Annals of Mathematics and Computer Science
ISSN: 2789-7206
Vol 16 (2023) 112-122

OPTIMAL SOLUTION TO THE ASSIGNMENT PROBLEM

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ABSTRACT. An assignment mix is a specific form of the transportation mix that has significant applications in the real world. The assignment mix arises from various situations for which we ought to decide the best route to dispense 'n' tasks to 'n' persons in a piece wise linear function. In the present paper, we lay out three different approaches for cracking the assignment mix perfectly. These approaches rely on an iterative cycle of pair-wise diagonal interchanges between the assigned and unassigned cells. This paper describes the application of an iterative cycle of pair-wise diagonal interchanges to elicit a feasible solution in which the best solution is guaranteed.

1. Introduction and Preliminaries

One of the significant problems in mathematical optimization (or operational research) remains the assignment mix solution. It is a sort of transportation mix in which the resources are persons and the purposes are jobs. Additionally, one of the most basic applications of linear programming is the assignment mix, in which our target remains to distribute n jobs among n persons while keeping costs to a minimum and profits to a maximum level. An assignment may be persons to jobs like salesmen to ships, blocks to rooms, operators to machines, drivers to trucks, problems to research teams, and so on.

In this problem, every source has a supply (because each assignee must be given exactly one task) and every destination has a demand (because each task must be given to exactly one assignee). Effectiveness is required in the assignments to ensure the best possible solution. The problem of assignment mix solutions arises since the availability of resources, like the working hours of persons, duration of operating machines, etc., varies.

Koing established and presented a procedure for resolving the assignment pattern and titled his technique Hungarian in 1931. James Munkers, in 1957 [6], developed the algorithm known as the Kuhn-Munkers algorithm or the Munkers assignment algorithm, which has been generalized for the transportation problem.

Date: Received: May 26, 2023; Accepted: Jun 30, 2023.

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2010 *Mathematics Subject Classification.* Primary 90B80.

Key words and phrases. Assignment problem, Cost Minimization (or Maximization), Median approach method (MAM), Variable approach method (VAM), Weight approach method (WAM), Optimality, The net difference in cost (${d_{ij}}^p$), The net difference in the weight (${d_{w_{ij}}}^p$).

H.W. Kuhn [1] produced the first specialized approach for the solution of the assignment problem in 1955, which was later extended to the most common network flow problems. D.P. Bertsekas [2] examined the problem in 1981 and devised a new algorithm to solve it. Thompson (1981) proposed a repetitive method for obtaining the solution of the assignment problems. In 2007, S. Anshuman and T. Rudrajit [4] used the genetic algorithm and simulated annealing to solve the assignment problem. A much simpler heuristic approach is proposed here to find an optimal solution with fewer iterations and very simple computations.

There are four techniques for illuminating assignment problems in the literature so far: (1) Enumeration Method, (2) Simplex Method, (3) Transportation Method, and (4) Hungarian Technique.

The present paper develops three enumeration methods to assign cost and, afterward, to check if the solution is optimal by the way of optimality, which is introduced subsequently.

Notation: ${d_{ij}}^p$= net difference in cost for ith row and jth column cell, for each i = 1,2. . . n -1, p = 1,2. . . n − i , for each i.

${d_{w_{ij}}}^p$ = The net difference in the weight for ith row and jth column cell where, i =1,2. . . n-1, p = 1,2. . . n − i, for each i.

1.1. **Mathematical Preliminaries.** Given n persons (rows) and n jobs (columns), and each person's effectiveness (in terms of costs, profits, time, etc.) for each job, the concern is to assign each person to just one task in order to optimize the given measure of effectiveness.

Resources	**Activities (jobs)**				**Supply**
	J_1	J_2	....	J_n	
W_1	C_{11}	C_{12}	....	C_{1n}	1
W_2	C_{21}	C_{22}	....	C_{2n}	1
....	....	....	....	....	....
W_n	C_{n1}	C_{n2}	....	C_{nn}	1
Demand	1	1	....	1	N

Let x_{ij} denoted the assignment of the facility (Resources) i to job (Activities) j such that

$$x_{ij}=\begin{cases} 1 & facility\ i\ assigned\ to\ activity\ j \\ 0 & otherwise \end{cases}$$

Then, the mathematical model of the assignment problem can be stated as follows:

The objective function is to Minimize $\mathbb{Z}=\sum_{i=1}^{n}\sum_{j=1}^{n} C_{\mathrm{ij}}x_{\mathrm{ij}}$

Subject to constraints: $\sum_{j=1}^{n} x_{\mathrm{ij}} =\ 1$ for all i(resource availability)

$\sum_{i=1}^{n} x_{\mathrm{ij}} =\ 1$ for all j (activity requirement)

And here, $x_{\mathrm{ij}} = 0\ \forall i\ \&\ j$, where C_{ij} represents the cost of assignment of resource i to activity j.

2. Approaches

2.1. Median Approach Method (MAM).

Step 1: Construct the assignment table for the given problem. If the problem has not equal rows and columns then add the necessary number of dummy rows or columns and also the cost must be zero for those dummy rows or columns.

Step 2: Find the median of each row and each column and write them against the corresponding row and corresponding column.

Step 3: Assign the priorities to the largest or smallest median of the rows or columns subsequently. Choose the smallest or largest cost element and make an assignment by making a square ($\square$) around it for the minimization or maximization problem, and then cross ($\times$) the row and column against the assignment cell.

Step 4: Repeat step 3 until each row and column contains exactly one assignment.

Step 5: Apply the optimality test.

2.2. Variable Approach Method (VAM).

Step 1: Construct the assignment table for the given problem. If the problem has not equal rows and columns then add the necessary number of dummy rows or columns and also the cost must be zero for those dummy rows or columns.

Step 2: Choose the minimum or maximum cost cell of each row and give the assignment of the cost cell with the square ($\square$) and cross ($\times$) the row and column against the assignment cell.

Step 3: Repeat step-2 until each row and column contains exactly one assignment.

Step 4: Apply the optimality test.

Test of Optimality for MAM & VAM:

(1) Beginning with the assigned cost value of the row, construct a cycle having two corners are allocated assigned cost values in the corresponding row and column, and the remaining two corners containing the cost value of the matrix.

(2) Allot plus (+) and minus (-) sign alternatively of each corner cell of the cycle, starting with plus (+) sign for the assigned cost value.

(3) Calculate the net difference in cost ($d_{ij}{}^{p}$) for all assigned cells. Locate $d_{ij}{}^{p} \leqslant 0$/ $d_{ij}{}^{p} \geqslant 0$ for the minimization or maximization problem. Continue this process until all assigned cells are resolved.

(4) If the condition is not satisfied (${d_{ij}}^p \leqslant 0/\ {d_{ij}}^p \geqslant 0$) i.e., any assigned cells (${d_{ij}}^p$) value is positive or negative of minimization or maximization problem, then select the highest positive or lowest negative value of (${d_{ij}}^p$) for minimization or maximization problem and pairwise diagonally interchange the cells of the ${d_{ij}}^p$. I.e., interchange the allocated cells of the i$^{\text{th}}$ and (i+p)$^{\text{th}}$ row. This is called the revised cost matrix.

2.3. **Weight Approach Method (WAM).**

2.3.1. *Weight in the Assignment problem.* Now suppose w_{ij} be the assigned weights are the costs C_{ij} for each i= 1, 2, 3,. . . ,n and j= 1,2,3,. . . ,n. We assigned the weights to the costs C_{ij} from the highest to lowest assignment cost for each row and each column. That is, we assigned weight 1 to the highest (maximum) cost, subsequent weights to the subsequent costs, and n to the lowest cost for each n row. Similarly, we assigned weight 1 to the highest (maximum) cost, subsequent weights to the subsequent costs, and n to the lowest cost for each n column.

We note that weights can possibly be assigned chronologically or randomly in case of tied costs. We denote the weights assigned to the cost C_{ij} by w^r_{ij} considering the cell in a row and w^c_{ij} considering it in a column. We consider the sum of the weights for each cell in the matrix.

We define the sum of weights S_{ij}=w^r_{ij}+w^c_{ij} for each cell in the assignment cost matrix. We define the net difference in the weights $d_{w^p_{ij}}$ for each occupied cell using the sum of weights S_{ij}.

2.3.2. *Algorithm.*

Step 1: Construct the assignment table for the given problem. If the problem has not equal rows and columns then add the necessary number of dummy rows or columns and also the cost must be zero for those dummy rows/ columns.

Step 2: Assign weights of the given cost matrix and then obtain the sum of weight S_{ij} for each cell.

Step 3: Identify the maximum/ minimum S_{ij} of each row and assign the cell by circling ($\bigcirc$) and cross off ($\times$) that row and column against the assignment cell of the minimization/ maximization problem.

Step 4: Repeat step-3 until each row and column contains exactly one assignment.

Step 5: Apply the optimality test.

2.3.3. *Test of Optimality.*

(1) Beginning with the assigned cell of the row, construct a cycle with two corners that are allocated cells with a sum of weights S_{ij} in the corresponding row and column, and the remaining two corners having the sum of weights S_{ij}.

(2) Allot plus (+) and minus (-) sign alternatively of each corner of S_{ij} of the cycle, starting with plus (+) sign for the assigned cell of S_{ij}.

(3) