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NOVEL MULTI-WAVE SOLUTIONS FOR THE FRACTIONAL ORDER DUAL-MODE NONLINEAR SCHRÖDINGER EQUATION

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ABSTRACT. This paper considers the fractional order dual-mode nonlinear Schrödinger equation (FDMNLSE) with cubic law nonlinearity. This model dissects the absorption or enlargement of dual waves in the occurrence of non-linearity and distribution influences. Also, it has many implementations in fiber optics and nonlinear physics. This work gives the fractional derivative in terms of time and space conformable sense. We will analyze the periodic cross-kink wave solution strategy, M-shaped rational soliton technique, M-shaped with one and two kinks procedure, the interaction between periodic and M-shaped method, interaction with M-shaped, I-kink, and rogue wave approach and their applications for this equation are obtained using logarithmic transformation. The analytical outcomes are analyzed by the graphical illustration displaying the suggested techniques' reliability and virtue.

1. Introduction

The examination of nonlinear partial differential equations (NLPDEs) is used to depict nonlinear-complex phenomena in different specializations of nonlinear sciences, such as plasma physics, fluid dynamics, hydrodynamics, and many more [1, 2].

In the current century, mathematical prototypes with differential equations (DEs) have been standard in the past decades to model scientific or daily life problems, seen in chemistry, biology, engineering, etc., and even real-world problems. One of the most well-known prototypes is the Korteweg-de Vries equation (KdVE) represents wave motion in shallow canals. The second one happening in this region is a nonlinear Schrödinger equation (NLSE) known as the nonlinear evolution equation (NLEE). The study of NLEEs is an issue of active interest in various specializations, including chemistry, engineering, and physics. The exact solutions to NLEEs supply insightful points and physical explanations of considerable problems of draw that govern the real world.

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Nowadays, in fractional calculus, various shapes of operators like Atangana-Baleanu derivatives, Beta-time derivatives [3], the Riemann Liouville derivatives [4], the Caputo Fabrizio derivatives [5], and the conformable fractional derivatives(CFDs) [6] have been created to research the fractional NLPDEs. To portray the problems of the real world, this prototype shape has notable matter and is being used in the areas of applied sciences and engineering to solve complicated nonlinear issues. The performance of conformable derivatives is easier and quite fruitful. Thus it helps us to understand the nature of perceptible systems.

The primary focal point of nonlinear sciences is soliton and their interaction solutions. Many researchers are interested in exploring rational solutions and their interactions with kink and periodic waves because of their implementations in different specializations, such as physics, biology, chemistry, atmosphere, engineering, oceanography, neuroscience, etc.

Over the past few years, several efficient analytical techniques for NLEEs have been suggested as Solitary wave solution [7], the quasi-periodic solution [8], the Ricatti-Bernoulli Sub ODE method (RBSOM) [9], cnoidal and snoidal wave solutions [10], Lie symmetry analysis [11], the extended modified auxiliary equation mapping method [12], Unified method [13], the improved projective Riccati equations method [14], and much more.

The FDMNLSE is described as

$$i(D_t^{2\gamma}W - s^2 D_x^{2\gamma}W) + (D_t^{\gamma}\left\{F\left(|W|^2\right)W\right\} - \pi s D_x^{\gamma}\left\{F\left(|W|^2\right)W\right\})$$

$$+ (D_t^{\gamma}\left\{\frac{1}{2}D_x^{2\gamma}W\right\} - \theta s D_x^{\gamma}\left\{\frac{1}{2}D_x^{2\gamma}W\right\}) = 0. \tag{1.1}$$

$W = W(x,t)$ is a complex function that stands for the envelope field with the propagation distance x and the temporal variable t. $s \geq 0$, $|\pi| \leq \pm 1$, and $|\theta| \leq \pm 1$ symbolize an interaction phase speed, nonlinearity factor, and dispersive factor, respectively. Eq.(1.1) is used for evaluating the absorption or enlargement of dual waves in the occurrence of nonlinearity and distribution effects. Those equations are also in charge of investigating the effects of interplay phase speed s. Also, dual-mode class equations (DMEs) have lately lured more examination in the nonlinear sciences field. The reason is that DMEs in the current design explore extemporaneous wave associations. We observe considerably different works in the literature relating to dual-mode KdVE [15, 20].

The principal objective of this paper highlights utilizing conformable fractional derivatives with the periodic cross-kink wave solution strategy, M-shaped rational solitons technique, M-shaped with one and two kinks procedure, the interaction between periodic and M-shaped method, interaction with M-shaped, I-kink, and rogue wave approaches to extract new optical solutions.

