Annals of Mathematics and Computer Science ISSN: 2789-7206
Vol 19 (2023) 10-33 https://doi.org/10.56947/amcs.v19.213

CLIMATIC ANALYSIS BASED ON INTERVAL-VALUED COMPLEX PYTHAGOREAN FUZZY GRAPH STRUCTURE

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Abstract. The Interval-valued Complex Pythagorean Fuzzy Set (IVCPFS), an extension of the Pythagorean Fuzzy Set (PFS), offers a more accurate description of uncertainty than traditional fuzzy sets. It has numerous uses in fuzzy control. In this research study, we show the concept of Interval-Valued Complex Pythagorean Fuzzy Graph Structure (IVCPFGS), as well as the results of some regular IVCPFGS and totally regular IVCPFGS, along with the degree of vertex present. We also discussed the subdivision of an IVCPFGS, the ψ-complement IVCPFGS, and the strength of a Path in an IVCPFGS. Finally, we discuss a real-world example based on temperature variation and climatic data analysis in an environment with an interval-valued complex Pythagorean fuzzy Structure to demonstrate the applicability of the generated results.

1. Introduction

In order to address the difficulties associated with dealing with uncertainty in fuzzy sets, L.A. Zadeh [35] created fuzzy sets in 1965. Since then, other real-world problems involving ambiguous and uncertain situations have been studied in order to resolve by other researchers using fuzzy sets and fuzzy logic. The extension of fuzzy sets that the author started in Turksen [27] in 1986 is called interval-valued fuzzy sets. In order to accommodate for uncertainty, it includes the values of number intervals rather than using numbers as the membership function. Normally, it is denoted by the symbol $[\mu^-_{AL}(x), \mu^+_{AU}]$. To express the degree of membership of the fuzzy set A, use the formula $0 \leq \mu^-_{AL}(x) + \mu^+_{AU}(x) \leq 1$. An additional element was included to fuzzy sets by T. Atanassov [3], namely a non-membership function that is represented by intuitionistic fuzzy sets. He also expanded the concept of intuitionistic fuzzy sets to include interval valued intuitionistic fuzzy sets [4]. The representation of uncertainty using interval-valued intuitionistic fuzzy sets is preferable than that of conventional fuzzy sets. The aspect of fuzzy control that requires the greatest processing is defuzzification, which is used in fuzzy control in a variety of ways. For the purpose of interpreting the degree of true and false membership functions, it is defined as a pair of intervals

Date: Received: Sep 12, 2023; Accepted: Oct 17, 2023.
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2010 *Mathematics Subject Classification.* Primary 05C72, 05C78; Secondary 05C99.

Key words and phrases. degree of a vertex in IVCPFGS, regular in IVCPFGS, totally regular IVCPFGS, subdivision of an IVCPFGS, ψ-complement IVCPFGS, Application Climate Morphology.

$[\mu^-, \mu^+], 0 \leq \mu^- + \mu^+ \leq 1$ and $[\lambda^-, \lambda^+], 0 \leq \lambda^- + \lambda^+ \leq 1$ with $0 \leq \mu^+ + \lambda^+ \leq 1$. The handling of uncertain information is nevertheless permitted despite the aforementioned restrictions, which only permit holding incomplete data. Pythagorean fuzzy set (PFS), a new generalisation of IFS, was introduced by Yager[[29], [30], [31]]. It abides by the rule that $\mu^2 + \lambda^2 \leq 1$ and is made up of both membership and non-membership degrees. It has a larger range in order to handle the uncertainty. Since $0.8 + 0.9 > 1$, the IFN cannot support this condition when a decision maker gives evaluation data with membership degrees of 0.8 and 0.9, respectively. In this manner is the importance of PFS defined. On the other hand, $(0.8)^2+(0.9)^2 \leq 1$. Inaccurate, inconsistent, and incomplete periodic information, however, cannot be handled by FSs, IFSs, or PFSs. Although these theories are applicable to many different branches of science, they all share a common weakness: they cannot be used to model two-dimensional phenomena. The fuzzy set (FS) theory is a practical tool for addressing uncertainties that occur in a variety of spheres of life. However, in some cases, this theory is unable to simulate the imperfect and erratic information of a two-dimensional or periodic nature found in the real world. In order to address this issue, Ramot [17] developed the idea of a complex fuzzy set (CFS) in 2012. This concept is a valuable generalisation of FS in which the membership grade is constrained to only accept values from the complex plane's unit circle and is of the form $re^{i\theta}$, where r is the amplitude term and θ is the phase term. Because it can better manage periodic difficulties or reoccurring problematic phenomenon, the phase term of CFS is crucial. The inclusion of the phase term in CFS assures that there may be situations in which the second dimension is necessary. CFS is distinguished from all other groups of material that are currently available by this phrase. A CF representation of solar activity is one potential application that illustrates the new idea. applications for time series forecasting and signal processing. The concepts of complex intuitionistic were first introduced by Alkouri and Salleh [1] in 2012, using the degree of complex valued non-membership functions to translate the concepts of CFS to complex intuitionistic fuzzy sets (CIFSs). Recently, the concepts of Complex Interval-Valued Intuitionistic Fuzzy Sets (CIVIFSs) and its Aggregation Operator were introduced by Harish Garg and Dimple Rani [6]. For the purpose of interpreting the complex degree of true and false membership functions, it is defined as a pair of intervals $[\mu^- e^{i\alpha^-}, \mu^+ e^{i\alpha^+}], 0 \leq \mu^- + \mu^+ \leq 1, 0 \leq \alpha^- + \alpha^+ \leq 2\pi$ and $[\lambda^- e^{i\beta^-}, \lambda^+ e^{i\beta^-}], 0 \leq \lambda^- + \lambda^+ \leq 1, 0 \leq \beta^- + \beta^+ \leq 2\pi$ with $0 \leq \mu^+ + \lambda^+ \leq 1$ and $0 \leq \alpha^+ + \beta^+ \leq 1$. Despite the aforementioned limitations, which only permit keeping partial data, handling of unclear information is still allowed. A novel generalisation of CIVIFS called Interval-Valued Complex Pythagorean Fuzzy Set (IVCPFS) is what we've developed. To deal with the unpredictability, it has a wider range. The CIVIFS cannot support this condition when a decision maker provides evaluation data because $\mu^+ = 0.6$ and $\lambda^+ = 0.7$ and $\alpha^+ = 1.4\pi$ and $\beta^+ = 1.2\pi$ then $\mu^+ + \lambda^+ > 1$ and $\alpha^+ + \beta^+ > 2\pi$. In this way, the significance of IVCPFS is defined. However, it applied to IVCPFS when ${\mu^+}^2 + {\lambda^+}^2 > 1$ and ${\alpha^+}^2 + {\beta^+}^2 > 2\pi$ were multiplied together. This Flowchart-1 depicts the development of IVCPFS in our usage. Figure-1 CS Crisp set, FS Fuzzy set, IFS Intuition-

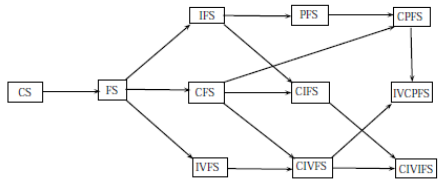

FIGURE 1. Flowchar

istic Fuzzy Set, IVFS Interval-Valued Fuzzy Set, CFS Complex Fuzzy set, PFS Pythagorean Fuzzy Set, CIFS Complex Intuitionistic Fuzzy Set, CIVFS Complex Interval-Valued Fuzzy Set, CPFS Complex Pythagorean Fuzzy Set, CIVIFS Complex Interval-Valued Intuitionistic Fuzzy Set, IVCPFS Interval-Valued Complex Pythagorean Fuzzy Set. A graph structure can be created by generalising an undirected graph, and this structure can then be used to investigate various types of structures like graphs and signed graphs. The concept of graph structures was initially introduced in Sampath Kumar's [22] work from 2006. T. Dinesh and T. V. Ramakrishnan [5] initially proposed the concept of a fuzzy graph structure in 2011. Muhammad Akram [[11],[15]] recently introduced the idea operations on intuitionistic fuzzy graph structures and extension of competition graphs under complex fuzzy environment.

(1) Introduced are the concepts of IVCPFS.
(2) More specifically, we demonstrate the idea of IVCPFGS, as well as the results of some regular IVCPFGS and totally regular IVCPFGS, as well as the degree of vertex present.
(3) It was also discussed how an IVCPFGS is divided, how an IVCPFGS is ψ-complemented, and how strong a Path is within an IVCPFGS.
(4) We also discuss about using interval-valued complex pythagorean fuzzy sets to analyse the morphology of the climate in kale.
(5) Furthermore, Kale Climate Morphology IVCPFGS Application.

The paper is structured as follows: 1. The Preliminaries Section provides the fundamental definitions that are required. The IVCPFGS are offered, applied to vertex degree and total degree analysis, and certain IVCPFGS regular and entirely regular IVCPFGS characteristics. In section 3. subdivision of an IVCPFGS, the IVCPFGS that ψ-complements it, and the strength of a Path within an IVCPFGS. Within section 4. The capacity to predict atmospheric temperature is critical for many applications, which brings us to our main conclusion. Because of how constantly changing the atmosphere is, it is difficult to make any form of reliable weather predictions. The IVCPFGS concept can be used to analyse the climate change data in sections 5 and 6.

2. PRELIMINARIES

The description of some basic definitions and properties in this area will aid in the development of the research work.

Definition 2.1. Let's say that conversation is the universe Y. Interval-Valued Complex Pythagorean Fuzzy Set (IVCPFS) A defined on Y is the object of the form.

$$A = \{(r, [\mu^-_{A_1}(r)e^{i\alpha^-_{A_1}(r)}, \mu^+_{A_1}(r)e^{i\alpha^+_{A_1}(r)}],\ [\mu^-_{A_2}(r)e^{i\alpha^-_{A_2}(r)}, \mu^+_{A_2}(r)e^{i\alpha^+_{A_2}(r)}] : r \in Y\}$$

where $i = \sqrt{-1},\ \mu^-_{A_1}(r), \mu^+_{A_1}(r), \mu^-_{A_2}(r), \mu^+_{A_2}(r) \in [0,1],\ \alpha^-_{A_1}(r), \alpha^+_{A_1}(r)$, $\alpha^-_{A_2}(r), \alpha^+_{A_2}(r) \in [0, 2\pi], 0 \leq (\mu^+_{A_1}(r))^2+(\mu^+_{A_2}(r))^2 \leq 1,\ 0 \leq (\alpha^+_{A_1}(r))^2+(\alpha^+_{A_1}(r))^2 \leq 2\pi$.

Definition 2.2. Let $A = \{(r, [\mu^-_{A_1}(r)e^{i\alpha^-_{A_1}(r)}, \mu^+_{A_1}(r)e^{i\alpha^+_{A_1}(r)}]$, $[\mu^-_{A_2}(r)e^{i\alpha^-_{A_2}(r)}, \mu^+_{A_2}(r)e^{i\alpha^+_{A_2}(r)}] : r \in Y\}$ and $B = \{(r, [\mu^-_{B_1}(r)e^{i\alpha^-_{B_1}(r)}$, $\mu^+_{B_1}(r)e^{i\alpha^+_{B_1}(r)}], [\mu^-_{B_2}(r)e^{i\alpha^-_{B_2}(r)}, \mu^+_{B_2}(r)e^{i\alpha^+_{B_2}(r)}] : r \in Y\}$ be the Two IVCPFSs in Y, then

- $A \subseteq B$ if and only if $\mu^-_{A_1}(r) \leq \mu^-_{B_1}(r),\ \mu^+_{A_1}(r) \leq \mu^+_{B_1}(r)$ and $\mu^-_{A_2}(r) \leq \mu^-_{B_2}(r),\ \mu^+_{A_2}(r) \leq \mu^+_{B_2}(r)$ for amplitude terms and $\alpha^-_{A_1}(r) \leq \alpha^-_{B_1}(r),\ \alpha^+_{A_1}(r) \leq \alpha^+_{A_1}(r)$ and $\alpha^-_{A_2}(r) \leq \alpha^-_{B_2}(r),\ \alpha^+_{A_2}(r) \leq \alpha^+_{A_2}(r)$ for phase terms, for all $r \in Y$;
- $A = B$ if and only if $\mu^-_{A_1}(r) = \mu^-_{B_1}(r),\ \mu^+_{A_1}(r) = \mu^+_{B_1}(r)$ and $\mu^-_{A_2}(r) = \mu^-_{B_2}(r),\ \mu^+_{A_2}(r) = \mu^+_{B_2}(r)$ for amplitude terms and $\alpha^-_{A_1}(r) = \alpha^-_{B_1}(r),\ \alpha^+_{A_1}(r) = \alpha^+_{A_1}(r)$ and $\alpha^-_{A_2}(r) = \alpha^-_{B_2}(r),\ \alpha^+_{A_2}(r) = \alpha^+_{A_2}(r)$ for phase terms, for all $r \in Y$;
- $\bar{A} = \{(r, [\mu^-_{A_2}(r)e^{i\alpha^-_{A_2}(r)}, \mu^+_{A_2}(r)e^{i\alpha^+_{A_2}(r)}], [\mu^-_{A_1}(r)e^{i\alpha^-_{A_1}(r)}, \mu^+_{A_1}(r)e^{i\alpha^+_{A_1}(r)}] : r \in Y\}$

For simplicity, the $([\mu^-_{A_1}(r)e^{i\alpha^-_{A_1}(r)}, \mu^+_{A_1}(r)e^{i\alpha^+_{A_1}(r)}], [\mu^-_{A_2}(r)e^{i\alpha^-_{A_2}(r)}, \mu^+_{A_2}(r)e^{i\alpha^+_{A_2}(r)}])$ is called the IVCPFS, where, $\mu^+_{A_1}, \mu^+_{A_2} \in [0,1]$ such that ${\mu^+_{A_1}}^2 + {\mu^+_{A_2}}^2 \leq 1$ and $\alpha^+_{A_1}, \alpha^+_{A_2} \in [0, 2\pi]$ such that ${\alpha^+_{A_1}}^2 + {\alpha^+_{A_2}}^2 \leq 2\pi$.

Definition 2.3. A Interval-valued complex Pythagorean fuzzy relation in Y is described as a IVCPFS X in $Y \times Y$ and is characterised by

$$\begin{aligned} X = \{&(rs, [\mu^-_{X_1}(rs)e^{i\alpha^-_{X_1}(rs)}, \mu^+_{X_1}(rs)e^{i\alpha^+_{X_1}(rs)}], \\ &[\mu^-_{X_2}(rs)e^{i\alpha^-_{X_2}(rs)}, \mu^+_{X_2}(rs)e^{i\alpha^+_{X_2}(rs)}]) : rs \in Y \times Y\} \end{aligned}$$

where the Inter-valued complex Pythagorean fuzzy membership and non-membership functions of X are mapping to [0,1], such that $0 \leq {\mu^+_{X_1}}^2(rs) + {\mu^+_{X_2}}^2(rs) \leq 1$ and $0 \leq {\alpha^+_{X_1}}^2(rs) + {\mu^+_{X_2}}^2(rs) \leq 2\pi$ for all $rs \in Y \times Y$.

Definition 2.4. On a non-empty set X, a Interval-valued complex Pythagorean fuzzy graph is a pair $G = (A, B)$, where A and B are complex Pythagorean fuzzy sets on X and a Interval-valued complex Pythagorean fuzzy relation on X, respectively, such that:

$$
\begin{aligned}
(i)\mu^-_{B_1}(rs)e^{i\alpha^-_{B_1}(rs)} &\leq \min\{\mu^-_{A_1}(r), \mu^-_{A_1}(s)\}e^{i\min\{\alpha^-_{A_1}(r),\alpha^-_{A_1}(s)\}}\\
(ii)\mu^+_{B_1}(rs)e^{i\alpha^+_{B_1}(rs)} &\leq \min\{\mu^+_{A_1}(r), \mu^+_{A_1}(s)\}e^{i\min\{\alpha^+_{A_1}(r),\alpha^+_{A_1}(s)\}}\\
(iii)\mu^-_{B_2}(rs)e^{i\alpha^-_{B_2}(rs)} &\leq \max\{\mu^-_{A_2}(r), \mu^-_{A_2}(s)\}e^{i\max\{\alpha^-_{A_2}(r),\alpha^-_{A_2}(s)\}}\\
(iv)\mu^+_{B_2}(rs)e^{i\alpha^+_{B_2}(rs)} &\leq \max\{\mu^+_{A_2}(r), \mu^+_{A_2}(s)\}e^{i\max\{\alpha^+_{A_2}(r),\alpha^+_{A_2}(s)\}}
\end{aligned}
$$

$0 \leq {\mu^+_{B_1}}^2(rs)+{\mu^+_{B_2}}^2(rs) \leq 1$ and $0 \leq {\alpha^+_{B_1}}^2(rs)+{\alpha^+_{B_2}}^2(rs) \leq 2\pi$ for all $rs \in Y\times Y$.

3. Regular IVCPFGS

The idea of regular IVCPFGS is introduced in this section. Examples are also given to clarify some of the standard IVCPFGS properties.

Definition 3.1. Let $\tilde{G} = \{\mu, \lambda_1, \lambda_2, ..., \lambda_k\}$ is referred to as an IVCPFGS of graph structure (GS) $G^* = \{Q, R_1, R_2, ..., R_k\}$ if $\mu = (\mu_1, \mu_2) = ([\mu_1^- e^{i\alpha_1^-}, \mu_1^+ e^{i\alpha_1^+}],$ $[\mu_2^- e^{i\alpha_2^-}, \mu_2^+ e^{i\alpha_2^+}])$ is an IVCPFS on Q and $\lambda_J = (\lambda_{1J}, \lambda_{2J}) = ([\lambda^-_{1J}e^{i\beta^-_{1J}}, \lambda^+_{1J}e^{i\beta^+_{1J}}],$ $[\lambda^-_{2J}e^{i\beta^-_{2J}}, \lambda^+_{2J}e^{i\beta^+_{2J}}])$ are IVCPFSs on Q and R_J such that

$$
\begin{aligned}
(i)\lambda^-_{1J}(a,b)e^{i\beta^-_{1J}(a,b)} &\leq \min\{\mu_1^-(a), \mu_1^-(b)\}e^{i\min\{\alpha_1^-(a),\alpha_1^-(b)\}}\\
(ii)\lambda^+_{1J}(a,b)e^{i\beta^+_{1J}(a,b)} &\leq \min\{\mu_1^+(a), \mu_1^+(b)\}e^{i\min\{\alpha_1^+(a),\alpha_1^+(b)\}}\\
(iii)\lambda^-_{2J}(a,b)e^{i\beta^-_{2J}(a,b)} &\leq \max\{\mu_2^-(a), \mu_2^-(b)\}e^{i\max\{\alpha_2^-(a),\alpha_2^-(b)\}}\\
(iv)\lambda^+_{2J}(a,b)e^{i\beta^+_{2J}(a,b)} &\leq \max\{\mu_2^+(a), \mu_2^+(b)\}e^{i\max\{\alpha_2^+(a),\alpha_2^+(b)\}}
\end{aligned}
$$

$0 \leq (\lambda^+_{1J}(a,b))^2 + (\lambda^+_{2J}(a,b))^2 \leq 1$ and $0 \leq (\beta^+_{1J}(ab))^2 + (\beta^+_{2J}(ab))^2 \leq 2\pi\ \forall\ ab \in R_J, J = 1, 2, ..., k$.

Note: λ^-_{1J}, λ^+_{1J}, λ^-_{2J} and λ^+_{2J} are function from R_J to [0,1] such that $\lambda^-_{1J}(a,b) \leq \lambda^+_{1J}(a,b)$ and $\lambda^-_{2J}(a,b) \leq \lambda^+_{2J}(a,b)$ and $\beta^-_{1J}(a,b) \leq \beta^+_{1J}(a,b)$ and $\beta^-_{2J}(a,b) \leq \beta^+_{2J}(a,b)$ for all $(a,b) \in R_J, J = 1, 2, ..., k$.

Example 3.2. An IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ of a GS $G^* = (Q, R_1, R_2)$ given figure-2 is a IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ such that $\mu = \{u_1([0.4e^{i1.2\pi}, 0.6e^{i1.6\pi}],$ $[0.3e^{i1.2\pi}, 0.7e^{i1.4\pi}]), u_2([0.5e^{i0.8\pi}, 0.6e^{i1.0\pi}], [0.4e^{i0.8\pi}, 0.6e^{i1.2\pi}])\ u_3([0.5e^{i0.8\pi}, 0.8e^{i1.0\pi}],$ $[0.4e^{i1.0\pi}, 0.7e^{i1.2\pi}]), u_4([0.3e^{i0.6\pi}, 0.5e^{i1.8\pi}], [0.4e^{i1.4\pi}, 0.5e^{i1.2\pi}])\}$.

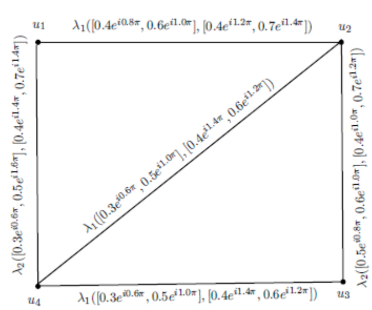

FIGURE 2. $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ is IVCPFGS of G_1^*

Definition 3.3. Let $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ be an IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$. The $\lambda_{1J}^- - degree$ of vertex u is the sum of $\lambda_{1J}^- - edge$ starting from u. it is denoted by $d_{\lambda_{1J}^-}(u)$. Thus, $d_{\lambda_{1J}^-}(u) = \sum_{(u,v)\in R_J} \lambda_{1J}^-(u,v)e^{i\sum_{(u,v)\in R_J} \beta_{1J}^-(u,v)}$, the $\lambda_{1J}^+ - degree$ of vertex u is the sum of $\lambda_{1J}^+ - edge$ starting from u. it is denoted by $d_{\lambda_{1J}^+}(u)$. Thus, $d_{\lambda_{1J}^+}(u) = \sum_{(u,v)\in R_J} \lambda_{1J}^+(u,v)e^{i\sum_{(u,v)\in R_J} \beta_{1J}^+(u,v)}$ and the $\lambda_{2J}^- - degree$ of vertex u is the sum of $\lambda_{2J}^- - edge$ starting from u. it is denoted by $d_{\lambda_{2J}^-}(u)$. Thus, $d_{\lambda_{2J}^-}(u) = \sum_{(u,v)\in R_J} \lambda_{2J}^-(u,v)$
$e^{i\sum_{(u,v)\in R_J} \beta_{2J}^-(u,v)}$, the $\lambda_{2J}^+ - degree$ of vertex u is the sum of $\lambda_{2J}^+ - edge$ starting from u. it is denoted by $d_{\lambda_{2J}^+}(u)$.
Thus, $d_{\lambda_{2J}^+}(u) = \sum_{(u,v)\in R_J} \lambda_{2J}^+(u,v)e^{i\sum_{(u,v)\in R_J} \beta_{2J}^+(u,v)}$

Definition 3.4. The $\lambda_J - degree$ of vertex u is defined as
$d_{\lambda_J}(u) = ([d_{\lambda_{1J}}^-(u), d_{\lambda_{1J}}^+(u)], [d_{\lambda_{2J}}^-(u), d_{\lambda_{2J}}^+(u)])$ (or)
$d_{\lambda_J}(u) = ([\sum_{(u,v)\in R_J} \lambda_{1J}^-(u,v), \sum_{(u,v)\in R_J} \lambda_{1J}^+(u,v)],$
$[\sum_{(u,v)\in R_J} \lambda_{2J}^-(u,v), \sum_{(u,v)\in R_J} \lambda_{2J}^+(u,v)]),$
for all $J = 1, 2, ..., k$.

Definition 3.5. A IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ is $\lambda_J - strong$ if

$$\begin{aligned}
\lambda_{1J}^-(u,v)e^{i\beta_{1J}^-(u,v)} &= \min\{\mu_1^-(u), \mu_1^-(v)\}e^{i\min\{\alpha_1^-(u),\alpha_1^-(v)\}},\\
\lambda_{1J}^+(u,v)e^{i\beta_{1J}^+(u,v)} &= \min\{\mu_1^+(u), \mu_1^+(v)\}e^{i\min\{\alpha_1^+(u),\alpha_1^+(v)\}},\\
\lambda_{2J}^-(u,v)e^{i\beta_{2J}^-(u,v)} &= \max\{\mu_2^-(u), \mu_2^-(v)\}e^{i\max\{\alpha_2^-(u),\alpha_2^-(v)\}},\\
\lambda_{2J}^+(u,v)e^{i\beta_{2J}^+(u,v)} &= \max\{\mu_2^+(u), \mu_2^+(v)\}e^{i\max\{\alpha_2^+(u),\alpha_2^+(v)\}},\\
&\quad \text{for all } (r,s) \in R_J, J = 1, 2, ..., k.
\end{aligned}$$

If $\tilde{G}$ is $\lambda_J - strong$ for all $J = 1, 2, ..., k$, then $\tilde{G}$ is called $\lambda_J - strong$ IVCPFGS.

