem 2.1. *Let q be a univalent function in $\mathbb{E}$ such that*
(i) $\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right) > 0$,
(ii) $\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)} + \frac{1}{\alpha}q(z)\right) > 0$.
If $f \in \Sigma_p$ satisfies

$$-\frac{z^{p+1}({\mathcal{D}_*}^{\lambda,p}f(z))'}{\left(z^p{\mathcal{D}_*}^{\lambda,p}f(z)\right)^{\mu-1}} + \alpha\left(1 + (2-\mu)p + \frac{z({\mathcal{D}_*}^{\lambda,p}f(z))''}{({\mathcal{D}_*}^{\lambda,p}f(z))'} - (\mu-1)\frac{z({\mathcal{D}_*}^{\lambda,p}f(z))'}{({\mathcal{D}_*}^{\lambda,p}f(z))}\right) \prec q(z) + \alpha\frac{zq'(z)}{q(z)} \tag{2.1}$$

then

$$-\frac{z^{p+1}({\mathcal{D}_*}^{\lambda,p}f(z))'}{\left(z^p{\mathcal{D}_*}^{\lambda,p}f(z)\right)^{\mu-1}} \prec q(z),$$

where α is non zero complex number.

Proof. On writing $-\frac{z^{p+1}({\mathcal{D}_*}^{\lambda,p}f(z))'}{\left(z^p{\mathcal{D}_*}^{\lambda,p}f(z)\right)^{\mu-1}} = u(z)$, in (2.1), we obtain:

$$u(z) + \alpha\frac{zu'(z)}{u(z)} \prec q(z) + \alpha\frac{zq'(z)}{q(z)},$$

Let us define the following functions θ and ϕ as follows:

$$\theta(w) = w$$

and

$$\phi(w) = \frac{\alpha}{w}.$$

Therefore,

$$Q(z) = \phi(q(z))zq'(z) = \alpha\frac{zq'(z)}{q(z)}$$

and

$$h(z) = \theta(q(z)) + Q(z) = q(z) + \alpha\frac{zq'(z)}{q(z)}.$$

On differentiating, we obtain $\dfrac{zQ'(z)}{Q(z)} = 1 + \dfrac{zq''(z)}{q'(z)} - \dfrac{zq'(z)}{q(z)}$ and

$$\frac{zh'(z)}{Q(z)} = 1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)} + \frac{1}{\alpha}q(z).$$

In view of given conditions, we see that $Q(z)$ is starlike and $\Re\left(\dfrac{zh'(z)}{Q(z)}\right) > 0$. Therefore, the proof, now follows from lemma 1.1. $\square$

3. Special Cases and Applications

Selecting the dominant $q(z) = \left(\dfrac{1+z}{1-z}\right)^{\delta}, 0 < \delta \leq 1$ in Theorem 2.1, we notice that

$$\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right) = \Re\left(\frac{1+z^2}{1-z^2}\right) > 0.$$

Thus for a positive real number α, q satisfies the conditions (i) and (ii) of Theorem 2.1. Therefore, we immediately arrive at the following result.

Corollary 3.1. *Let α be a positive real number. If $f \in \Sigma_p$ satisfies*

$$-\frac{z^{p+1}(\mathcal{D}_*{}^{\lambda,p}f(z))'}{\left(z^p\mathcal{D}_*{}^{\lambda,p}f(z)\right)^{\mu-1}} + \alpha\left(1 + (2-\mu)p + \frac{z(\mathcal{D}_*{}^{\lambda,p}f(z))''}{(\mathcal{D}_*{}^{\lambda,p}f(z))'} - (\mu-1)\frac{z(\mathcal{D}_*{}^{\lambda,p}f(z))'}{(\mathcal{D}_*{}^{\lambda,p}f(z))}\right)$$
$$\prec \left(\frac{1+z}{1-z}\right)^{\delta} + 2\alpha\delta\left(\frac{z}{1-z^2}\right)$$

then

$$-\frac{z^{p+1}(\mathcal{D}_*{}^{\lambda,p}f(z))'}{\left(z^p\mathcal{D}_*{}^{\lambda,p}f(z)\right)^{\mu-1}} \prec \left(\frac{1+z}{1-z}\right)^{\delta}$$