The paper is assembled as follows. Section 2 defines some properties of the CFD. Section 3 presents the periodic cross-kink wave solution strategy, M-shaped rational solitons technique, M-shaped with one and two kinks procedure, the interaction between periodic and M-shaped method, interaction with M-shaped, I-kink, and rogue wave approaches and their applications. Conclusions are offered in section 4.

2. The some properties of the CFD

A CFD is well-defined as [21, 22]:

$$\wp_x^\gamma(q(x)) = \lim_{\epsilon\to 0}\frac{q(x+\epsilon x^{1-\gamma}) - q(x)}{\epsilon}.$$

Theorem 2.1. *Let $\gamma \in (0,1]$, and $k = k(x)$, $l = l(x)$ and γ be CFD at $x > 0$, then*

p1) $\wp_x^\gamma(x^n) = nx^{n-\gamma}, \quad n \in \mathbb{R}$
p2) $\wp_x^\gamma(\omega) = 0$, in which ω is any arbitrary constant.
p3) $\wp_x^\gamma(\varkappa_1 k(x) + \varkappa_2 l(x)) = \varkappa_1\wp_x^\gamma(k(x)) + \varkappa_2\wp_x^\gamma(l(x))$, $\forall \varkappa_1, \varkappa_2 \in \mathbb{R}$.
p4) $\wp_x^\gamma(k(x).l(x)) = k(x)\wp_x^\gamma(l(x)) + l(x)\wp_x^\gamma(k(x))$.
p5) $\wp_x^\gamma(\frac{k(x)}{l(x)}) = \frac{l(x)\wp_x^\gamma(k(x)) - k(x)\wp_x^\gamma(l(x))}{l^2(x)}$. If $k(x)$ is differentiable, then $\wp_x^\gamma k(x) = x^{1-\gamma}\frac{dk(x)}{dx}$.

Theorem 2.2. *Assume $k(x)$ is a differentiable and also $\gamma-$differentiable with $\gamma \in (0,1]$. Let a function $l(x)$ be defined in the same range of $k(x)$ and also differentiable, then*

$$\wp_x^\gamma(k(x).l(x)) = x^{1-\gamma}l'(t)k'(s(t)).$$

3. Algorithms and execution of the proposed methods

In cubic law, $F(W) = W$, thus Eq.(1.1) become like this

$$i(D_t^{2\gamma}W - s^2D_x^{2\gamma}W) + (D_t^\gamma\left\{|W|^2W\right\} - \pi s D_x^\gamma\left\{|W|^2W\right\})$$
$$+ (D_t^\gamma\left\{\frac{1}{2}D_x^{2\gamma}W\right\} - \theta s D_x^\gamma\left\{\frac{1}{2}D_x^{2\gamma}W\right\}) = 0. \tag{3.1}$$

Let us define wave transformation as

$$\begin{aligned} W(x,t) &= e^{i\alpha\phi}V(\xi), \\ \xi(x,t) &= \frac{(x^\gamma - at^\gamma)}{\gamma}, \quad \phi(x,t) = \frac{(x^\gamma + \eta t^\gamma)}{\gamma}. \end{aligned} \tag{3.2}$$

Putting the (3.2) into Eq.(3.1), the imaginary and real parts are got respectively as:

$$(-2\eta^2 - 2s^2 + \theta s\alpha - \alpha\eta)\alpha^2V + (-2\pi s + 2\eta)\alpha V^3 + (2a^2 - 2s^2 + \alpha\eta - 2sa - 3\alpha\theta s)V'' = 0, \tag{3.3}$$

$$(4\eta a + 4s^2 + 3\alpha\theta s - 2s\eta + \alpha a)\alpha V - 2(a + \pi s)V^3 - (\theta s + a)V'' = 0. \tag{3.4}$$

From the Eq.(3.4), $\pi = \theta = -\frac{a}{s}$, $\eta = \frac{\alpha a - 2s^2}{2a-\alpha}$, and plugging them into Eq.(3.3), then Eq.(3.3) is carried to a nonlinear ordinary differential equation (NODE) as follows

$$\begin{gathered}(2a-\alpha)(a+s)(a-s)aV'' \\ +(-5\alpha a s^2+2s^4+2s^2a^2+s^2\alpha^2+\alpha a^3)\alpha^2 V+(2a-\alpha)(a+s)(a-s)\alpha V^3=0. \end{gathered} \tag{3.5}$$

For the goal of discovering soliton solutions, we utilize the following alteration (Ma et al.[23]):

$$V(\xi) = 2(\ln\kappa)_\xi = 2\frac{\kappa_\xi}{\kappa}. \tag{3.6}$$

By placing Eq.(3.6) into Eq.(3.5) to obtain the subsequent bilinear form:

$$\begin{gathered}4a^4\kappa^2\kappa_{\xi\xi\xi} - 2\alpha a^3\kappa^2\kappa_{\xi\xi\xi} - 4a^2s^2\kappa^2\kappa_{\xi\xi\xi} + 2a\alpha s^2\kappa^2\kappa_{\xi\xi\xi} - 12a^4\kappa\kappa_\xi\kappa_{\xi\xi} \\ +6\alpha a^3\kappa\kappa_\xi\kappa_{\xi\xi} + 12a^2s^2\kappa\kappa_\xi\kappa_{\xi\xi} - 6\alpha a s^2\kappa\kappa_\xi\kappa_{\xi\xi} + 8a^4\kappa_\xi^3 + 12\alpha a^3\kappa_\xi^3 \\ -8a^2s^2\kappa_\xi^3 - 12a\alpha s^2\kappa_\xi^3 + 2\alpha^3a^3\kappa^2\kappa_\xi + 4a^2s^2\alpha^2\kappa^2\kappa_\xi - 10\alpha^3 a s^2\kappa^2\kappa_\xi \\ + 2\kappa^2\alpha^4s^2\kappa_\xi + 4\kappa^2\alpha^2s^4\kappa_\xi - 8\alpha^2a^2\kappa_\xi^3 + 8\alpha^2s^2\kappa_\xi^3 = 0. \end{gathered} \tag{3.7}$$

Using the Eq.(3.7), we discuss the following procedures [24, 25].

3.1. **Periodic cross-kink wave solution:**

To get the periodic cross-kink wave solution of Eq.(3.1), we consider the successive transformation

$$\kappa = \exp(-k_1\xi - k_2) + r_0\exp(k_1\xi+k_2) + r_1\cos(k_3\xi+k_4) + r_2\cosh(k_5\xi+k_6) + h_0, \tag{3.8}$$

in which k_i, r_i are constants. Putting Eq.(3.8) into Eq.(3.7) and solving all the coefficients of $\exp(-k_1\xi - k_2)$, $\exp(k_1\xi + k_2)$, $\sinh(k_5\xi + k_6)$, $\cosh(k_5\xi + k_6)$, $\sin(k_3\xi + k_4)$, $\cos(k_3\xi + k_4)$, $\exp(-2k_1\xi - 2k_2)$, $\exp(2k_1\xi + 2k_2)$, $\exp(3k_1\xi + 3k_2)$, $\exp(-3k_1\xi - 3k_2)$ to zero, we reach

$$\begin{aligned} a &= -2\alpha, \quad h_0 = 0, \quad k_2 = k_2, \quad k_3 = 0, \quad k_4 = k_4, \quad k_6 = k_6, \quad r_0 = r_0, \\ r_1 &= 0, \quad r_2 = r_2, \quad k_1 = k_5 = \pm\sqrt{-\frac{8\alpha^4 - 19\alpha^2 s^2 - 2s^4}{80\alpha^2 - 20s^2}}. \end{aligned} \tag{3.9}$$

Thanks to the Eqs.(3.8),(3.9) and by using the (3.2),(3.6), we explore the desired periodic cross-kink wave solution of Eq.(3.1)

$$W_1(x,t) = \frac{2\begin{pmatrix} -\sqrt{-\frac{8\alpha^4-19\alpha^2s^2-2s^4}{80\alpha^2-20s^2}}\exp\left(-\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right)\sqrt{-\frac{8\alpha^4-19\alpha^2s^2-2s^4}{80\alpha^2-20s^2}} - k_2\right) \\ +r_0\sqrt{-\frac{8\alpha^4-19\alpha^2s^2-2s^4}{80\alpha^2-20s^2}}\exp\left(\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right)\sqrt{-\frac{8\alpha^4-19\alpha^2s^2-2s^4}{80\alpha^2-20s^2}} + k_2\right) \\ +r_2\sinh\left(\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right)\sqrt{-\frac{8\alpha^4-19\alpha^2s^2-2s^4}{80\alpha^2-20s^2}} + k_6\right)\sqrt{-\frac{8\alpha^4-19\alpha^2s^2-2s^4}{80\alpha^2-20s^2}} \end{pmatrix}}{\begin{pmatrix} \exp\left(-\xi\sqrt{-\frac{8\alpha^4-19\alpha^2s^2-2s^4}{80\alpha^2-20s^2}} - k_2\right) \\ +r_0\exp\left(\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right)\sqrt{-\frac{8\alpha^4-19\alpha^2s^2-2s^4}{80\alpha^2-20s^2}} + k_2\right) \\ +r_2\cosh\left(\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right)\sqrt{-\frac{8\alpha^4-19\alpha^2s^2-2s^4}{80\alpha^2-20s^2}} + k_6\right) \end{pmatrix}} \times e^{i\alpha\frac{(x^\gamma+\eta t^\gamma)}{\gamma}} \tag{3.10}$$

3.2. **M-Shaped with I-kink solution:**
For M-Shaped with I-kink solution of Eq.(3.1), we suppose the following ansatz transformation

$$\kappa = (k_1\xi + k_2)^2 + (k_3\xi + k_4)^2 + h_1 \exp(k_5\xi + k_6), \tag{3.11}$$

in which k_i and h_1 are some parameters. Putting Eq.(3.11) into Eq.(3.7) and compute all the coefficients of $\exp(k_5\xi + k_6)$, $\exp(2k_5\xi + 2k_6)$, and $\exp(3k_5\xi + 3k_6)$ to zero, we attain

$$\begin{aligned} s &= \frac{1}{2\alpha}(\alpha(-2\alpha a^2 + 5\alpha^2 a + 8ak_5^2 - \alpha^3 - 4\alpha k_5^2 + (4a^4\alpha^2 - 28a^3\alpha^3 - 96a^3\alpha k_5^2 \\ &\quad +29a^2\alpha^4 + 128a^2\alpha^2 k_5^2 + 64a^2k_5^4 - 10a\alpha^5 - 56a\alpha^3k_5^2 - 64a\alpha k_5^4 + \alpha^6 \\ &\quad +8\alpha^4k_5^2 + 16\alpha^2k_5^4)^{\frac{1}{2}}))^{\frac{1}{2}}, \\ h_1 &= h_1, \quad k_1 = \pm Ik_3, \quad k_2 = \pm Ik_4. \end{aligned} \tag{3.12}$$

Thanks to the Eqs.(3.11),(3.12) and by using the (3.2),(3.6), we reach the required M-Shaped with I-kink solution of Eq.(3.1)

$$W_2(x,t) = \frac{2\begin{pmatrix} 2I(Ik_3\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right) + Ik_4)k_3 + 2(k_3\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right) + k_4)k_3 \\ +h_1k_5\exp(k_5\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right) + k_6) \end{pmatrix}}{\begin{pmatrix} \left(Ik_3\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right) + Ik_4\right)^2 + \left(k_3\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right) + k_4\right)^2 \\ +h_1\exp(k_5\left(\frac{(x^\gamma - at^\gamma)}{\gamma}\right) + k_6) \end{pmatrix}} \times e^{i\alpha\frac{(x^\gamma + \eta t^\gamma)}{\gamma}}. \tag{3.13}$$

3.3. **Rational M-shaped soliton:**
For this approach, κ is considered as follows

$$\kappa = (k_1\xi + k_2)^2 + (k_3\xi + k_4)^2 + k_5. \tag{3.14}$$

Here, k_i is any parameter. Substituting Eq.(3.14) into Eq.(3.7) and by solving the all coefficients of different power of ξ to zero, we have

$$\begin{aligned} a &= -8\alpha, \quad s = \frac{\sqrt{-169 + 17\sqrt{113}}\alpha}{2}, \quad k_1 = k_1, \quad k_2 = k_2, \\ k_3 &= \frac{(k_2k_4 + \sqrt{-k_2^2k_5 - k_4^2k_5 - k_5^2})k_1}{k_2^2 + k_5}, \quad k_4 = k_4, \quad k_5 = k_5. \end{aligned} \tag{3.15}$$

Thanks to the Eqs.(3.14),(3.15) and by using the (3.2),(3.6), we discover the desired rational M-shaped soliton solution of Eq.(3.1)

$$W_3(x,t)=\frac{2\left(\begin{array}{c}2\left(k_1\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_2\right)k_1\\ +\frac{2\left(\frac{\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)(k_2k_4+\sqrt{-k_2^2k_5-k_4^2k_5-k_5^2})k_1}{k_2^2+k_5}+k_4\right)(k_2k_4+\sqrt{-k_2^2k_5-k_4^2k_5-k_5^2})k_1}{k_2^2+k_5}\end{array}\right)}{\left(\begin{array}{c}\left(k_1\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_2\right)^2\\ +\left(\frac{\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)\left(k_2k_4+\sqrt{-k_2^2k_5-k_4^2k_5-k_5^2}\right)k_1}{k_2^2+k_5}+k_4\right)^2+k_5\end{array}\right)}\times e^{i\alpha\frac{(x^\gamma+\eta t^\gamma)}{\gamma}}. \tag{3.16}$$

Also, if we choose $\pm\frac{I}{2}\sqrt{17\sqrt{113}+169}\alpha$ for s, Eq.(3.16) holds.

3.4. **Interaction with M-shaped, I-kink, and rogue wave solution:**

To uncover interaction with M-shaped, I-kink, and rogue wave solution of Eq.(3.1), we employ the next transformation

$$\kappa=(k_1\xi+k_2)^2+(k_3\xi+k_4)^2+\exp(k_5\xi+k_6)+r_0\cosh(k_7\xi+k_8)+k_9. \tag{3.17}$$

However, k_i, and r_0 are some constants. Putting Eq.(3.17) into Eq.(3.7) and equating all the coefficients of $\exp(\xi k_5+k_6)$, $\exp(2\xi k_5+2k_6)$, $\exp(3\xi k_5+3k_6)$, $\sinh(\xi k_7+k_8)$, $\cosh(\xi k_7+k_8)$ to zero, we attain,

$$\begin{aligned} a &= -2\alpha,\quad s=\pm\frac{\sqrt{-19+5\sqrt{17}}\alpha}{2},\quad k_1=\pm Ik_3,\quad k_2=k_2,\quad r_0=r_0\\ k_3 &= k_3,\quad k_4=k_4,\quad k_5=0,\quad k_6=k_6,\quad k_7=0,\quad k_8=k_8,\quad k_9=k_9. \end{aligned} \tag{3.18}$$

Thanks to the Eqs.(3.17),(3.18) and by using the (3.2),(3.6), we find the required interaction with M-shaped, I-kink, and rogue wave solution of Eq.(3.1)

$$W_4(x,t)=\frac{2\left(2I\left(Ik_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_2\right)k_3+2\left(k_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_4\right)k_3\right)}{\left(\begin{array}{c}\left(Ik_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_2\right)^2\\ +\left(k_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_4\right)^2+\exp(k_6)+r_0\cosh(k_8)+k_9\end{array}\right)}\times e^{i\alpha\frac{(x^\gamma+\eta t^\gamma)}{\gamma}}. \tag{3.19}$$

Also, if we choose $\pm\frac{I}{2}\sqrt{5\sqrt{17}+19}\alpha$ for s, Eq.(3.19) holds.

3.5. **Interaction between periodic and M-shaped:**

We regard the subsequent transformation to obtain interaction between periodic and M-shaped of Eq.(3.1)

$$\kappa=(k_1\xi+k_2)^2+(k_3\xi+k_4)^2+r_0\cos(k_5\xi+k_6)+k_7. \tag{3.20}$$

However, k_i and r_0 are any parameters. Putting Eq.(3.20) into Eq.(3.7) and by equating all the coefficients of $\sin(\xi k_5+k_6)$, $\cos(\xi k_5+k_6)$ to zero, we get

$$\begin{aligned} a &= -2\alpha,\quad s=\pm\frac{\sqrt{-19+5\sqrt{17}}\alpha}{2},\quad k_1=\pm Ik_3,\quad k_2=k_2,\\ k_3 &= k_3,\quad k_4=k_4,\quad k_5=0,\quad k_6=k_6,\quad k_7=k_7,\quad r_0=r_0. \end{aligned} \tag{3.21}$$

Thanks to the Eqs.(3.20),(3.21) and by using the (3.2),(3.6), we uncover the desired interaction between periodic and M-Shaped function solutions of Eq.(3.1)

$$W_5(x,t)=\frac{2\left(2I(Ik_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_2\right)k_3+2\left(k_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_4\right)k_3)}{\left(\begin{array}{c}\left(Ik_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_2\right)^2\\+\left(k_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_4\right)^2+k_7+r_0\cos(k_6)\end{array}\right)}\times e^{i\alpha\frac{(x^\gamma+\eta t^\gamma)}{\gamma}}. \tag{3.22}$$

Also, if we choose $\pm\frac{I}{2}\sqrt{5\sqrt{17}+19}\alpha$ for s, then Eq.(3.22) holds.

3.6. **M-shaped interaction with II-kink:**

To calculate the M-Shaped interaction with the II-kinks solution of Eq.(3.1), we utilize the subsequent modification

$$\kappa=(k_1\xi+k_2)^2+(k_3\xi+k_4)^2+h_1\exp(-k_5\xi-k_6)+h_2\exp(k_7\xi+k_8), \tag{3.23}$$

in which k_i,and h_i and are any parameters. By putting Eq.(3.23) in Eq.(3.7) and solving all the coefficients of $\exp(-k_5\xi-k_6), \exp(k_7\xi+k_8)$, $\exp(-k_5\xi+k_7\xi-k_6+k_8)$, $\exp(-3k_5\xi-3k_6)$, $\exp(2k_7\xi+2k_8)$, $\exp(3k_7\xi+3k_8)$, $\exp(-2k_5\xi-2k_6)$, $\exp(-k_5\xi+2k_7\xi-k_6+2k_8)$, $\exp(-2k_5\xi+k_7\xi-2k_6+k_8)$ to zero, we get

$$a=\frac{\alpha}{2},\ k_1=k_1,\ k_2=k_2,\ k_3=k_3,\ s=-\frac{\alpha}{2}. \tag{3.24}$$

