

A SUBCLASS OF BAZILEVIČ FUNCTIONS ASSOCIATED WITH COMBINATION OF OPOOLA AND SWAMY OPERATORS AND SATISFYING SUBORDINATE CONDITION

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ABSTRACT. This investigation involves the application of Horadam polynomials and subordination to a subclass of Bazilevič functions of type α for the purpose of obtaining some initial bounds for functions in the class. In addition, the famous classical Fekete-Szegő inequality for our new class is established. Furthermore, the connections between our results with some existing results are stated as corollaries.

1. INTRODUCTION AND PRELIMINARIES

Let $\mathcal{A}$ be the class of functions $k(\eta)$ which are analytic in the open unit disk $\Delta := \{\eta : \eta \in \mathbb{C} \text{ and } |\eta| < 1\}$ and of the series form

$$k(\eta) = \eta + \sum_{r=2}^{\infty} m_r \eta^r \quad (\eta \in \Delta). \quad (1.1)$$

Also, let $\mathbb{R} = (-\infty, \infty)$ be the set of real numbers, $\mathbb{C}$ be the set of complex numbers and

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

be the set of natural numbers, so that $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

Let " $\prec$ " denote subordination, then for two functions $k_1, k_2 \in \mathcal{A}$, $k_1(\eta) \prec k_2(\eta)$ if there is an analytic function

$$u(\eta) = \sum_{r=1}^{\infty} c_r \eta^r \quad (u(0) = 0, |u(\eta)| < 1, \eta \in \Delta) \quad (1.2)$$

such that $k_1(\eta) = k_2(u(\eta))$ and $\eta \in \Delta$. Suppose $k_2(\eta)$ is univalent in Δ , then

$$k_1(\eta) \prec k_2(\eta) \text{ if and only if } k_1(0) = k_2(0) \text{ and } k_1(\Delta) \subset k_2(\Delta).$$

Date: Received: Jun 12, 2023; Accepted: Jul 3, 2023.

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2010 *Mathematics Subject Classification.* Primary: 30C45; Secondary: 33C47.

Key words and phrases. Analytic functions, Bazilevič functions, Fekete-Szegő inequality, Horadam polynomials, Opoola differential operator, Ruscheweyh operator, Swamy multiplier operator.

For function $k(\eta)$ of the form (1.1), we recall the operator

$$\begin{aligned} & \mathcal{RE}_{\alpha,\beta,\delta}^n(\sigma, \varphi, t)k(\eta) \\ &= \eta + \sum_{r=2}^{\infty} [(1-\delta)B_r(n) + \delta A_r(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t)^n] m_r \eta^r = \mathcal{J}(\eta) \end{aligned} \quad (1.3)$$

where $\mathcal{RE}_{\alpha,\beta,\delta}^n(\sigma, \varphi, t)k(\eta)$, $B_r(n)$ and $A_r(\rho, \beta, n)$ are as defined in [23]. We also note that the restrictions imposed on the involved parameters are that $\beta \geq 0$; $\alpha \in \mathbb{R}$; $t, (\alpha + \beta) > 0$; $n \in \mathbb{N}_0$ and $0 \leq \sigma \leq \varphi$. It is worth mentioning that (1.3) is a combination of the Ruscheweyh operator [26] and a new generalized operator involving convolution, in which Opoola differential operator in [14] and Swamy generalized multiplier operator in [27] are prominent. For other related and recent works on Opoola differential operator, interested readers are enjoined to see [2, 13, 18, 22, 23, 24, 25].

With the aid of the generalized differential operator $\mathcal{RE}_{\alpha,\beta,\delta}^n(\sigma, \varphi, t)$, the authors in [23] introduced a subclass $\mathcal{M}_{\alpha,\beta,\delta}^{m,\lambda,\mu,\sigma,\varpi,t}(\rho)$ of the class of analytic functions in the open unit disk Δ and investigated the class for characterization, coefficient estimates and distortion theorems where the class consisted of functions in $\mathcal{A}$ having negative coefficients.

In what follows, we give the following new definitions by making use of the operator defined by (1.3).