Setting $\lambda = 0$ and $\mu = 1 = p$, in above corollary, we get

Example 3.2. If $f(z) \in \Sigma$ satisfies the following differential subordination

$$-z^2f'(z) + \alpha\left(2 + \frac{zf''(z)}{f'(z)}\right) \prec \left(\frac{1+z}{1-z}\right)^{\delta} + 2\alpha\delta\left(\frac{z}{1-z^2}\right)$$

then $-z^2f'(z) \prec \left(\dfrac{1+z}{1-z}\right)^{\delta}$ *i.e.* $f(z) \in \mathcal{MC}(\alpha)$.

Taking $\mu = 2, p = 1$ and $\lambda = 0$ in Corollary 3.1, we have the following result.

Example 3.3. If $f(z) \in \Sigma$ satisfies

$$-(1+\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right) \prec \left(\frac{1+z}{1-z}\right)^{\delta} + 2\alpha\delta\left(\frac{z}{1-z^2}\right)$$

then

$$-\frac{zf'(z)}{f(z)} \prec \left(\frac{1+z}{1-z}\right)^{\delta}.$$

Considering the dominant $q(z) = \dfrac{1+(1-2\beta)z}{1-z}$, $0 \le \beta < 1$, in Theorem 2.1, we obtain

$$\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right) = \Re\left(\frac{1+(1-2\beta)z^2}{(1-z)(1+(1-2\beta)z)}\right) > 0,$$

Also for $\alpha > 0$

$$\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)} + \frac{q(z)}{\alpha}\right) = \Re\left(\frac{1+(1-2\beta)z^2}{(1-z)(1+(1-2\beta)z)} + \frac{1+(1-2\beta)z}{\alpha}\right) > 0,$$

Both the conditions of Theorem 2.1 are satisfied. Hence, we get the following result.

Corollary 3.4. *Let α be a positive real number. If $f \in \Sigma_p$ satisfies*

$$-\frac{z^{p+1}(\mathcal{D}_*{}^{\lambda,p}f(z))'}{\left(z^p\mathcal{D}_*{}^{\lambda,p}f(z)\right)^{\mu-1}} + \alpha\left(1 + (2-\mu)p + \frac{z(\mathcal{D}_*{}^{\lambda,p}f(z))''}{(\mathcal{D}_*{}^{\lambda,p}f(z))'} - (\mu-1)\frac{z(\mathcal{D}_*{}^{\lambda,p}f(z))'}{(\mathcal{D}_*{}^{\lambda,p}f(z))}\right)$$
$$\prec \frac{1+(1-2\beta)z}{1-z} + \frac{2\alpha(1-\beta)z}{(1-z)(1+(1-2\beta)z)},\ 0 \le \beta < 1,$$

then

$$-\frac{z^{p+1}(\mathcal{D}_*{}^{\lambda,p}f(z))'}{\left(z^p\mathcal{D}_*{}^{\lambda,p}f(z)\right)^{\mu-1}} \prec \frac{1+(1-2\beta)z}{1-z}$$

Setting $\lambda = 0$ and $\mu = 1 = p$, in above corollary, we get

Example 3.5. If $f(z) \in \Sigma$ satisfies the following differential subordination

$$-z^2f'(z) + \alpha\left(2 + \frac{zf''(z)}{f'(z)}\right) \prec \frac{1+(1-2\beta)z}{1-z} + \frac{2\alpha(1-\beta)z}{(1-z)(1+(1-2\beta)z)},\ 0 \le \beta < 1,$$

then $-z^2f'(z) \prec \dfrac{1+(1-2\beta)z}{1-z}$ *i.e.* $f(z) \in \mathcal{MC}(\alpha)$.

Taking $\mu = 2, p = 1$ and $\lambda = 0$ in Corollary 3.1, we have the following result

Example 3.6. If $f(z) \in \Sigma$ satisfies

$$-(1+\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right) \prec \frac{1+(1-2\beta)z}{1-z} + \frac{2\alpha(1-\beta)z}{(1-z)(1+(1-2\beta)z)},\ 0 \le \beta < 1,$$

then

$$-\frac{zf'(z)}{f(z)} \prec \frac{1+(1-2\beta)z}{1-z}.$$

For the dominant $q(z) = e^z$, we have

$$\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right) = 1$$

Also for $\alpha > 0$

$$\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)} + \frac{q(z)}{\alpha}\right) = 1 + \frac{e^z}{\alpha},$$

We notice that q satisfies the conditions (i) and (ii) of Theorem 2.1. We obtain the following result.

Corollary 3.7. *Let α be a positive real number. If $f \in \Sigma_p$ satisfies*

$$-\frac{z^{p+1}({\mathcal{D}_*}^{\lambda,p}f(z))'}{\left(z^p{\mathcal{D}_*}^{\lambda,p}f(z)\right)^{\mu-1}} + \alpha\left(1 + (2-\mu)p + \frac{z({\mathcal{D}_*}^{\lambda,p}f(z))''}{({\mathcal{D}_*}^{\lambda,p}f(z))'} - (\mu-1)\frac{z({\mathcal{D}_*}^{\lambda,p}f(z))'}{({\mathcal{D}_*}^{\lambda,p}f(z))}\right) \prec e^z + \alpha z,$$

then

$$-\frac{z^{p+1}({\mathcal{D}_*}^{\lambda,p}f(z))'}{\left(z^p{\mathcal{D}_*}^{\lambda,p}f(z)\right)^{\mu-1}} \prec e^z$$

Setting $\lambda = 0$ and $\mu = 1 = p$, in above corollary, we get

Example 3.8. If $f(z) \in \Sigma$ satisfies the following differential subordination

$$-z^2 f'(z) + \alpha\left(2 + \frac{zf''(z)}{f'(z)}\right) \prec e^z + \alpha z,$$

then $-z^2 f'(z) \prec e^z$ *i.e.* $f(z) \in \mathcal{MC}(\alpha)$.

Taking $\mu = 2, p = 1$ and $\lambda = 0$ in Corollary 3.1, we have the following result

Example 3.9. If $f(z) \in \Sigma$ satisfies

$$-(1+\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right) \prec e^z + \alpha z,$$

then

$$-\frac{zf'(z)}{f(z)} \prec e^z.$$

When we select the dominant $q(z) = \dfrac{\alpha'(1-z)}{\alpha' - z}$, $\alpha' > 1$ in Theorem 2.1, a simple calculation yields that

$$\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right) = \frac{1}{1-z} + \frac{z}{\alpha' - z}$$

and

$$\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)} + \frac{q(z)}{\alpha}\right) = \frac{1}{1-z} + \frac{z}{\alpha' - z} + \frac{\alpha'(1-z)}{\alpha(\alpha' - z)},$$

Thus for a positive real number α, we notice that q satisfies the conditions (i) and (ii) of Theorem 2.1. Hence, we deduce the following result.

Corollary 3.10. *Let α be a positive real number. If $f \in \Sigma_p$ satisfies*

$$-\frac{z^{p+1}({\mathcal{D}_*}^{\lambda,p}f(z))'}{\left(z^p{\mathcal{D}_*}^{\lambda,p}f(z)\right)^{\mu-1}}+\alpha\left(1+(2-\mu)p+\frac{z({\mathcal{D}_*}^{\lambda,p}f(z))''}{({\mathcal{D}_*}^{\lambda,p}f(z))'}-(\mu-1)\frac{z({\mathcal{D}_*}^{\lambda,p}f(z))'}{({\mathcal{D}_*}^{\lambda,p}f(z))}\right)$$
$$\prec \frac{\alpha'(1-z)}{\alpha'-z}+\frac{\alpha z(1-\alpha')}{(1-z)(\alpha'-z)},\ \alpha'>1$$

then

$$-\frac{z^{p+1}({\mathcal{D}_*}^{\lambda,p}f(z))'}{\left(z^p{\mathcal{D}_*}^{\lambda,p}f(z)\right)^{\mu-1}}\prec \frac{\alpha'(1-z)}{\alpha'-z}$$