Thanks to the Eqs.(3.23),(3.24) and by using the (3.2), (3.6), we explore the desired M-Shaped interaction with the II-kinks solution of Eq.(3.1)

$$W_6(x,t)=\frac{2\left(\begin{array}{c}2(k_1\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_2)k_1+2(k_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_4)k_3\\-h_1k_5\exp(-k_5\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)-k_6)+h_2k_7\exp(k_7\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_8)\end{array}\right)}{\left(\begin{array}{c}(k_1\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_2)^2+(k_3\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_4)^2\\+h_1\exp(-k_5\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)-k_6)+h_2\exp(k_7\left(\frac{(x^\gamma-at^\gamma)}{\gamma}\right)+k_8)\end{array}\right)}\times e^{i\alpha\frac{(x^\gamma+\eta t^\gamma)}{\gamma}}. \tag{3.25}$$

4. Conclusions

This research deals with the FDMNLSE with cubic law nonlinearity to analyze the propagation of two waves moving simultaneously with the relations of embedded phase speed. The fractional convert with the CFD was applied to the prototype for altering it into an NODE, particularly the periodic cross-kink wave, M-shaped rational solitons, M-shaped with one and two kinks, the interaction between periodic and M-shaped, interaction with M-shaped, I-kink, and rogue waves were assembled with their 3d, contour, and 2d geometric miniatures. According to the outcomes, the free parameters controlled the wave propagation of solitons. The noted solitons may also describe wave propagation demeanors in physical systems with nonlinear transmission lines, memory effect, and transient effects in telecommunication and electrical systems. We saw that the offered techniques are

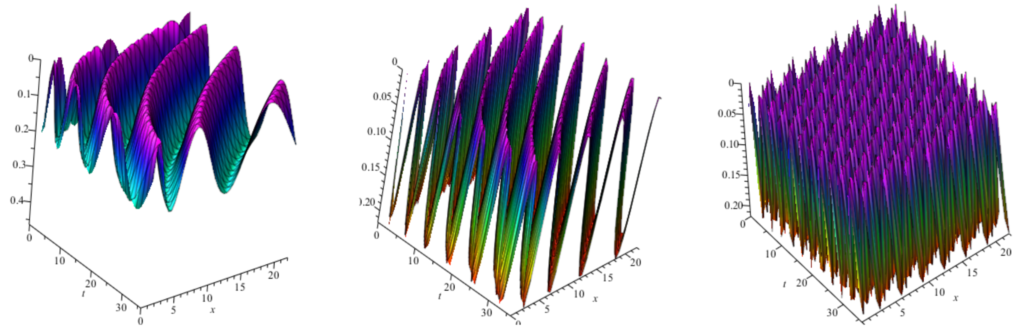

FIGURE 1. The 3d plots for the real part of the $W_1(x,t)$ in Eq.(3.10)by selecting the values $\gamma = 0.5,\ \gamma = 0.75,\ \gamma = 0.99$, respectively, $s = 1,\ a = -1,\ \alpha = -1,\ \eta = 1,\ r_0 = 8,\ r_2 = 10,\ k_2 = 6,\ k_6 = 12$.

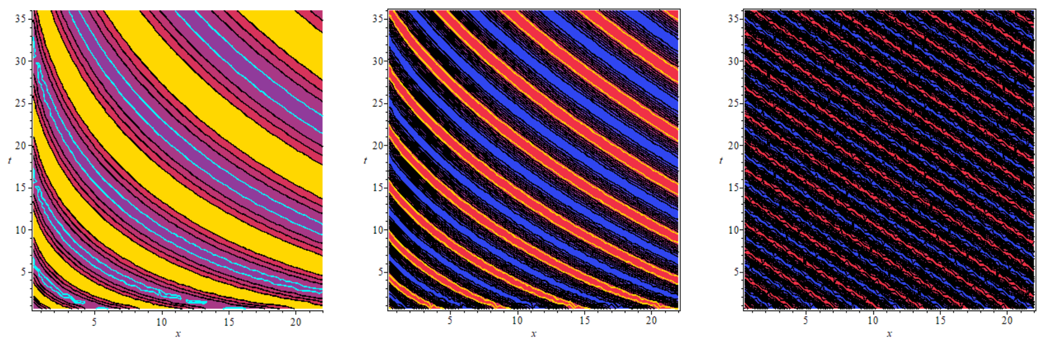

FIGURE 2. The contour plots for the real part of the $W_1(x,t)$ in Eq.(3.10) by selecting the values $\gamma = 0.5,\ \gamma = 0.75,\ \gamma = 0.99$, respectively, $s = 1,\ a = -1,\ \alpha = -1,\ \eta = 1,\ r_0 = 8,\ r_2 = 10,\ k_2 = 6,\ k_6 = 12$.