Example 3.6. Consider a IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$. Then the degree of a $\lambda_J - strong$ vertex is shown in Figure-2 for all $J = 1, 2$.
The degree of a $\lambda_1 - strong$ vertex u_i, i=1,2,3,4 is

$$\begin{aligned}
d_{\lambda_1}(u_1) &= ([d_{\lambda_{11}^-}(u_1), d_{\lambda_{11}^+}(u_1)], [d_{\lambda_{21}^-}(u_1), d_{\lambda_{21}^+}(u_1)])\\
d_{\lambda_1}(u_1) &= ([0.4e^{i0.8\pi}, 0.6e^{i1.0\pi}], [0.4e^{i1.2\pi}, 0.7e^{i1.4\pi}])\\
d_{\lambda_1}(u_2) &= ([0.7e^{i1.4\pi}, 1.1e^{i2.0\pi}], [0.8e^{i2.6\pi}, 1.3e^{i2.6\pi}])\\
d_{\lambda_1}(u_3) &= ([0.3e^{i0.6\pi}, 0.5e^{i1.0\pi}], [0.4e^{i1.4\pi}, 0.6e^{i1.2\pi}])\\
d_{\lambda_1}(u_4) &= ([0.6e^{i1.2\pi}, 1.0e^{i2.0\pi}], [0.8e^{i2.8\pi}, 1.2e^{i2.4\pi}])
\end{aligned}$$

The degree of a $\lambda_2 - strong$ vertex u_i, i=1,2,3,4 is

$$\begin{aligned}
d_{\lambda_2}(u_1) &= ([d_{\lambda_{12}^-}(u_1), d_{\lambda_{12}^+}(u_1)], [d_{\lambda_{22}^-}(u_1), d_{\lambda_{22}^+}(u_1)])\\
d_{\lambda_2}(u_1) &= ([0.3e^{i0.6\pi}, 0.5e^{i1.6\pi}], [0.4e^{i1.4\pi}, 0.7e^{i1.4\pi}])\\
d_{\lambda_2}(u_2) &= ([0.5e^{i0.8\pi}, 0.6e^{i1.0\pi}], [0.4e^{i1.0\pi}, 0.7e^{i1.2\pi}])\\
d_{\lambda_2}(u_3) &= ([0.5e^{i0.8\pi}, 0.6e^{i1.0\pi}], [0.4e^{i1.0\pi}, 0.7e^{i1.2\pi}])\\
d_{\lambda_2}(u_4) &= ([0.3e^{i0.6\pi}, 0.5e^{i1.6\pi}], [0.4e^{i1.4\pi}, 0.7e^{i1.4\pi}])
\end{aligned}$$

Theorem 3.7. *Let $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ be an IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$. Then*

$$\sum_{i=1}^{n} d_{\lambda_J}(u_i) = ([2 \sum_{(u_i,v)\in R_J} \lambda_{1J}^{-}(u_i, v) e^{i2\sum_{(u_i,v)\in R_J} \beta_{1J}^{-}(u_i,v)}, 2 \sum_{(u_i,v)\in R_J} \lambda_{1J}^{+}(u_i, v) e^{i2\sum_{(u_i,v)\in R_J} \beta_{1J}^{+}(u_i,v)}], [2 \sum_{(u_i,v)\in R_J} \lambda_{2J}^{-}(u_i, v) e^{i2\sum_{(u_i,v)\in R_J} \beta_{2J}^{-}(u_i,v)}, 2 \sum_{(u_i,v)\in R_J} \lambda_{2J}^{+}(u_i, v) e^{i2\sum_{(u_i,v)\in R_J} \beta_{2J}^{+}(u_i,v)}])$$

is $\lambda_J - strong$ IVCPFGS for all $J = 1, 2, ..., k$.

Proof. Let $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ be an IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$. Let us consider $\tilde{G}$ has a n vertices such that $\mu_1^{-}(u_i)e^{i\alpha_1^{-}(u_i)}, \mu_1^{+}(u_i)e^{i\alpha_1^{+}(u_i)}$, $\mu_2^{-}(u_i)e^{i\alpha_2^{-}(u_i)}, \mu_2^{+}(u_i)e^{i\alpha_2^{+}(u_i)}$ for all $(i = 1, 2, ..., n)$. Since each edge is incident on two vertices, we can infer from the definition-3.3,3.5 of degree of any $\lambda_J - strong$ vertex of an IVCPFGS of GS that the sum of all $\lambda_J - strong$ vertices in the IVCPFGS $\tilde{G}$ is twice that of the degree of membership of the sum of all $\lambda_{1J}^{-} - edges$ for all $J = 1, 2, ..., k$ and twice the degree of membership of sum of $\lambda_{1J}^{+} - edges$ for all $J = 1, 2, ..., k$ in $\tilde{G}$ and twice the degree of membership of sum of $\lambda_{2J}^{-} - edges$ for all $J = 1, 2, ..., k$ in $\tilde{G}$ and twice the degree of membership of sum of $\lambda_{2J}^{+} - edges$ for all $J = 1, 2, ..., k$ in $\tilde{G}$. Thus, $\sum_{i=1}^{n} d_{\lambda_J}(u_i) = ([2\sum_{(u_i,v)\in R_J} \lambda_{1J}^{-}(u_i, v)e^{i2\sum_{(u_i,v)\in R_J} \beta_{1J}^{-}(u_i,v)}, 2\sum_{(u_i,v)\in R_J} \lambda_{1J}^{+}(u_i, v) e^{i2\sum_{(u_i,v)\in R_J} \beta_{1J}^{+}(u_i,v)}], [2\sum_{(u_i,v)\in R_J} \lambda_{2J}^{-}(u_i, v)e^{i2\sum_{(u_i,v)\in R_J} \beta_{2J}^{-}(u_i,v)}, 2\sum_{(u_i,v)\in R_J} \lambda_{2J}^{+}(u_i, v)e^{i2\sum_{(u_i,v)\in R_J} \beta_{2J}^{+}(u_i,v)}])$ □

Example 3.8. Next, we show the example of the above theorem-3.7. Let us Consider a IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ as shown in figure-2. Then $\sum_{i=1}^{4} d_{\lambda_J}(u_i) = ([2\sum_{(u_i,v)\in R_J} \lambda_{1J}^{-}(u_i, v)e^{i2\sum_{(u_i,v)\in R_J} \beta_{1J}^{-}(u_i,v)}, 2\sum_{(u_i,v)\in R_J} \lambda_{1J}^{+}(u_i, v)e^{i2\sum_{(u_i,v)\in R_J} \beta_{1J}^{+}(u_i,v)}], [2\sum_{(u_i,v)\in R_J} \lambda_{2J}^{-}(u_i, v)e^{i2\sum_{(u_i,v)\in R_J} \beta_{2J}^{-}(u_i,v)}, 2\sum_{(u_i,v)\in R_J} \lambda_{2J}^{+}(u_i, v)e^{i2\sum_{(u_i,v)\in R_J} \beta_{2J}^{+}(u_i,v)}])$
is
$\lambda_J - strong$ IVCPFGS for all $J = 1, 2$.
Twice the degree of sum of $\lambda_1 - edges$ in $\tilde{G}$ is

$$\begin{aligned} 2 \sum_{(u_i,v)\in R_1} \lambda_{11}^{-}(u_i, v)e^{i2\sum_{(u_i,v)\in R_1} \beta_{11}^{-}(u_i,v)} &= 2(\lambda_{11}^{-}(u_1, u_2) + \lambda_{11}^{-}(u_2, u_4) + \lambda_{11}^{-}(u_3, u_4)) \\ &\quad e^{i2(\beta_{11}^{-}(u_1,u_2)+\beta_{11}^{-}(u_2,u_4)+\beta_{11}^{-}(u_3,u_4))} \\ &= 2(0.4 + 0.3 + 0.3)e^{i2(0.8\pi+0.6\pi+0.6\pi)} \\ &= 2.0e^{i4.0\pi} \end{aligned}$$

$$\begin{aligned}
2\sum_{(u_i,v)\in R_1}\lambda^{+}_{11}(u_i,v)e^{i2\sum_{(u_i,v)\in R_1}\beta^{+}_{11}(u_i,v)} &= 2(\lambda^{+}_{11}(u_1,u_2)+\lambda^{+}_{11}(u_2,u_4)+\lambda^{+}_{11}(u_3,u_4))\\
&\quad e^{i2(\beta^{+}_{11}(u_1,u_2)+\beta^{+}_{11}(u_2,u_4)+\beta^{+}_{11}(u_3,u_4))}\\
&= 2(0.6+0.5+0.5)e^{i2(1.0\pi+1.0\pi+1.0\pi)}\\
&= 3.2e^{i6.0\pi}
\end{aligned}$$

$$\begin{aligned}
2\sum_{(u_i,v)\in R_1}\lambda^{-}_{21}(u_i,v)e^{i2\sum_{(u_i,v)\in R_1}\beta^{-}_{21}(u_i,v)} &= 2(\lambda^{-}_{21}(u_1,u_2)+\lambda^{-}_{21}(u_2,u_4)+\lambda^{-}_{21}(u_3,u_4))\\
&\quad e^{i2(\beta^{-}_{21}(u_1,u_2)+\beta^{-}_{21}(u_2,u_4)+\beta^{-}_{21}(u_3,u_4))}\\
&= 2(0.4+0.4+0.4)e^{i2(1.2\pi+1.4\pi+1.4\pi)}\\
&= 2.4e^{i8.0\pi}
\end{aligned}$$

$$\begin{aligned}
2\sum_{(u_i,v)\in R_1}\lambda^{+}_{21}(u_i,v)e^{i2\sum_{(u_i,v)\in R_1}\beta^{+}_{21}(u_i,v)} &= 2(\lambda^{+}_{21}(u_1,u_2)+\lambda^{+}_{21}(u_2,u_4)+\lambda^{+}_{21}(u_3,u_4))\\
&\quad e^{i2(\beta^{+}_{21}(u_1,u_2)+\beta^{+}_{21}(u_2,u_4)+\beta^{+}_{21}(u_3,u_4))}\\
&= 2(0.7+0.6+0.6)e^{i2(1.4\pi+1.2\pi+1.2\pi)}\\
&= 3.8e^{i7.6\pi}
\end{aligned}$$

Degree of $\lambda_1 - strong$ vertices in IVCPFGS is given by Example-3.6.

$$\begin{aligned}
\sum_{i=1}^{4} d_{\lambda_1}(u_i) &= ([\sum_{i=1}^{4} d_{\lambda^{-}_{1J}}(u_i), \sum_{i=1}^{4} d_{\lambda^{+}_{1J}}(u_i)], [\sum_{i=1}^{4} d_{\lambda^{-}_{2J}}(u_i), \sum_{i=1}^{4} d_{\lambda^{+}_{2J}}(u_i)])\\
&= ([2.0e^{i4.0\pi}, 3.2e^{i6.0\pi}], [2.4e^{i8.0\pi}, 3.8e^{i7.6\pi}])
\end{aligned}$$

$$\begin{aligned}
\sum_{i=1}^{4} d_{\lambda_1}(u_i) &= ([2\sum_{(u_i,v)\in R_1}\lambda^{-}_{11}(u_i,v)e^{i2\sum_{(u_i,v)\in R_1}\beta^{-}_{11}(u_i,v)}, 2\sum_{(u_i,v)\in R_1}\lambda^{+}_{11}(u_i,v)\\
&\quad e^{i2\sum_{(u_i,v)\in R_1}\beta^{+}_{11}(u_i,v)}], [2\sum_{(u_i,v)\in R_1}\lambda^{-}_{21}(u_i,v)e^{i2\sum_{(u_i,v)\in R_1}\beta^{-}_{21}(u_i,v)},\\
&\quad 2\sum_{(u_i,v)\in R_1}\lambda^{+}_{21}(u_i,v)e^{i2\sum_{(u_i,v)\in R_1}\beta^{+}_{21}(u_i,v)}])
\end{aligned}$$

Similarly, we calculate

$$\begin{aligned}
\sum_{i=1}^{4} d_{\lambda_2}(u_i) &= ([2\sum_{(u_i,v)\in R_2}\lambda^{-}_{12}(u_i,v)e^{i2\sum_{(u_i,v)\in R_2}\beta^{-}_{12}(u_i,v)}, 2\sum_{(u_i,v)\in R_2}\lambda^{+}_{12}(u_i,v)\\
&\quad e^{i2\sum_{(u_i,v)\in R_2}\beta^{+}_{12}(u_i,v)}], [2\sum_{(u_i,v)\in R_2}\lambda^{-}_{22}(u_i,v)e^{i2\sum_{(u_i,v)\in R_2}\beta^{-}_{22}(u_i,v)},\\
&\quad 2\sum_{(u_i,v)\in R_1}\lambda^{+}_{22}(u_i,v)e^{i2\sum_{(u_i,v)\in R_1}\beta^{+}_{22}(u_i,v)}])\\
&= ([1.6e^{i2.8\pi}, 2.2e^{i5.2\pi}], [1.6e^{i4.8\pi}, 2.8e^{i5.2\pi}])
\end{aligned}$$

Definition 3.9. Let $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ be an IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$. If $d_{\lambda_J}(u_i) = ([a_1, b_1], [a_2, b_2])$ for all $u_i \in Q$ for each vertex of $\lambda^-_{1J} - degree$ of vertex has the same degree a_1 and for each vertex of $\lambda^+_{1J} - degree$ of vertex has the same degree b_1 and for each vertex of $\lambda^-_{2J} - degree$ of vertex has the same degree a_2 and for each vertex of $\lambda^+_{2J} - degree$ of vertex has the same degree b_2. Then $\tilde{G}$ is said to be λ_J regular IVCPFGS for all $J = 1, 2, ..., k$.

Example 3.10. An IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ of a GS $G^* = (Q, R_1, R_2)$ given figure-2 is a IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ such that $\mu = \{u_1([0.4e^{i1.2\pi}, 0.6e^{i1.6\pi}], [0.3e^{i1.2\pi}, 0.7e^{i1.4\pi}]), u_2([0.5e^{i0.8\pi}, 0.6e^{i1.0\pi}], [0.4e^{i0.8\pi}, 0.6e^{i1.2\pi}])$ $u_3([0.4e^{i1.2\pi}, 0.6e^{i1.6\pi}], [0.3e^{i1.2\pi}, 0.7e^{i1.4\pi}]),\ u_4([0.5e^{i0.8\pi}, 0.6e^{i1.0\pi}], [0.4e^{i0.8\pi}, 0.6e^{i1.2\pi}])\}$. The degree of a $\lambda_J - strong$ vertex $u_i,\ i = 1, 2, 3, 4$ and $J = 1, 2$.

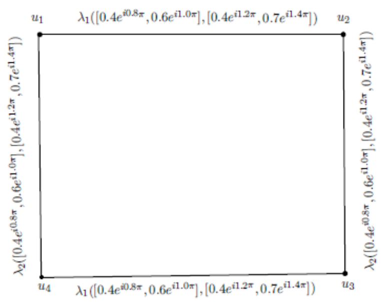

FIGURE 3. $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ is IVCPFGS of G^*_1

$$d_{\lambda_J}(u_i) = ([0.4e^{i0.8\pi}, 0.6e^{i1.0\pi}], [0.4e^{i1.2\pi}, 0.7e^{i1.4\pi}]), \forall i = 1, 2, 3, 4 \text{ and } J = 1, 2.$$

Hence, $\tilde{G}$ is λ_J regular IVCPFGS for all $J = 1, 2$.

Theorem 3.11. *Let $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ be an IVCPFGS of GS $G^* = (V, R_1, R_2, ..., R_k)$. Then G^* be an odd λ_J−cycle, $\tilde{G}$ is λ_J regular if and only if $\lambda^-_{1J}(u, v), \lambda^+_{1J}(u, v)$ and $\lambda^-_{2J}(u, v), \lambda^+_{2J}(u, v)$ are constant for all $\lambda_J - edges$ in R_J.*

Proof. Let $\lambda_J(u, v) = ([a_1, b_1][a_2, b_2])$ are constant function for all $(u, v) \in R_J$
ie, $d_{\lambda_J}(u_i) = ([2a_1, 2b_1], [2a_2, 2b_2])$ for all $J = 1, 2, ..., k$.
So, $\tilde{G}$ is λ_J regular. Conversely,
Suppose that $\tilde{G}$ be a λ_J regular. and Let λ_J edges of R_J be $e_1, e_2, ..., e_{2n+1}$.
Let $\lambda_J(e_1) = ([r_1, s_1][r_2, s_2])$.
Since, $\tilde{G}$ be a λ_J regular.

$$\begin{aligned} \lambda^-_{1J}(e_2) &= r - r_1, \\ \lambda^-_{1J}(e_3) &= r - (r - r_1) = r_1, ..., \\ \lambda^-_{1J}(e_m) &= \begin{cases} r_1 \ if \ m \text{ is odd} \\ r - r_1 \ if \ m \text{ is even} \end{cases} \end{aligned}$$

Similarly, we can solve $\lambda^+_{1J}(e_i), \lambda^-_{2J}(e_i)$ and $\lambda^+_{2J}(e_i)$ for all $i = 2, 3, ..., m$
Hence, $\lambda^-_{1J}(e_1) = \lambda^-_{1J}(e_{2n+1}),\ \lambda^+_{1J}(e_1) = \lambda^+_{1J}(e_{2n+1}), \lambda^-_{2J}(e_1) = \lambda^-_{2J}(e_{2n+1})$ and

$\lambda_{2J}^{+}(e_1) = \lambda_{2J}^{+}(e_{2n+1})$.
So, if e_1 and e_{2n+1} are incident with u. Then
$d_{\lambda_{1J}^{-}}(u) = r_1 + r_1 = 2r_1,\ d_{\lambda_{1J}^{+}}(u) = s_1 + s_1 = 2s_1, d_{\lambda_{2J}^{-}}(u) = r_2 + r_2 = 2r_2$ and $d_{\lambda_{2J}^{+}}(u) = s_2 + s_2 = 2s_2$.
$\Rightarrow d_{\lambda_{1J}^{-}}(u) = r,\ d_{\lambda_{1J}^{+}}(u) = s,\ d_{\lambda_{1J}^{-}}(u) = r'$ and $d_{\lambda_{1J}^{-}}(u) = s'$
$\Rightarrow 2r_1 = r,\ 2s_1 = s,\ 2r_2 = r'$ and $2s_2 = s'$
$\Rightarrow r_1 = \frac{r}{2},\ s_1 = \frac{s}{2},\ r_2 = \frac{r'}{2}$ and $s_2 = \frac{s'}{2}$
$\Rightarrow \lambda_{1J}^{-}(u,v), \lambda_{1J}^{+}(u,v)$ and $\lambda_{2J}^{-}(u,v), \lambda_{2J}^{+}(u,v)$ are constant for all $\lambda_J - edges$ in R_J. $\square$

Definition 3.12. Let $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ be an IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$. The total degree of λ_J vertex is defined as

$$td_{\lambda_J}(u) = ([td_{\lambda_{1J}^{-}}(u), td_{\lambda_{1J}^{+}}(u)], [td_{\lambda_{2J}^{-}}(u), td_{\lambda_{2J}^{-}}(u)])$$

$$\begin{aligned}
td_{\lambda_{1J}^{-}}(u) &= (\sum_{(u,v)\in R_J} \lambda_{1J}^{-}(u,v) + \mu_1^{-}(u))e^{i\sum_{(u,v)\in R_J}\beta_{1J}^{-}(u,v)+\alpha_1^{-}(u)},\\
td_{\lambda_{1J}^{+}}(u) &= (\sum_{(u,v)\in R_J} \lambda_{1J}^{+}(u,v) + \mu_1^{+}(u))e^{i\sum_{(u,v)\in R_J}\beta_{1J}^{+}(u,v)+\alpha_1^{+}(u)},\\
td_{\lambda_{2J}^{-}}(u) &= (\sum_{(u,v)\in R_J} \lambda_{2J}^{-}(u,v) + \mu_2^{-}(u))e^{i\sum_{(u,v)\in R_J}\beta_{2J}^{-}(u,v)+\alpha_1^{-}(u)},\\
td_{\lambda_{2J}^{+}}(u) &= (\sum_{(u,v)\in R_J} \lambda_{2J}^{+}(u,v) + \mu_2^{+}(u))e^{i\sum_{(u,v)\in R_J}\beta_{2J}^{+}(u,v)+\alpha_2^{+}(u)}
\end{aligned}$$

For each vertex of λ_{1J}^{-} total degree of vertex has the same degree n_1 and for each vertex of λ_{1J}^{+} total degree of vertex has the same degree m_1 and for each vertex of λ_{2J}^{-} total degree of vertex has the same degree n_1 and for each vertex of λ_{2J}^{+} total degree of vertex has the same degree m_2. Then $\tilde{G}$ is said to be totally λ_J regular IVCPFGS for all $J = 1, 2, ..., k$.

Example 3.13. Consider IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ of a GS $G^* = (Q, R_1, R_2)$ given figure-2. The total degree of a $\lambda_J - strong$ vertex $u_i,\ i = 1, 2, 3, 4$ and $J = 1, 2$.

$$\begin{aligned}
td_{\lambda_1}(u_1) &= ([0.8e^{i2.0\pi}, 1.2e^{i2.6\pi}], [0.7e^{i2.4\pi}, 1.4e^{i2.8\pi}]),\\
td_{\lambda_1}(u_2) &= ([0.9e^{i1.6\pi}, 1.2e^{i2.0\pi}], [0.8e^{i2.0\pi}, 1.3e^{i2.6\pi}]),\\
td_{\lambda_1}(u_3) &= ([0.8e^{i2.0\pi}, 1.2e^{i2.6\pi}], [0.7e^{i2.4\pi}, 1.4e^{i2.8\pi}]),\\
td_{\lambda_1}(u_4) &= ([0.9e^{i1.6\pi}, 1.2e^{i2.0\pi}], [0.8e^{i2.0\pi}, 1.3e^{i2.6\pi}]),\\
td_{\lambda_2}(u_1) &= ([0.8e^{i2.0\pi}, 1.2e^{i2.6\pi}], [0.7e^{i2.4\pi}, 1.4e^{i2.8\pi}]),\\
td_{\lambda_2}(u_2) &= ([0.9e^{i1.6\pi}, 1.2e^{i2.0\pi}], [0.8e^{i2.0\pi}, 1.3e^{i2.6\pi}]),\\
td_{\lambda_2}(u_3) &= ([0.8e^{i2.0\pi}, 1.2e^{i2.6\pi}], [0.7e^{i2.4\pi}, 1.4e^{i2.8\pi}]),\\
td_{\lambda_2}(u_4) &= ([0.9e^{i1.6\pi}, 1.2e^{i2.0\pi}], [0.8e^{i2.0\pi}, 1.3e^{i2.6\pi}])
\end{aligned}$$

Hence, $\tilde{G}$ is not totally λ_J regular IVCPFGS for all $J = 1, 2$.

Remark 3.14. If $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ is λ_J regular, then $a_1 = r_1(n-1)$, $b_1 = s_1(n-1)$, $a_2 = r_2(n-1)$ and $b_2 = s_2(n-1)$ where $r_1 = \mu_1^-(u_i),\ s_1 = \mu_1^+(u_i),\ r_2 = \mu_2^-(u_i)$ and $s_2 = \mu_2^+(u_i)\ \forall\ i = 1, 2, ..., n$ and $u_i \in Q,\ n$ is the number of vertices in $\tilde{G}$ and all arcs of $\tilde{G}$ are strong arcs.

Theorem 3.15. *For any λ_J strong IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ with $r_1 = \mu_1^-(u_i)e^{i\alpha_1^-(u_i)},\ s_1 = \mu_1^+(u_i)e^{i\alpha_1^+(u_i)},\ r_2 = \mu_2^-(u_i)e^{i\alpha_2^-(u_i)}$ and $s_2 = \mu_2^+(u_i)e^{i\alpha_2^+(u_i)}\ \forall\ i = 1, 2, ..., n$ is λ_J regular if and only if $\tilde{G}$ is totally λ_J regular.*

Proof. Let us consider that $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ with $r_1 = \mu_1^-(u_i)e^{i\alpha_1^-(u_i)},\ s_1 = \mu_1^+(u_i)e^{i\alpha_1^+(u_i)},\ r_2 = \mu_2^-(u_i)e^{i\alpha_2^-(u_i)}$ and $s_2 = \mu_2^+(u_i)e^{i\alpha_2^+(u_i)}\ \forall\ i = 1, 2, ..., n.$
We know that the degree of all λ_J-strong vertices in $\tilde{G}$ is $a_1 = r_1(n-1)$, $b_1 = s_1(n-1)$, $a_2 = r_2(n-1)$ and $b_2 = s_2(n-1)$. Therefore, consider for any

$$\begin{aligned}
& td_{\lambda_J}(u_i) = ([(\sum_{(u_i,u_{i+1})\in R_J} \lambda_{1J}^-(u_i,u_{i+1}) + \mu_1^-(u_i))e^{i\sum_{(u_i,u_{i+1})\in R_J}\beta_{1J}^-(u_i,u_{i+1})+\alpha_1^-(u_i)}, \\
& (\sum_{(u_i,u_{i+1})\in R_J} \lambda_{1J}^+(u_i,u_{i+1}) + \mu_1^+(u_i))e^{i\sum_{(u_i,u_{i+1})\in R_J}\beta_{1J}^+(u_i,u_{i+1})+\alpha_1^+(u_i)}], \\
& [(\sum_{(u_i,u_{i+1})\in R_J} \lambda_{2J}^-(u_i,u_{i+1}) + \mu_2^-(u_i))e^{i\sum_{(u_i,u_{i+1})\in R_J}\beta_{2J}^-(u_i,u_{i+1})+\alpha_2^-(u_i)}, \\
& (\sum_{(u_i,u_{i+1})\in R_J} \lambda_{2J}^+(u_i,u_{i+1}) + \mu_2^+(u_i))e^{i\sum_{(u_i,u_{i+1})\in R_J}\beta_{2J}^+(u_i,u_{i+1})+\alpha_2^+(u_i)}]) \\
= \ & ([a_1 + r_1, b_1 + s_1], [a_2 + r_2, b_2 + s_2]),\ \forall\ u_i \in\ Q, J = 1, 2, ..., k
\end{aligned}$$

Hence, $\tilde{G}$ is totally λ_J regular IVCPFGS.
Conversely, suppose $\tilde{G}$ is totally λ_J regular IVCPFGS. Then for u_i

$$\begin{aligned}
& td_{\lambda_J}(u_i) = ([td_{\lambda_{1J}^-}(u_i), td_{\lambda_{1J}^+}(u_i)], [td_{\lambda_{2J}^-}(u_i), td_{\lambda_{2J}^+}(u_i)]) \\
= \ & ([td_{\lambda_{1J}^-}(u_i) = (\sum_{(u_i,u_{i+1})\in R_J} \lambda_{1J}^-(u_i,u_{i+1}) + \mu_1^-(u_i))e^{i\sum_{(u_i,u_{i+1})\in R_J}\beta_{1J}^-(u_i,u_{i+1})+\alpha_1^-(u_i)}, \\
& td_{\lambda_{1J}^+}(u_i) = (\sum_{(u_i,u_{i+1})\in R_J} \lambda_{1J}^+(u_i,u_{i+1}) + \mu_1^+(u_i))e^{i\sum_{(u_i,u_{i+1})\in R_J}\beta_{1J}^+(u_i,u_{i+1})+\alpha_1^+(u_i)}] \\
& [td_{\lambda_{2J}^-}(u_i) = (\sum_{(u_i,u_{i+1})\in R_J} \lambda_{2J}^-(u_i,u_{i+1}) + \mu_2^-(u_i))e^{i\sum_{(u_i,u_{i+1})\in R_J}\beta_{2J}^-(u_i,u_{i+1})+\alpha_2^-(u_i)}, \\
& td_{\lambda_{2J}^+}(u_i) = (\sum_{(u_i,u_{i+1})\in R_J} \lambda_{2J}^+(u_i,u_{i+1}) + \mu_2^+(u_i))e^{i\sum_{(u_i,u_{i+1})\in R_J}\beta_{2J}^+(u_i,u_{i+1})+\alpha_2^+(u_i)}] \\
= \ & ([n_1 = d_{\lambda_{1J}^-}(u_i) + r_1, m_1 = d_{\lambda_{1J}^+}(u_i) + s_1], \\
& [n_2 = d_{\lambda_{2J}^-}(u_i) + r_2, m_2 = d_{\lambda_{2J}^+}(u_i) + s_2]) \\
= \ & ([n_1 - r_1, m_1 - s_1], [n_2 - r_2, m_2 - s_2]),\ \forall u_i \in Q,\ J = 1, 2, ..., k.
\end{aligned}$$

Since, $r_1 = \mu_1^-(u_i)e^{i\alpha_1^-(u_i)},\ s_1 = \mu_1^+(u_i)e^{i\alpha_1^+(u_i)},\ r_2 = \mu_2^-(u_i)e^{i\alpha_2^-(u_i)}$ and $s_2 = \mu_2^+(u_i)e^{i\alpha_2^+(u_i)}\ \forall\ i = 1, 2, ..., n$
Hence, $\tilde{G}$ is λ_J regular IVCPFGS. □

Theorem 3.16. *If an IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ is both λ_J regular and totally λ_J regular IVCPFGS. Then $\mu_1^-(u_i)e^{i\alpha_1^-(u_i)}$, $\mu_1^+(u_i)e^{i\alpha_1^+(u_i)}$, $\mu_2^-(u_i)e^{i\alpha_2^-(u_i)}$ and $\mu_2^+(u_i)e^{i\alpha_2^+(u_i)}$ is a constant for all i.*

Proof. Let us consider that $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ is both λ_J regular and totally λ_J regular IVCPFGS with n vertices. For λ_J regular IVCPFGS $\tilde{G}$,

$$
\begin{aligned}
d_{\lambda_J}(u_i) &= ([d_{\lambda_{1J}^-}(u_i), d_{\lambda_{1J}^+}(u_i)], [d_{\lambda_{2J}^-}(u_i), d_{\lambda_{2J}^+}(u_i)]) \\
&= ([a_1, b_1], [a_2, b_2]),\ \forall\ J = 1, 2, ..., k. \\
&\quad \text{and for totally } \lambda_J \text{ regular IVCPFGS, for any } u_i, \\
td_{\lambda_J}(u_i) &= ([td_{\lambda_{1J}^-}(u_i), td_{\lambda_{1J}^+}(u_i)], [td_{\lambda_{2J}^-}(u_i), td_{\lambda_{2J}^+}(u_i)]) \\
td_{\lambda_{1J}^-}(u_i) &= d_{\lambda_{1J}^-}(u_i) + \mu_1^-(u_i)e^{i\alpha_1^-(u_i)} \\
&= n_1 = a_1 + \mu_1^-(u_i)e^{i\alpha_1^-(u_i)} \\
&= \mu_1^-(u_i)e^{i\alpha_1^-(u_i)} = n_1 - a_1 = a \\
\text{Similarly,} & \\
td_{\lambda_{1J}^+}(u_i) &= m_1 - b_1 = b \\
td_{\lambda_{2J}^-}(u_i) &= n_2 - a_2 = c \\
td_{\lambda_{2J}^+}(u_i) &= m_2 - b_2 = d
\end{aligned}
$$

$\therefore\ td_{\lambda_J}(u_i) = ([a, b], [c, d])$ is constant for all i. □

Theorem 3.17. *The λ_J-degree of λ_J regular IVCPFGS$\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ of $G^* = (Q, R_1, R_2, ..., R_k)$ is $([\frac{na_1}{2}, \frac{nb_1}{2}], [\frac{na_2}{2}, \frac{nb_2}{2}])$ where n is the number of vertices in Q. ie, $d_{\lambda_J}(u_i) = ([\frac{na_1}{2}, \frac{nb_1}{2}], [\frac{na_2}{2}, \frac{nb_2}{2}])$*

Proof. The λ_J-degree of $\tilde{G}$ is

$$
\begin{aligned}
& d_{\lambda_J}(u_i) \\
= \; & ([\sum_{(u,v)\in R_J} \lambda_{1J}^-(u,v)e^{i\sum_{(u,v)\in R_J}\beta_{1J}^-(u,v)}, \sum_{(u,v)\in R_J} \lambda_{1J}^+(u,v)e^{i\sum_{(u,v)\in R_J}\beta_{1J}^+(u,v)}], \\
& [\sum_{(u,v)\in R_J} \lambda_{2J}^-(u,v)e^{i\sum_{(u,v)\in R_J}\beta_{2J}^-(u,v)}, \sum_{(u,v)\in R_J} \lambda_{2J}^+(u,v)e^{i\sum_{(u,v)\in R_J}\beta_{2J}^+(u,v)}]) \\
& \text{Let } d_{\lambda_J}(u_i) = ([a_1, b_1], [a_2, b_2]) \\
& \text{But } \sum_{u_i\in Q} d_{\lambda_J}(u_i) = \\
& ([\sum_{u_i\in Q} d_{\lambda_{1J}^-}(u_i), \sum_{u_i\in Q} d_{\lambda_{1J}^+}(u_i)], [\sum_{u_i\in Q} d_{\lambda_{2J}^-}(u_i), \sum_{u_i\in Q} d_{\lambda_{2J}^+}(u_i)]) \\
= \; & ([2\sum_{(u_i,v)\in R_J} \lambda_{1J}^-(u_i,v)e^{i2\sum_{(u_i,v)\in R_J}\beta_{1J}^-(u_i,v)}, \\
& 2\sum_{(u_i,v)\in R_J} \lambda_{1J}^+(u_i,v)e^{i2\sum_{(u_i,v)\in R_J}\beta_{1J}^+(u_i,v)}], \\
& [2\sum_{(u_i,v)\in R_J} \lambda_{2J}^-(u_i,v)e^{i2\sum_{(u_i,v)\in R_J}\beta_{2J}^-(u_i,v)}, \\
& 2\sum_{(u_i,v)\in R_J} \lambda_{2J}^+(u_i,v)e^{i2\sum_{(u_i,v)\in R_J}\beta_{2J}^+(u_i,v)}]) \\
& \text{Let n number of vertices in Q} \\
= \; & ([\frac{na_1}{2}, \frac{nb_1}{2}], [\frac{na_2}{2}, \frac{nb_2}{2}])
\end{aligned}
$$

□

Theorem 3.18. *If $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ is a totally λ_J regular IVCPFGS of GS $G^* = (Q, R_1, R_2, ..., R_k)$. Then $([2d_{\lambda_{1J}^-}(u_i, v) + \mu_1^-(u_i)e^{i\alpha_1^-(u_i)}, 2d_{\lambda_{1J}^+}(u_i, v) + \mu_1^+(u_i)e^{i\alpha_1^+(u_i)}], [2d_{\lambda_{2J}^-}(u_i, v) + \mu_2^-(u_i)e^{i\alpha_2^-(u_i)}, 2d_{\lambda_{2J}^+}(u_i, v) + \mu_2^+(u_i)e^{i\alpha_2^+(u_i)}]) = ([nn_1, nm_1], [nn_2, nm_2])$.*

Proof. Since, $\tilde{G}$ is a totally λ_J regular IVCPFGS.

$$\begin{aligned}
& td_{\lambda_J}(u_i) = \\
& ([td_{\lambda^-_{1J}}(u_i), td_{\lambda^+_{1J}}(u_i)], [td_{\lambda^-_{2J}}(u_i), td_{\lambda^+_{2J}}(u_i)]) \\
= \; & ([n_1, m_1], [n_2, m_2]), \; \forall \; u_i \in Q, J = 1, 2, ..., k. \\
& \text{So}([\sum_{u_i \in Q} n_1, \sum_{u_i \in Q} m_1], [\sum_{u_i \in Q} n_2, \sum_{u_i \in Q} m_2] \\
= \; & ([\sum_{u_i} d_{\lambda^-_{1J}}(u_i), \sum_{u_i} d_{\lambda^+_{1J}}(u_i)], [\sum_{u_i} d_{\lambda^-_{2J}}(u_i), \sum_{u_i} d_{\lambda^+_{2J}}(u_i)] \\
& ([nn_1, nm_1], [nn_2, nm_2]) = \\
& ([2d_{\lambda^-_{1J}}(u_i, v) + \mu^-_1(u_i)e^{i\alpha^-_1(u_i)}, 2d_{\lambda^+_{1J}}(u_i, v) + \mu^+_1(u_i)e^{i\alpha^+_1(u_i)}], \\
& [2d_{\lambda^-_{2J}}(u_i, v) + \mu^-_2(u_i)e^{i\alpha^-_2(u_i)}, 2d_{\lambda^+_{2J}}(u_i, v) + \mu^+_2(u_i)e^{i\alpha^+_2(u_i)}])
\end{aligned}$$

□

Theorem 3.19. *If $\tilde{G} = (\mu, \lambda_1, \lambda_2, ..., \lambda_k)$ is a λ_J regular and totally λ_J regular IVCPFGS. Then $\mu(u_i) = ([n(n_1 - a_1), n(m_1 - b_1)], [n(n_2 - a_2), n(m_2 - b_2)])$.*

Proof. From above theorem-3.17

$$\begin{aligned}
d_{\lambda_J}(u_i) &= ([\frac{na_1}{2}, \frac{nb_1}{2}], [\frac{na_2}{2}, \frac{nb_2}{2}]) \\
ie, 2d_{\lambda_J}(u_i) &= ([na_1, nb_1], [na_2, nb_2])
\end{aligned}$$

From above Theorem-3.18

$$\begin{aligned}
& ([nn_1, nm_1], [nn_2, nm_2]) = \\
& ([2d_{\lambda^-_{1J}}(u_i, v) + \mu^-_1(u_i)e^{i\alpha^-_1(u_i)}, 2d_{\lambda^+_{1J}}(u_i, v) + \mu^+_1(u_i)e^{i\alpha^+_1(u_i)}], \\
& [2d_{\lambda^-_{2J}}(u_i, v) + \mu^-_2(u_i)e^{i\alpha^-_2(u_i)}, 2d_{\lambda^+_{2J}}(u_i, v) + \mu^+_2(u_i)e^{i\alpha^+_2(u_i)}]) \\
& \mu(u_i) = ([nn_1 - na_1, nm_1 - nb_1], [nn_2 - na_2, nm_2 - nb_2]) \\
& = ([n(n_1 - a_1), n(m_1 - b_1)], [n(n_2 - a_2), n(m_2 - b_2)]).
\end{aligned}$$

□

4. Some result on IVCPFGS

This section introduces the terms strength of a $\lambda_J - path$, subdivision of an IVCPFGS, and ψ-complement IVCPFGS along with definitions that are important for comprehending the key findings. We further analysis a number of IVCPFGS traits using examples.

Definition 4.1. Let $\tilde{G} = \{\mu, \lambda_1, \lambda_2, ..., \lambda_k\}$ be a IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$.Then the strength of a $\lambda_J - path$ in $\tilde{G}$, $Path_{\lambda_J} = u_1, u_2, ..., u_n$ are denoted by $P_{\lambda^-_{1J}e^{i\beta^-_{1J}}}$, $P_{\lambda^+_{1J}e^{i\beta^+_{1J}}}$, $P_{\lambda^-_{2J}e^{i\beta^-_{2J}}}$ and $P_{\lambda^+_{2J}e^{i\beta^+_{2J}}}$, respectively and defined as

$$\begin{aligned} P_{\lambda_J} &= ([\wedge_{i=2}^{n} P_{\lambda^-_{1J}}(u_{i-1},u_i)e^{i\wedge_{i=2}^{n}\beta^-_{1J}(u_{i-1},u_i)}, \wedge_{i=2}^{n} P_{\lambda^+_{1J}}(u_{i-1},u_i)e^{i\wedge_{i=2}^{n}\beta^+_{1J}(u_{i-1},u_i)}], \\ &\quad [\vee_{i=2}^{n} P_{\lambda^-_{2J}}(u_{i-1},u_i)e^{i\vee_{i=2}^{n}\beta^-_{2J}(u_{i-1},u_i)}, \vee_{i=2}^{n} P_{\lambda^+_{2J}}(u_{i-1},u_i)e^{i\vee_{i=2}^{n}\beta^+_{2J}(u_{i-1},u_i)}]), \\ &\quad \forall u_{i-1}, u_i \in Q,\ \ J = 1,2,...,k. \end{aligned}$$

Example 4.2. An IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ of a GS $G^* = (Q, R_1, R_2)$ given figure-4is a IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ such that $\mu = \{u_1([0.3e^{i1.2\pi}, 0.6e^{i1.6\pi}],$ $[0.4e^{i1.2\pi}, 0.7e^{i1.4\pi}]),\ u_2([0.4e^{i0.8\pi}, 0.6e^{i1.0\pi}], [0.5e^{i0.8\pi}, 0.6e^{i1.2\pi}])$ $u_3([0.5e^{i0.8\pi}, 0.8e^{i1.2\pi}], [0.5e^{i1.0\pi}, 0.7e^{i1.4\pi}])\}$ Here, $Path_{\lambda_2} = u_1, u_2, u_3$ and $\lambda_J -$

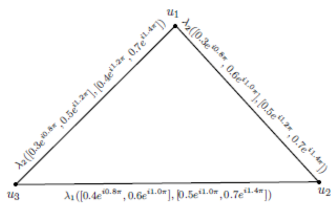

FIGURE 4. $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ is IVCPFGS of G_1^*

path is $P_{\lambda_2} = ([0.3e^{i0.8\pi}, 0.5e^{i1.0\pi}], [0.5e^{i1.2\pi}, 0.7e^{i1.4\pi}])$

Definition 4.3. Let $\tilde{G} = \{\mu, \lambda_1, \lambda_2, ..., \lambda_k\}$ be a IVCPFGS of GS $G^* = \{Q, R_1, R_2,$ $..., R_k\}$. The subdivision of an IVCPFGS $\tilde{G}$ is thus denoted by $\tilde{G_{sd}} = \{\mu_{sd}, \lambda_{sd1}, \lambda_{sd2},$ $..., \lambda_{sdk}\}$ and is obtained by inserting a new vertex u_k into every edge $(vi, vj) \in R_J$ of G^* such that the vertex v_k and the edges v_iu_k and u_kv_j are defined as follows:

$$\begin{aligned} \mu^-_{sd1}(u_i) &= \lambda^-_{1J}(u_j,u_k),\ \ \alpha^-_{sd1}(u_i) = \beta^-_{1J}(u_j,u_k), \\ \mu^+_{sd1}(u_i) &= \lambda^+_{1J}(u_j,u_k),\ \ \alpha^+_{sd1}(u_i) = \beta^+_{1J}(u_j,u_k), \\ \mu^-_{sd2}(u_i) &= \lambda^-_{2J}(u_j,u_k),\ \ \alpha^-_{sd2}(u_i) = \beta^-_{2J}(u_j,u_k), \\ \mu^+_{sd2}(u_i) &= \lambda^+_{2J}(u_j,u_k),\ \ \alpha^-_{sd2}(u_i) = \beta^+_{2J}(u_j,u_k),\ \forall\ u_i \in Q' \end{aligned}$$

$$\begin{aligned} \lambda^-_{sd1J}(u_i,u_j)e^{i\beta^-_{sd1J}(u_i,u_j)} &\leq \min\{\mu^-_{sd1}(u_i), \mu^-_{sd1}(u_j)\}e^{i\min\{\alpha^-_{sd1}(u_i),\alpha^-_{sd1}(u_j)\}}, \\ \lambda^+_{sd1J}(u_i,u_j)e^{i\beta^+_{sd1J}(u_i,u_j)} &\leq \min\{\mu^+_{sd1}(u_i), \mu^+_{sd1}(u_j)\}e^{i\min\{\alpha^+_{sd1}(u_i),\alpha^+_{sd1}(u_j)\}}, \\ \lambda^-_{sd2J}(u_i,u_j)e^{i\beta^-_{sd2J}(u_i,u_j)} &\leq \max\{\mu^-_{sd2}(u_i), \mu^-_{sd2}(u_j)\}e^{i\max\{\alpha^-_{sd2}(u_i),\alpha^-_{sd2}(u_j)\}}, \\ \lambda^+_{sd2J}(u_i,u_j)e^{i\beta^+_{sd2J}(u_i,u_j)} &\leq \max\{\mu^+_{sd2}(u_i), \mu^+_{sd2}(u_j)\}e^{i\max\{\alpha^+_{sd2}(u_i),\alpha^+_{sd2}(u_j)\}}, \\ &\forall\ u_i, u_j \in R_J \end{aligned}$$

Example 4.4. An IVCPFGS $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ of a GS $G^* = (Q, R_1, R_2)$ given figure-6. Then the subdivision of an IVCPFGS $\tilde{G} = \{\mu_{sd}, \lambda_{sd1}, \lambda_{sd1}\}$ such that $\mu_{sd} = \{u_1([0.5e^{i0.4\pi}, 0.6e^{i0.5\pi}], [0.4e^{i0.5\pi}, 0.7e^{i0.7\pi}]),\ u_2([0.5e^{i0.4\pi}, 0.7e^{i0.7\pi}]$ $, [0.4e^{i0.5\pi}, 0.6e^{i0.5\pi}]), u_3([0.5e^{i0.8\pi}, 0.8e^{i1.2\pi}], [0.5e^{i1.0\pi}, 0.7e^{i1.4\pi}]),$ $u_4([0.4e^{i0.4\pi}, 0.6e^{i0.5\pi}], [0.3e^{i0.5\pi}, 0.4e^{i1.0\pi}]), u_5([0.4e^{i0.5\pi}, 0.5e^{i0.5\pi}], [0.3e^{i0.5\pi}, 0.5e^{i0.4\pi}]),$ $v_1([0.5e^{i0.4\pi}, 0.5e^{i0.5\pi}], [0.4e^{i0.5\pi}, 0.4e^{i0.5\pi}]), v_2([0.5e^{i0.4\pi}, 0.5e^{i0.5\pi}], [0.5e^{i0.3\pi}, 0.5e^{i0.4\pi}]),$

$v_3([0.5e^{i0.4\pi}, 0.5e^{i0.4\pi}], [0.4e^{i0.5\pi}, 0.4e^{i0.5\pi}]), v_4([0.3e^{i0.3\pi}, 0.6e^{i0.5\pi}], [0.2e^{i0.5\pi}, 0.7e^{i1.4\pi}]),$
$v_5([0.3e^{i0.3\pi}, 0.6e^{i0.5\pi}], [0.2e^{i0.4\pi}, 0.5e^{i1.0\pi}]), v_6([0.4e^{i0.4\pi}, 0.4e^{i0.5\pi}], [0.4e^{i0.5\pi}, 0.5e^{i0.5\pi}])\}$
$\lambda_{sd2} =$

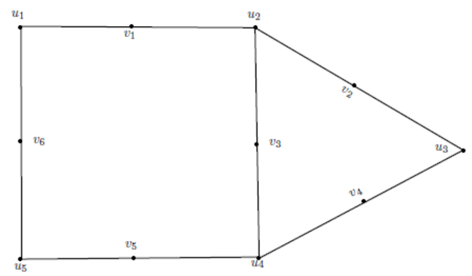

FIGURE 5. $\tilde{G_{sd}} = (\mu_{sd}, \lambda_{sd1}, \lambda_{sd2})$ is subdivision of an IVCPFGS of G_1^*

$\{u_1v_1([0.5e^{i0.4\pi}, 0.5e^{i0.5\pi}], [0.4e^{i0.5\pi}, 0.7e^{i0.7\pi}]), u_2v_1([0.5e^{i0.4\pi}, 0.5e^{i0.5\pi}], [0.4e^{i0.5\pi}, 0.6e^{i0.5\pi}]),$
$u_2v_2([0.5e^{i0.4\pi}, 0.5e^{i0.5\pi}], [0.5e^{i0.4\pi}, 0.6e^{i0.6\pi}]), u_2v_3([0.5e^{i0.4\pi}, 0.5e^{i0.4\pi}], [0.4e^{i0.5\pi}, 0.6e^{i0.5\pi}]),$
$u_4v_3([0.4e^{i0.4\pi}, 0.5e^{i0.4\pi}], [0.4e^{i0.5\pi}, 0.4e^{i0.6\pi}]), u_1v_6([0.4e^{i0.4\pi}, 0.4e^{i0.5\pi}], [0.4e^{i0.5\pi}, 0.7e^{i0.7\pi}])$
$, u_5v_6([0.4e^{i0.4\pi}, 0.4e^{i0.5\pi}], [0.4e^{i0.5\pi}, 0.5e^{i0.5\pi}])\},$
$\lambda_{sd2} = \{u_3v_4([0.3e^{i0.3\pi}, 0.6e^{i0.5\pi}], [0.5e^{i1.0\pi}, 0.7e^{i1.4\pi}]), u_4v_4([0.3e^{i0.3\pi}, 0.6e^{i0.5\pi}],$
$[0.3e^{i0.5\pi}, 0.7e^{i1.4\pi}]), u_4v_5([0.3e^{i0.3\pi}, 0.6e^{i0.5\pi}], [0.3e^{i0.4\pi}, 0.5e^{i1.0\pi}]),$
$u_5v_5([0.3e^{i0.3\pi}, 0.5e^{i0.5\pi}], [0.3e^{i0.5\pi}, 0.5e^{i1.0\pi}])\}$

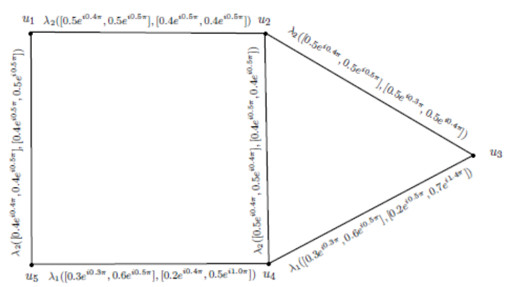

FIGURE 6. $\tilde{G} = (\mu, \lambda_1, \lambda_2)$ is IVCPFGS of G_1^*

Theorem 4.5. *Let $\tilde{G} = \{\mu, \lambda_1, \lambda_2, ..., \lambda_k\}$ be a λ_J- strong IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$ and $\tilde{G_{sd}} = \{\mu_{sd}, \lambda_{sd1}, \lambda_{sd2}, ..., \lambda_{sdk}\}$ be an subdivision of an IVCPFGS $\tilde{G}$. Then $S_{\lambda^-_{sd1J}}(G_{sd}) \leq 2S_{\lambda^-_{1J}}(G)$, $S_{\lambda^+_{sd1J}}(G_{sd}) \leq 2S_{\lambda^+_{sd1J}}(G)$ and $S_{\lambda^-_{sd2J}}(G_{sd}) \leq 2S_{\lambda^-_{2J}}(G)$, $S_{\lambda^+_{sd2J}}(G_{sd}) \leq 2S_{\lambda^+_{sd2J}}(G)$ where $S_{\lambda^-_{1J}}, S_{\lambda^+_{1J}}, S_{\lambda^-_{2J}}, S_{\lambda^+_{2J}}$ are λ_J-size of $\tilde{G}$.*

Proof. Let $\tilde{G} = \{\mu, \lambda_1, \lambda_2, ..., \lambda_k\}$ be a λ_J- strong IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$ and $\tilde{G_{sd}} = \{\mu_{sd}, \lambda_{sd1}, \lambda_{sd2}, ..., \lambda_{sdk}\}$ be an subdivision of an IVCPFGS $\tilde{G}$. The λ_J-size of $\tilde{G}$ is $S_{\lambda_J}(G) = ([S_{\lambda^-_{1J}}(G), S_{\lambda^+_{1J}}(G)], [S_{\lambda^-_{2J}}(G), S_{\lambda^+_{2J}}(G)])$, where Consider,