Setting $\lambda=0$ and $\mu=1=p$, in above corollary, we get

Example 3.11. If $f(z)\in\Sigma$ satisfies the following differential subordination

$$-z^2f'(z)+\alpha\left(2+\frac{zf''(z)}{f'(z)}\right)\prec\frac{\alpha'(1-z)}{\alpha'-z}+\frac{\alpha z(1-\alpha')}{(1-z)(\alpha'-z)},\ \alpha'>1,$$

then $-z^2f'(z)\prec\dfrac{\alpha'(1-z)}{\alpha'-z}$ *i.e.* $f(z)\in\mathcal{MC}(\alpha)$.

Taking $\mu=2, p=1$ and $\lambda=0$ in Corollary 3.1, we have the following result

Example 3.12. If $f(z)\in\Sigma$ satisfies

$$-(1+\alpha)\frac{zf'(z)}{f(z)}+\alpha\left(1+\frac{zf''(z)}{f'(z)}\right)\prec\frac{\alpha'(1-z)}{\alpha'-z}+\frac{\alpha z(1-\alpha')}{(1-z)(\alpha'-z)},\ \alpha'>1,$$

then

$$-\frac{zf'(z)}{f(z)}\prec\frac{\alpha'(1-z)}{\alpha'-z}.$$

Taking the dominant $q(z)=1+az$, $0\le a<1$, in Theorem 2.1, we obtain

$$\left(1+\frac{zq''(z)}{q'(z)}-\frac{zq'(z)}{q(z)}\right)=\frac{1}{1+az},$$

Also for $\alpha>0$

$$\left(1+\frac{zq''(z)}{q'(z)}-\frac{zq'(z)}{q(z)}+\frac{q(z)}{\alpha}\right)=\frac{1}{1+az}+\frac{1+az}{\alpha},$$

Both the conditions of Theorem 2.1 are satisfied. Hence, we get the following result.

Corollary 3.13. *Let α be a positive real number. If $f \in \Sigma_p$ satisfies*

$$-\frac{z^{p+1}({\mathcal{D}_*}^{\lambda,p}f(z))'}{\left(z^p{\mathcal{D}_*}^{\lambda,p}f(z)\right)^{\mu-1}}+\alpha\left(1+(2-\mu)p+\frac{z({\mathcal{D}_*}^{\lambda,p}f(z))''}{({\mathcal{D}_*}^{\lambda,p}f(z))'}-(\mu-1)\frac{z({\mathcal{D}_*}^{\lambda,p}f(z))'}{({\mathcal{D}_*}^{\lambda,p}f(z))}\right)$$
$$\prec 1+az+\frac{\alpha az}{1+az},\ 0\le a<1,$$

then

$$-\frac{z^{p+1}({\mathcal{D}_*}^{\lambda,p}f(z))'}{\left(z^p{\mathcal{D}_*}^{\lambda,p}f(z)\right)^{\mu-1}} \prec 1+az$$

Setting $\lambda = 0$ and $\mu = 1 = p$, in above corollary, we get

Example 3.14. If $f(z) \in \Sigma$ satisfies the following differential subordination

$$-z^2f'(z)+\alpha\left(2+\frac{zf''(z)}{f'(z)}\right) \prec 1+az+\frac{\alpha az}{1+az},\ 0 \le a < 1,$$

then $-z^2f'(z) \prec 1+az$ *i.e.* $f(z) \in \mathcal{MC}(\alpha)$.

Taking $\mu = 2, p = 1$ and $\lambda = 0$ in Corollary 3.1, we have the following result

Example 3.15. If $f(z) \in \Sigma$ satisfies

$$-(1+\alpha)\frac{zf'(z)}{f(z)}+\alpha\left(1+\frac{zf''(z)}{f'(z)}\right) \prec 1+az+\frac{\alpha az}{1+az},\ 0 \le a < 1,$$

then

$$-\frac{zf'(z)}{f(z)} \prec 1+az.$$

Acknowledgement. The authors are grateful to the referee for careful reading of the paper and valuable suggestions and comments.

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