straightforward and assertive, so the agreed form can be expanded for more nonlinear prototypes. The results are advantageous in manipulating the FDMNLSE in the nonlinear discipline. The got solutions are new for the FDMNLSE that are not declared by the other investigations. All solutions have been verified to be correct with the help of Maple software. The most charming section of the notified solutions is their usage in engineering and science. The new aspect of this examination is the solutions that were not reached early and demonstrated an excellent balance between the nonlinear physical elements. Moreover, the solutions can enable assessing the implementation of wave patterns in fibers when required in a configuration.

DECLARATIONS

Ethical Approval

Not applicable.

Competing interests

The authors declare no conflict of interest.

Authors' contributions

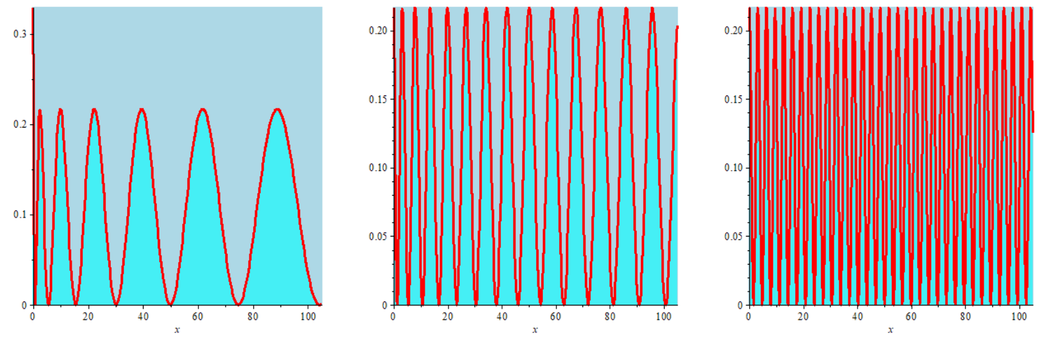

FIGURE 3. The 2d plots for the real part of the $W_1(x,t)$ in Eq.(3.10) by selecting the values $\gamma = 0.5,\ \gamma = 0.75,\ \gamma = 0.99$, respectively, $s = 1,\ a = -1,\ \alpha = -1,\ \eta = 1,\ r_0 = 8,\ r_2 = 10,\ k_2 = 6,\ k_6 = 12$.

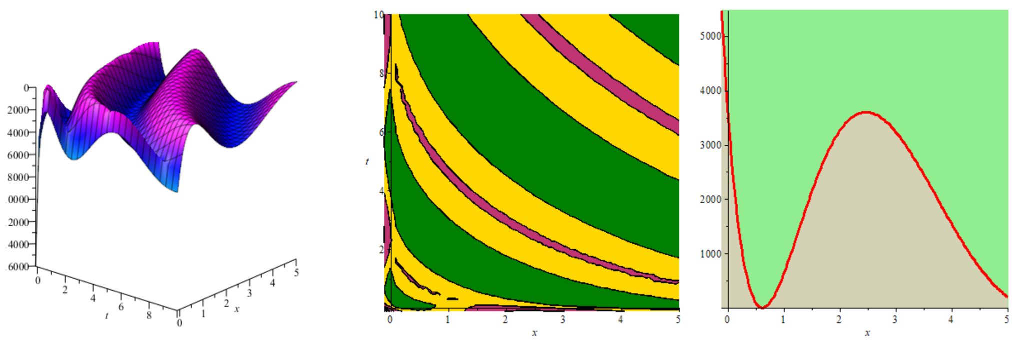

FIGURE 4. The 3d, contour, and 2d plots for the real part of the $W_2(x,t)$ in Eq.(3.13) by selecting the values $\gamma = 0.5,\ a = -1,\ \alpha = -1,\ \eta = 1,\ k_3 = 24,\ h_1 = 2.4,\ k_4 = 3.2,\ k_5 = 30,\ k_6 = 14$.