$S_{\lambda^z_{sdlJ}}(G_{sd})$

$$\begin{aligned}
&= \sum_{(u_i,u_j),(u_j,u_k)\in R_J} (\lambda^z_{sdlJ}(u_i,u_j) + \lambda^z_{sdlJ}(u_j,u_k)) \\
&\quad e^{i\sum_{(u_i,u_j),(u_j,u_k)\in R_J}(\beta^z_{sdlJ}(u_i,u_j)+\beta^z_{sdlJ}(u_j,u_k))} \\
&\leq (\sum_{(u_i,u_j)\in R_J} \lambda^z_{sdlJ}(u_i,u_j) + \sum_{(u_j,u_k)\in R_J} \lambda^z_{sdlJ}(u_j,u_k)) \\
&\quad e^{i\sum_{(u_i,u_j)\in R_J}\lambda^z_{sdlJ}(u_i,u_j)+\sum_{(u_j,u_k)\in R_J}\lambda^z_{sdlJ}(u_j,u_k)}, \\
&\leq (\sum_{u_j\in Q} \mu^z_l(u_j) + \sum_{u_j\in Q} \mu^z_l(u_j)) \\
&\quad e^{i\sum_{u_j\in Q}\alpha^z_l(u_j)+\sum_{u_j\in Q}\alpha^z_l(u_j)} \\
&\leq (2\sum_{(u_i,u_k)\in R_J} \lambda^z_{lJ}(u_i,u_k)) \\
&\quad e^{i2\sum_{(u_i,u_k)\in R_J}\beta^z_{lJ}(u_i,u_k)} \\
&\leq 2S_{\lambda^z_{lJ}}(G), \forall l = 1,2 \text{ and } z = -,+ \ and\ J = 1,2,...k.
\end{aligned}$$

Hence, $S_{\lambda^-_{sd1J}}(G_{sd}) \leq 2S_{\lambda^-_{1J}}(G)$, $S_{\lambda^+_{sd1J}}(G_{sd}) \leq 2S_{\lambda^+_{sd1J}}(G)$ and $S_{\lambda^-_{sd2J}}(G_{sd}) \leq 2S_{\lambda^-_{2J}}(G)$, $S_{\lambda^+_{sd2J}}(G_{sd}) \leq 2S_{\lambda^+_{sd2J}}(G)$ □

Definition 4.6. Let $\tilde{G} = \{\mu, \lambda_1, \lambda_2, ..., \lambda_k\}$ be a IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$ and $\tilde{G^\psi} = (\mu^\psi, \lambda^\psi_1, \lambda^\psi_2, ..., \lambda^\psi_k)$ is called ψ-complement IVCPFGS of GS. which is satisfies the condition.

$$\begin{aligned}
&(i) \quad \mu^{\psi^-}_1(u) = 1 - \mu^-_1(u), \text{ and } \alpha^{\psi^-}_1(u) = 2\pi - \alpha^-_1(u)\\
&(ii) \quad \mu^{\psi^+}_1(u) = 1 - \mu^+_1(u), \text{ and } \alpha^{\psi^+}_1(u) = 2\pi - \alpha^+_1(u)\\
&(iii) \quad \mu^{\psi^-}_2(u) = 1 - \mu^-_2(u), \text{ and } \alpha^{\psi^-}_2(u) = 2\pi - \alpha^-_2(u)\\
&(iv) \quad \mu^{\psi^+}_2(u) = 1 - \mu^+_2(u), \text{ and } \alpha^{\psi^+}_2(u) = 2\pi - \alpha^+_2(u), \forall u \in Q.
\end{aligned}$$

$$\begin{aligned}
&(v) \quad \lambda^{\psi^-}_{1J}(u,v)e^{i\beta^{\psi^-}_{1J}(u,v)} \geq \max\{\mu^{\psi^-}_1(u), \mu^{\psi^-}_1(v)\}e^{i\max\{\alpha^{\psi^-}_1(u),\alpha^{\psi^-}_1(v)\}}\\
&(vi) \quad \lambda^{\psi^+}_{1J}(u,v)e^{i\beta^{\psi^+}_{1J}(u,v)} \geq \max\{\mu^{\psi^+}_1(u), \mu^{\psi^+}_1(v)\}e^{i\max\{\alpha^{\psi^+}_1(u),\alpha^{\psi^+}_1(v)\}}\\
&(vii) \quad \lambda^{\psi^-}_{2J}(u,v)e^{i\beta^{\psi^-}_{2J}(u,v)} \geq \min\{\mu^{\psi^-}_2(u), \mu^{\psi^-}_2(v)\}e^{i\min\{\alpha^{\psi^-}_2(u),\alpha^{\psi^-}_2(v)\}}\\
&(viii) \quad \lambda^{\psi^+}_{2J}(u,v)e^{i\beta^{\psi^+}_{2J}(u,v)} \geq \min\{\mu^{\psi^+}_2(u), \mu^{\psi^+}_2(v)\}e^{i\min\{\alpha^{\psi^+}_2(u),\alpha^{\psi^+}_2(v)\}},\\
&\quad \forall (u,v) \in R_J, J = 1,2,...,k.
\end{aligned}$$

Note: $\lambda^{\psi^-}_{1J}$, $\lambda^{\psi^+}_{1J}$, $\lambda^{\psi^-}_{2J}$ and $\lambda^{\psi^+}_{2J}$ are ψ-complement such that $\lambda^{\psi^-}_{1J}(u,v) \geq \lambda^{\psi^+}_{1J}(u,v)$ and $\lambda^{\psi^-}_{2J}(u,v) \geq \lambda^{\psi^+}_{2J}(u,v)$ and $\beta^{\psi^-}_{1J}(u,v) \geq \beta^{\psi^+}_{1J}(u,v)$ and $\beta^{\psi^-}_{2J}(u,v) \geq \beta^{\psi^+}_{2J}(u,v)$ for all $(u,v) \in R_J, J = 1,2,...,k$.

Example 4.7. Let $\tilde{G} = \{\mu, \lambda_1, \lambda_2\}$ be a IVCPFGS of GS $G^* = \{Q, R_1, R_2\}$. Then $\tilde{G^\psi} = (\mu^\psi, \lambda^\psi_1, \lambda^\psi_2)$ is ψ-complement IVCPFGS of GS G^* as shown in Figure-4 in Example-4.2 and Figure-7 such that

$\mu^{\psi} = \{u_1([0.7e^{i0.8\pi}, 0.4e^{i0.4\pi}], [0.6e^{i0.6\pi}, 0.3e^{i0.6\pi}]), u_2([0.6e^{i1.2\pi}, 0.4e^{i1.0\pi}],$
$[0.5e^{i1.2\pi}, 0.4e^{i0.8\pi}]), u_3([0.5e^{i1.2\pi}, 0.2e^{i0.8\pi}], [0.5e^{i1.0\pi}, 0.3e^{i0.6\pi}])\}$

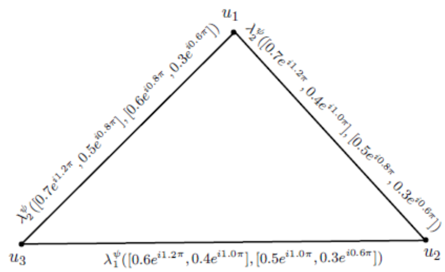

FIGURE 7. $\tilde{G^{\psi}} = (\mu^{\psi}, \lambda_1^{\psi}, \lambda_2^{\psi})$ is ψ-complement IVCPFGS of GS G^*

Theorem 4.8. *Let $\tilde{G} = \{\mu, \lambda_1, \lambda_2, ..., \lambda_k\}$ be a IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$. Then $\tilde{G^{\psi}} = (\mu^{\psi}, \lambda_1^{\psi}, \lambda_2^{\psi}, ..., \lambda_k^{\psi})$ is ψ-complement IVCPFGS of GS.*

Proof. Let $\tilde{G} = \{\mu, \lambda_1, \lambda_2, ..., \lambda_k\}$ be a IVCPFGS of GS $G^* = \{Q, R_1, R_2, ..., R_k\}$. Let $u, v \in Q$

$$\begin{aligned}
(i)\lambda_{1J}^{\psi^s}(u,v)e^{i\beta_{1J}^{\psi^s}(u,v)} &= (1-\lambda_{1J}^{s}(u,v))e^{i(2\pi-\beta_{1J}^{s}(u,v)} \\
&\geq (1-\min\{\mu_1^s(u),\mu_1^s(v)\})e^{i(2\pi-\min\{\alpha_1^s(u),\alpha_1^s(v)\})} \\
&= \max\{1-\mu_1^s(u), 1-\mu_1^s(v)\}e^{i(\max\{2\pi-\alpha_1^s(u),2\pi-\alpha_1^s(v)\}} \\
&= \max\{\mu_1^{\psi^s}(u),\mu_1^{\psi^s}(v)\}e^{i\max\{\alpha_1^{\psi^s}(u),\alpha_1^{\psi^s}(v)\}}
\end{aligned}$$

$\therefore$, $\lambda_{1J}^{\psi^s}(u,v)e^{i\beta_{1J}^{\psi^s}(u,v)} \geq \max\{\mu_1^{\psi^s}(u),\mu_1^{\psi^s}(v)\}e^{i\max\{\alpha_1^{\psi^s}(u),\alpha_1^{\psi^s}(v)\}}$

$$\begin{aligned}
(ii)\lambda_{2J}^{\psi^s}(u,v)e^{i\beta_{2J}^{\psi^s}(u,v)} &= (1-\lambda_{2J}^{s}(u,v))e^{i(2\pi-\beta_{2J}^{s}(u,v)} \\
&\geq (1-\max\{\mu_2^s(u),\mu_2^s(v)\})e^{i(2\pi-\max\{\alpha_2^s(u),\alpha_2^s(v)\})} \\
&= \min\{1-\mu_2^s(u), 1-\mu_2^s(v)\}e^{i(\min\{2\pi-\alpha_2^s(u),2\pi-\alpha_2^s(v)\}} \\
&= \min\{\mu_2^{\psi^s}(u),\mu_2^{\psi^s}(v)\}e^{i\min\{\alpha_2^{\psi^s}(u),\alpha_2^{\psi^s}(v)\}}
\end{aligned}$$

$\therefore$, $\lambda_{2J}^{\psi^s}(u,v)e^{i\beta_{2J}^{\psi^s}(u,v)} \geq \min\{\mu_2^{\psi^s}(u),\mu_2^{\psi^s}(v)\}e^{i\min\{\alpha_2^{\psi^s}(u),\alpha_2^{\psi^s}(v)\}}$,
$\forall s = -, +$and$(u,v) \in R_J$, $J = 1, 2, ..., k$.
Hence, $\tilde{G^{\psi}} = (\mu^{\psi}, \lambda_1^{\psi}, \lambda_2^{\psi}, ..., \lambda_k^{\psi})$ is ψ-complement IVCPFGS of GS. □

5. Application of Interval-Valued Complex Pythagorean Fuzzy Sets for Kale Climate Morphology

It is common knowledge that our Earth is 23.5° tilted and rotates once every day. There may be a December solstice and a June solstice during this revolution. Due to its location at the Equator point, Kale has continuous sunshine for a specific length of time as a result of Earth's rotation, during which time half of the planet is facing the sun. The summers are hot, dry, and clear here, while the winters are chilly, snowy, and partially cloudy. The average annual temperature ranges between $11°F$ and $81°F$, with lows of $-3°F$ and highs of $88°F$ being rare.