Definition 1.1. A function $k(\eta)$ of the form (1.1) is in the class $\mathcal{G}_{\alpha,\beta,\delta}^{n,\varphi,\sigma}(t, \mu, x)$ if it satisfies the subordination condition

$$\frac{J'(\eta)J(\eta)^{\mu-1}}{\eta^{\mu-1}} \prec \Pi(x, \eta) + 1 - a \quad (\eta \in \Delta)$$

where $0 \leq \mu \leq 1$ and

$$\Pi(x, \eta) + 1 - a = \frac{a + x\eta(b - ap)}{1 - px\eta - q\eta^2} \quad (a, b, p, q \in \mathbb{R})$$

is the generating function of the Horadam polynomials. All other parameters are as defined earlier.

Definition 1.2 ([9]). For $n \geq 3$, the Horadam polynomials sequence $h_n(x, a, b; p, q)$ or briefly denoted as $h_n(x)$ is defined by the recurrence relations

$$h_n(x) = pxh_{n-1}(x) + qh_{n-2}(x) \quad (x \in \mathbb{R}, n \in \mathbb{N} \setminus \{1, 2\}) \quad (1.4)$$

and having initial conditions

$$h_1(x) = a \quad \text{and} \quad h_2(x) = bx.$$

such that $a, b, p, q \in \mathbb{R}$ are constants. The characteristic equation of recurrence relation (1.4) is

$$t^2 - pxt - q = 0$$

and it has two real roots:

$$\alpha = \frac{px + \sqrt{p^2x^2 + 4q}}{2} \quad \text{and} \quad \beta = \frac{px - \sqrt{p^2x^2 + 4q}}{2}.$$

Remark 1.3. We note the following polynomials when the parameters a , b , p and q are varied in (1.4).

- (1) Setting $a = b = p = q = 1$ in (1.4) gives the Fibonacci polynomials $F_n(x)$.
- (2) Setting $a = q = 1$ and $b = p = 2$ in (1.4) gives the Lucas polynomials $L_n(x)$.
- (3) Setting $a = 2$ and $b = p = q = 1$ in (1.4) gives the Pell polynomials $P_n(x)$.
- (4) Setting $a = b = p = 2$ and $q = 1$ in (1.4) gives the Pell-Lucas polynomials $Q_n(x)$.
- (5) Setting $a = b = 1$, $p = 2$ and $q = -1$ in (1.4) gives the Chebyshev polynomials $T_n(x)$ of the first kind.
- (6) Setting $a = 1$, $b = p = 2$ and $q = -1$ in (1.4) gives the Chebyshev polynomials $U_n(x)$ of the second kind.

Remark 1.4. Some of the Horadam number sequence are shown in Table 1.

TABLE 1. Some of the Horadam number sequence

n	$h_n(x, a, b; p, q)$
1	a
2	bx
3	$bp^2x^2 + aq$
4	$bp^2x^3 + (apq + bq)x$
5	$bp^3x^4 + (ap^2q + 2bpq)x^2 + aq^2$
6	$bp^4x^5 + (ap^3q + 3bp^2q)x^3 + (2apq^2 + bq^2)x$
7	$bp^5x^6 + (ap^4q + 4bp^3q)x^4 + (3ap^2q^2 + 3bpq^2)x^2 + aq^3$
8	$bp^6x^7 + (ap^5q + 5bp^4q)x^5 + (4ap^3q^2 + 6bp^2q^2)x^3 + (3apq^3 + bq^3)x$
$\vdots$	$\vdots$
$\vdots$	$\vdots$

We note that if $x = 1$, then the Horadam number sequence

$$h_{n-1}(1) = ph_{n-3}(1), \quad h_0(1) = q \quad \text{and} \quad h_1(1) = b$$

are obtained and the generating function of Horadam polynomials $h_n(x)$ is given by

$$\Pi(x, \eta) = \sum_{n=1}^{\infty} h_n(x) \eta^{n-1} = \frac{a + x\eta(b - ap)}{1 - px\eta - q\eta^2}.$$