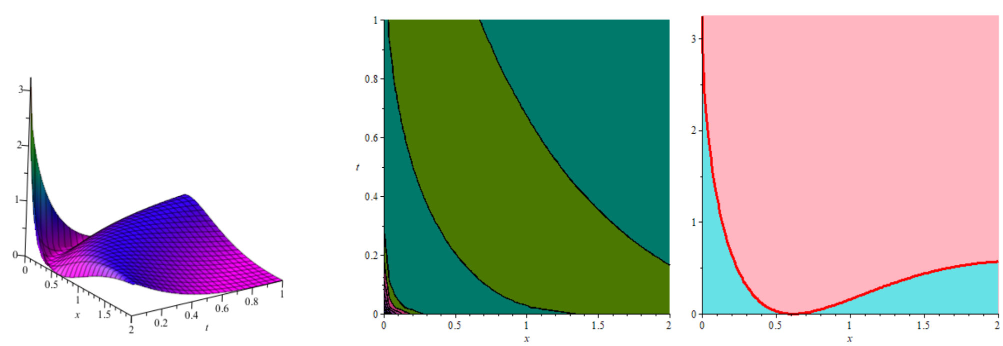

FIGURE 5. The 3d, contour, and 2d plots for the real part of the $W_3(x,t)$ in Eq.(3.16) by selecting the values $\gamma = 0.5,\ a = -1,\ \alpha = -1,\ \eta = 1,\ k_1 = 12,\ k_2 = 26,\ k_3 = 28,\ k_4 = 20,\ k_5 = 21$.

EY: Supervision, methodology, writing, review, and editing. BK: writing, review and editing.

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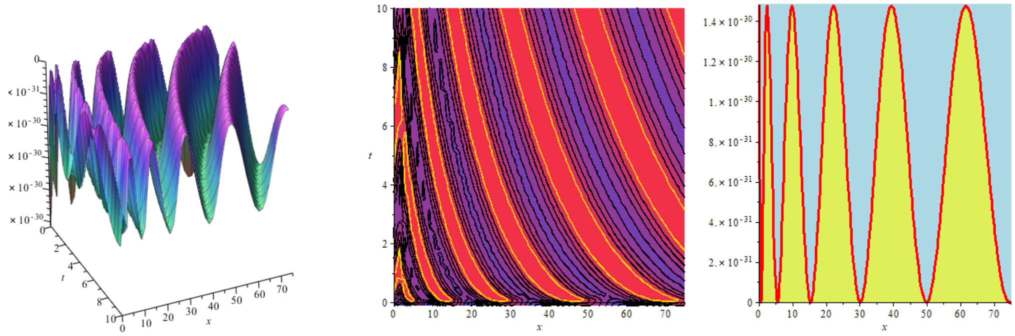

FIGURE 6. The 3d, contour, and 2d plots for the real part of the $W_4(x,t)$ in Eq.(3.19) by selecting the values $\gamma = 0.5,\ a = -1,\ \alpha = -1,\ \eta = 1,\ r_0 = 28,\ k_2 = 24,\ k_3 = 36,\ k_4 = 14,\ k_6 = 24,\ k_8 = 40,\ k_9 = 18$.

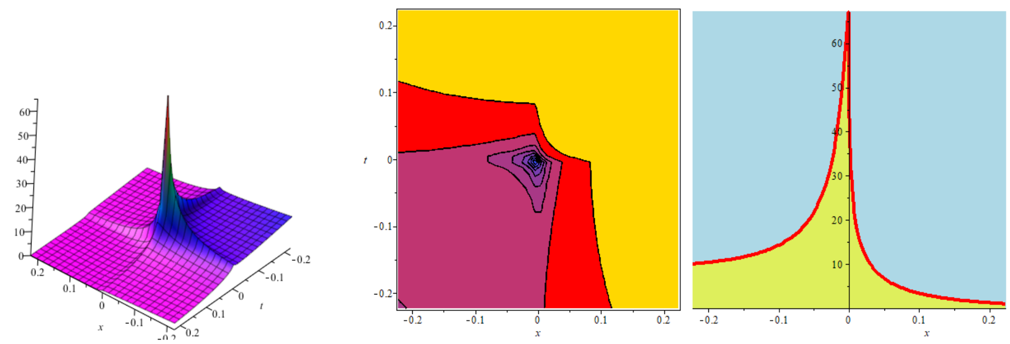

FIGURE 7. The 3d, contour, and 2d plots for the real part of the $W_5(x,t)$ in Eq.(3.22) by selecting the values $\gamma = 0.5,\ a = -1,\ \alpha = -1,\ \eta = 1,\ r_0 = 28,\ k_2 = 2.4,\ k_3 = 36,\ k_4 = 14,\ k_6 = 24,\ k_7 = 40$.

Availability of data and materials

Not applicable.

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