These circumstances correspond mathematically to the Interval-Valued Complex Pythagorean Fuzzy set. From June 13 to September 18, the warm season, which has an average daily high temperature of above $70°F$, lasts for 3.2 months. With an average high of $80°F$ and low of $51°F$, August is the hottest month of the year in Kale. From November 26 to March 8, the cold season, which has an average

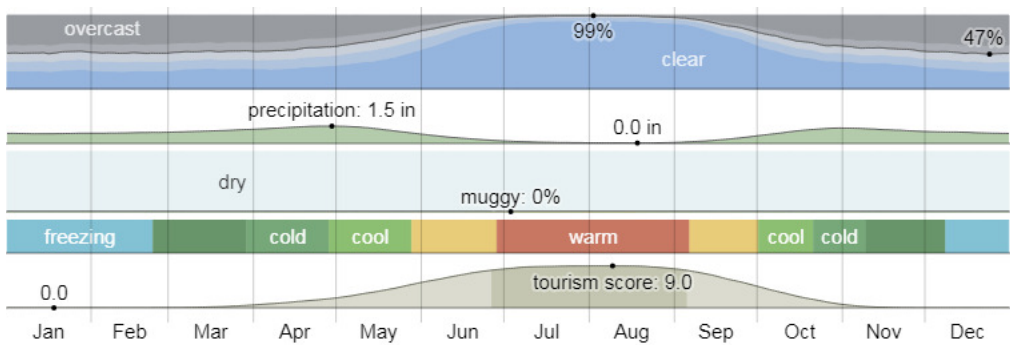

Figure 8. Climate summary for a year

daily high temperature below $37°F$, lasts for 3.4 months. With an average low of $12°F$ and high of $26°F$, January is the coldest month of the year in Kale. Let Q

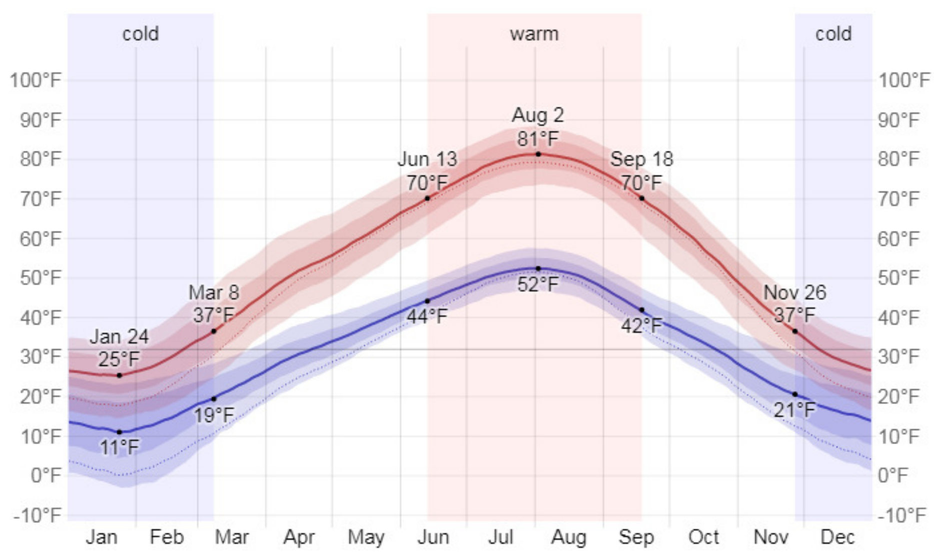

Figure 9. The thin dotted lines represent the equivalent average perceived temperatures while the daily high and low temperatures are shown.

represent the average annual temperature of Kale's different climates. Declare Y to be an Interval-Valued Complex Pythagorean Fuzzy set in Q. The amplitude term truth membership function lower and upper values are $\mu^{-}_{Q_1}$ and $\mu^{+}_{Q_1}$, respectively, while the phase term lower and upper values of the truth membership function are $\alpha^{-}_{Q_1}$, $\alpha^{+}_{Q_1}$, respectively, and they signify the summer season in Q. As a result, the amplitude term for the summertime temperature is between 44 and

81 degrees Fahrenheit. As a result, the range of 58 to 66 degrees Fahrenheit is the phase term for the midsummer average temperature. The lower and upper values of the amplitude term falsity membership function are $\mu^-_{Q_2}$ and $\mu^+_{Q_2}$, respectively, while the phase term lower and upper values of the falsity membership function are $\alpha^-_{Q_2}$, $\alpha^+_{Q_2}$, respectively, and they both denote the winter season in Q. The winter temperature therefore has an amplitude ranging between 11 and 37 degrees Fahrenheit. As a result, the phase term for the average temperature in midwinter is in the range of 19 to 23 degrees Fahrenheit. The following explanation is given in Figure-8, Figure-9, regarding the interpretation of this winter's snowfall and summer: Consider the temperature belonging to the set Y as the

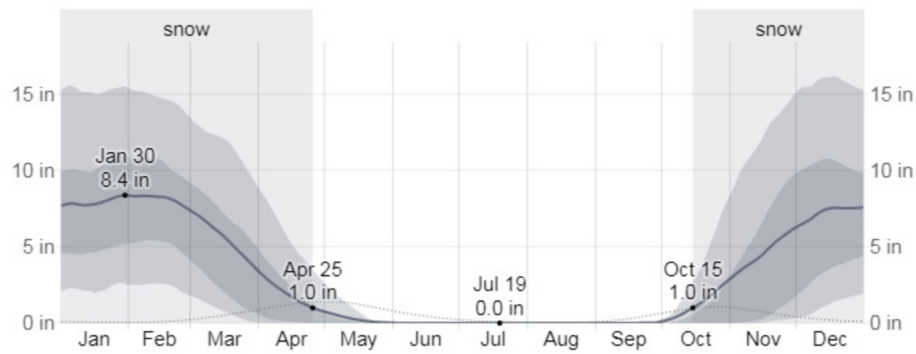

FIGURE 10. Average monthly liquid-equivalent snowfall, measured in inches.

IVCPFS percentage values for the sake of simplicity.

$$Q = ([0.44e^{i0.58\pi}, 0.81e^{i0.66\pi}], [0.11e^{i0.19\pi}, 0.37e^{i0.23\pi}])$$

Here, the set Q is satisfies the IVCPFS, such that

$${\mu^+_{Q_1}}^2 + {\mu^+_{Q_2}}^2 \leq 1 \text{ and } {\alpha^+_{Q_1}}^2 + {\alpha^+_{Q_2}}^2 \leq 2\pi.$$

For many applications, the predicting of atmospheric temperature is essential. The dynamic nature of the atmosphere makes it difficult to predict the weather with any degree of accuracy. By using the idea of IVCPFSs, one can analyse the data on climate changes.

6. IVCPFGS Application for Kale Climate Morphology

A day that has at least 0.04 inches of liquid or liquid-equivalent precipitation is considered to be wet. In Kale, the likelihood of rainy days fluctuates throughout the year. We categorise rainy days into those that only involve rain, those that only involve snow, and those that mix the two. The most frequent type of precipitation in Kale varies throughout the year based on this classification. This research of the temperature throughout that period of time when the environment changed can be examined. IVCPFS is previously explained in section-5. The average temperature of Kale's several climates is represented by a set Q from Jan. 24 to Mar. 8, Mar. 9 to Jun. 13, Jun. 14 to Aug. 2, Aug. 3 to Sep. 18, and Sep. 19 to Nov. 26 and as shown in Figure-9. On set Q, numerous relations can be

TABLE 1.

several climates	$([\mu^-_{Q_1}e^{i\alpha^-_{Q_1}}, \mu^+_{Q_1}e^{i\alpha^+_{Q_1}}], [\mu^-_{Q_2}e^{i\alpha^-_{Q_2}}, \mu^+_{Q_2}e^{i\alpha^+_{Q_2}}])$
Jan. 24 to Mar. 8 (A)	$([0.25e^{i0.31\pi}, 0.37e^{i0.35\pi}], [0.11e^{i0.15\pi}, 0.19e^{i0.16\pi}])$
Mar. 9 to Jun. 13 (B)	$([0.38e^{i0.48\pi}, 0.70e^{i0.58\pi}], [0.20e^{i0.27\pi}, 0.44e^{i0.32\pi}])$
Jun. 14 to Aug. 2 (C)	$([0.71e^{i0.59\pi}, 0.81e^{i0.66\pi}], [0.45e^{i0.47\pi}, 0.52e^{i0.49\pi}])$
Aug. 3 to Sep. 18 (D)	$([0.70e^{i0.74\pi}, 0.80e^{i0.76\pi}], [0.42e^{i0.45\pi}, 0.51e^{i0.46\pi}])$
Sep. 19 to Nov. 26 (E)	$([0.37e^{i0.48\pi}, 0.69e^{i0.57\pi}], [0.21e^{i0.28\pi}, 0.41e^{i0.32\pi}])$

TABLE 2. A and other period

	(A, C)	(A, E)
	$([0.23e^{i0.28\pi}, 0.34e^{i0.33\pi}], [0.45e^{i0.47\pi}, 0.52e^{i0.49\pi}])$	$([0.25e^{i0.31\pi}, 0.37e^{i0.35\pi}], [0.20e^{i0.28\pi}, 0.38e^{i0.30\pi}])$
	$([0.24e^{i0.30\pi}, 0.37e^{i0.34\pi}], [0.43e^{i0.45\pi}, 0.50e^{i0.46\pi}])$	$([0.25e^{i0.31\pi}, 0.37e^{i0.35\pi}], [0.20e^{i0.28\pi}, 0.40e^{i0.32\pi}])$
	$([0.24e^{i0.31\pi}, 0.36e^{i0.35\pi}], [0.44e^{i0.46\pi}, 0.50e^{i0.48\pi}])$	$([0.24e^{i0.30\pi}, 0.36e^{i0.34\pi}], [0.21e^{i0.28\pi}, 0.41e^{i0.32\pi}])$

TABLE 3. B and other period

	(B, D)	(B, E)
	$([0.38e^{i0.48\pi}, 0.70e^{i0.58\pi}], [0.41e^{i0.44\pi}, 0.50e^{i0.44\pi}])$	$([0.36e^{i0.47\pi}, 0.68e^{i0.56\pi}], [0.21e^{i0.28\pi}, 0.44e^{i0.32\pi}])$
	$([0.35e^{i0.46\pi}, 0.68e^{i0.56\pi}], [0.42e^{i0.45\pi}, 0.51e^{i0.46\pi}])$	$([0.37e^{i0.48\pi}, 0.69e^{i0.57\pi}], [0.20e^{i0.26\pi}, 0.43e^{i0.30\pi}])$
	$([0.38e^{i0.47\pi}, 0.69e^{i0.58\pi}], [0.40e^{i0.43\pi}, 0.50e^{i0.44\pi}])$	$([0.37e^{i0.48\pi}, 0.69e^{i0.57\pi}], [0.20e^{i0.26\pi}, 0.42e^{i0.31\pi}])$

defined. The relationships on Q should be defined as follows: R_1= Rain season, R_2= Mixed season, R_3= Snow season, such that $G^* = \{R_1, R_2, R_3\}$ is a GS. The season and daily chance of precipitation for kale are displayed by each component of the relationship. Given that the graph structure is $G^* = \{R_1, R_2, R_3\}$, only one relation may exist between the seasons of Snow, Mixed, and Daily Probability of Rain. It would then be considered a part of that connection, whose amplitude truth membership value is comparable to being the lowest compared to other relationships, as well as whose phase truth membership value is comparable to being the lowest compared to other relationships. It would therefore be considered a part of that relationship, whose amplitude falsity membership value is comparable high compared to that of other relationships and whose phase falsity membership value is comparable high compared to that of other relationships. These IVCPFSs are $\lambda_1, \lambda_2, \lambda_3$, respectively. $R_1 = \{AC, BE\}, R_2 = \{BD\}$ and $R_3 = \{AE\}$ And the corresponding IVCPFS are
$\lambda_1 = \{AC([0.23e^{i0.28\pi}, 0.34e^{i0.33\pi}], [0.45e^{i0.47\pi}, 0.52e^{i0.49\pi}]),$
$BE([0.36e^{i0.47\pi}, 0.68e^{i0.56\pi}], [0.21e^{i0.28\pi}, 0.44e^{i0.32\pi}])\},$
$\lambda_2 = \{BD([0.35e^{i0.46\pi}, 0.68e^{i0.56\pi}], [0.42e^{i0.45\pi}, 0.51e^{i0.46\pi}])\},$
$\lambda_3 = \{AE([0.24e^{i0.30\pi}, 0.36e^{i0.34\pi}], [0.21e^{i0.28\pi}, 0.41e^{i0.32\pi}])\}$. Therefore, the IVCPFGS represented in Figure-11 is $(\mu, \lambda_1, \lambda_2, \lambda_3)$. The season and daily chance of climate for kale are represented by each edge of the IVCPFGS in Figure-11. The ability to anticipate atmospheric temperature is crucial for many applications. It is challenging to make any kind of accurate weather predictions because the atmosphere is dynamic. Data on climate change can be analysed using the IVCPFGS concept.

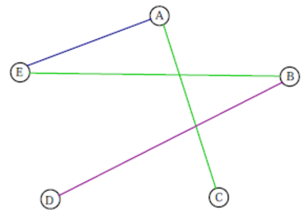

FIGURE 11. IVCPFGS

7. Conclusion

In this study, we demonstrate the idea of IVCPFGS, as well as the outcomes of some regular and totally regular IVCPFGS, as well as the degree of vertex present. The Subdivision of an IVCPFGS, the ψ-complement IVCPFGS, and the potency of a Path in an IVCPFGS were also topics of discussion. using IVCPFGS solve this problem. Our main finding is that many applications require the ability to estimate air temperature. It is challenging to produce any kind of accurate weather predictions because of how quickly the atmosphere changes. The climate change information in sections 5 and 6 can be analysed using the IVCPFGS concept.

Acknowledgement. We are very grateful for the reviewer's suggestion which we believe will further improve the quality of our article. I would like to express my sincere gratitude to Ms. Sowndharya for all of her help in getting my research done.

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