Also, the Binet's formula for the Horadam polynomials sequence $h_n(x)$ represented by the roots α and β is given by

$$h_n(x) = E_1 \alpha^{n-1} + E_2 \beta^{n-1}$$

where

$$E_1 = \frac{bx - a\beta}{\sqrt{p^2x^2 + 4q}} \quad \text{and} \quad E_2 = \frac{a\alpha - bx}{\sqrt{p^2x^2 + 4q}}.$$

The explicit formula for Horadam sequence is also given by

$$h_n = a \sum_{k=0}^{\infty} \left[\frac{n}{2} \right] \binom{n-k}{k} p^{n-2k} q^k + \left(\frac{b}{p} - a \right) \sum_{k=0}^{\infty} \left[\frac{n-1}{2} \right] \binom{n-k-1}{k} p^{n-2k} q^k$$

for $n \geq 1$ and $a, b, p, q \in \mathbb{Z}$, see [8] for more details.

Extensive study of orthogonal polynomials could be traced back to Legendre in 1784. Orthogonal polynomials are related to typically real functions, which play an important role in geometric function theory especially in estimating coefficient bounds. Certain orthogonal polynomials such as Chebyshev polynomials, Gegenbauer polynomials, Horadam polynomials and many others, have been applied in the study of certain subclasses of analytic, univalent and bi-univalent functions. For some examples see [1, 3, 11, 4, 6, 7, 8, 9, 10, 15, 16, 17, 19, 21, 28, 30, 29].

The goal of this present investigation is to derive some estimates for the coefficient bounds $|m_2|$ and $|m_3|$; and to consider the celebrated Fekete-Szegő problem (for instance see [5, 12, 13]) for functions of the form (1.1) and belonging to the class $\mathcal{G}_{\alpha, \beta, \delta}^{n, \varphi, \sigma}(t, \mu, x)$ in Definition 1.1.

2. MAIN RESULTS

Theorem 2.1. *If $k(\eta) \in \mathcal{G}_{\alpha, \beta, \delta}^{n, \varphi, \sigma}(t, \mu, x)$, then*

$$|m_2| \leq \frac{bx}{(\mu + 1)[1 - \delta]B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t)^n}$$

and

$$|m_3| \leq \frac{1}{(\mu + 2)[1 - \delta]B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t)^n} \times \left(bx + \left[bpx^2 + aq + \left(1 - \frac{\mu(\mu + 3)}{2(\mu + 1)} \right) \frac{b^2x^2}{\mu + 1} \right] \right).$$

Proof. If $k(\eta) \in \mathcal{G}_{\alpha, \beta, \delta}^{n, \varphi, \sigma}(t, \mu, x)$, then we have

$$\frac{\mathcal{J}'(\eta)\mathcal{J}(\eta)^{\mu-1}}{\eta^{\mu-1}} = 1 + h_2(x)u(\eta) + h_3(x)[u(\eta)]^2 + \dots \quad (2.1)$$

for $h_1(x) = a$. Replacing $\mathcal{J}(\eta)$ and $\mathcal{J}'(\eta)$ with their equivalent series and evaluating the LHS of (2.1), we have

$$\begin{aligned} & \left(\eta + \sum_{r=2}^{\infty} r [(1 - \delta)B_r(n) + \delta A_r(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t)^n] m_r \eta^r \right) \\ & \left(1 + \mu \sum_{r=2}^{\infty} [(1 - \delta)B_r(n) + \delta A_r(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t)^n] m_r \eta^{r-1} + \right. \\ & \left. \frac{\mu(\mu - 1)}{2} \sum_{r=2}^{\infty} [(1 - \delta)B_r(n) + \delta A_r(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t)^{2n}] m_r^2 \eta^{2(r-1)} + \dots \right) \\ & \hline & \left(\eta + \sum_{r=2}^{\infty} [(1 - \delta)B_r(n) + \delta A_r(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t)^n] m_r \eta^r \right) \\ & = 1 + h_2(x)u(\eta) + h_3(x)[u(\eta)]^2 + \dots \quad (2.2) \end{aligned}$$

where $u(\eta)$ is an analytic function in (1.2) such that

$$u(0) = 0 \quad \text{and} \quad |u(\eta)| = |c_1\eta + c_2\eta^2 + c_3\eta^3 + \dots| < 1 \quad (\eta \in \Delta). \quad (2.3)$$

Expressing (2.2) in terms of its summations gives

$$\begin{aligned}
& \left[\eta + 2[(1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n m_2 \eta^2 + \right. \\
& \quad \left. 3[(1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n m_3 \eta^3 + \dots \right] \times \\
& \quad \left[1 + \mu [1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t]^n m_2 \eta + \right. \\
& \quad \left. \mu [1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t]^n m_3 \eta^2 + \dots \right. \\
& \quad \left. + \frac{\mu(\mu - 1)}{2} [1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t]^{2n} m_2^2 \eta^2 + \dots \right] \\
& \quad = \left(1 + h_2(x)c_1 \eta + [h_2(x)c_2 + h_3(x)c_1^2] \eta^2 + \dots \right) \\
& \quad \times \left[\eta + [1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t]^n m_2 \eta^2 + \right. \\
& \quad \left. [1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t]^n m_3 \eta^3 + \dots \right] \quad (2.4)
\end{aligned}$$

so that

$$\begin{aligned}
& \eta + (\mu + 2) \left[(1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t \right]^n m_2 \eta^2 \\
& \quad \left[(\mu + 3) [1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t \right]^n m_3 + \\
& \quad \frac{\mu(\mu + 3)}{2} [1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t]^{2n} m_2^2 \right] \eta^3 + \dots \\
& \quad = \eta + \left[h_2(x)c_1 + [1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n m_2 \right] \eta^2 + \\
& \quad \left[h_2(x)c_2 + h_3(x)c_1^2 + h_2(x)c_1 [1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n m_2 + \right. \\
& \quad \left. [1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n m_3 \right] \eta^3 + \dots \quad (2.5)
\end{aligned}$$

Equating the like terms gives

$$(\mu + 1)[1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n m_2 = h_2(x)c_1 \quad (2.6)$$

so

$$m_2 = \frac{h_2(x)c_1}{(\mu + 1)[1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n} \quad (2.7)$$

and

$$|m_2| \leq \frac{bx}{(\mu + 1)[1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n}. \quad (2.8)$$

Also,

$$\begin{aligned}
& (\mu + 2) [(1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n m_3 \\
& = h_2(x)c_2 + h_3(x)c_1^2 + h_2(x)c_1 [(1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n m_2 \\
& \quad - \frac{\mu(\mu + 3)}{2} [(1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (r + \varphi - \sigma - 1)t]^{2n} m_2^2 \quad (2.9)
\end{aligned}$$

$$\begin{aligned}
(\mu + 2) [(1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n m_3 \\
= h_2(x)c_2 + \left[h_3(x) + \left(1 - \frac{\mu(\mu + 3)}{2(\mu + 1)} \right) \frac{h_2^2(x)}{\mu + 1} \right] c_1^2 \quad (2.10)
\end{aligned}$$

so

$$\begin{aligned}
m_3 = \frac{1}{(\mu + 2) [1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n} \times \\
\left(h_2(x)c_2 + \left[h_3(x) + \left(1 - \frac{\mu(\mu + 3)}{2(\mu + 1)} \right) \frac{h_2^2(x)}{\mu + 1} \right] c_1^2 \right) \quad (2.11)
\end{aligned}$$

and

$$\begin{aligned}
|m_3| \leq \frac{1}{(\mu + 2) [1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n} \times \\
\left(bx + \left[bpx^2 + aq + \left(1 - \frac{\mu(\mu + 3)}{2(\mu + 1)} \right) \frac{b^2x^2}{\mu + 1} \right] \right) \quad (2.12)
\end{aligned}$$

□

Putting $n = 0$, $b = 2$, $a = 1$, $p = 2$, $q = -1$ and $x = t$ in Theorem 2.1, then we obtain the following corollary.

Corollary 2.2. *If $k(\eta) \in \mathcal{G}_{\alpha, \beta, \delta}^{0, \varphi, \sigma}(t, \mu, t)$, then*

$$|m_2| \leq \frac{2t}{(\mu + 1)}$$

and

$$|m_3| \leq \frac{1}{(\mu + 2)} \left(2t + \left[4t^2 - 1 + \left(1 - \frac{\mu(\mu + 3)}{2(\mu + 1)} \right) \frac{4t^2}{\mu + 1} \right] \right).$$

This is the result obtained in [10].

Theorem 2.3. *If $k(\eta) \in \mathcal{G}_{\alpha, \beta, \delta}^{n, \varphi, \sigma}(t, \mu, x)$, then for real value ξ ,*

$$\begin{aligned}
|m_3 - \xi m_2^2| \leq \frac{1}{(\mu + 2) [1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n} \times \\
\left(bx + bpx^2 + aq + \left(1 - \frac{\mu(\mu + 3)}{2(\mu + 1)} \right) \frac{b^2x^2}{\mu + 1} - \right. \\
\left. \xi \frac{(\mu + 2) [1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n b^2x^2}{(\mu + 1)^2 [1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^{2n}} \right).
\end{aligned}$$

Proof. From (2.7) and (2.11) we have

$$\begin{aligned}
m_3 - \xi m_2^2 = \frac{1}{(\mu + 2) [(1 - \delta)B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n} \times \\
\left(h_2(x)c_2 + \left[h_3(x) + \left(1 - \frac{\mu(\mu + 3)}{2(\mu + 1)} \right) \frac{h_2^2(x)}{\mu + 1} \right] c_1^2 \right) \\
- \xi \frac{h_2^2(x)c_1^2}{(\mu + 1)^2 [(1 - \delta)B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^{2n}} \quad (2.13)
\end{aligned}$$

or

$$m_3 - \xi m_2^2 = \frac{1}{(\mu + 2)[1 - \delta]B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n} \times \left(h_2(x)c_2 + h_3(x)c_1^2 + \left(1 - \frac{\mu(\mu + 3)}{2(\mu + 1)}\right) \frac{h_2^2(x)}{\mu + 1} c_1^2 - \xi \frac{(\mu + 2)[1 - \delta]B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n}{(\mu + 1)^2[1 - \delta]B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^{2n}} h_2^2(x) c_1^2 \right) \quad (2.14)$$

and

$$|m_3 - \xi m_2^2| \leq \frac{1}{(\mu + 2)[1 - \delta]B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n} \times \left(bx + bpx^2 + aq + \left(1 - \frac{\mu(\mu + 3)}{2(\mu + 1)}\right) \frac{b^2x^2}{\mu + 1} - \xi \frac{(\mu + 2)[1 - \delta]B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n}{(\mu + 1)^2[1 - \delta]B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^{2n}} b^2x^2 \right). \quad (2.15)$$

□

Putting $\beta = 0$, $b = 2$, $a = 1$, $p = 2$, $q = -1$ and $x = t$ in Theorem 2.3 we obtain the following corollary.

Corollary 2.4. *If $k(\eta) \in \mathcal{G}_{\alpha,0,\delta}^{n,\varphi,\sigma}(t, \mu, t)$, then*

$$|m_3 - \xi m_2^2| \leq \frac{2t}{\mu + 2} + \frac{4t^2 - 1}{\mu + 2} + \frac{2(1 - \mu)t}{(\mu + 1)^2} - \xi 4t^2.$$

Putting $\mu = 1$ in Theorem 2.3 we obtain the following corollary.

Corollary 2.5. *If $k(\eta) \in \mathcal{G}(\varphi, \beta, 1, x)$, then*

$$|m_3 - \xi m_2^2| \leq \frac{1}{3[1 - \delta]B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n} \times \left(bx + bpx^2 + aq - \xi \frac{(\mu + 2)[1 - \delta]B_3(n) + \delta A_3(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^n}{(\mu + 1)^2[1 - \delta]B_2(n) + \delta A_2(\rho, \beta, n)(1 + (\varphi - \sigma + 1)t]^{2n}} b^2x^2 \right).$$

Acknowledgement. The authors would like to thank the editor and the anonymous reviewer(s) for their collective valuable suggestions that have helped to improved this work.

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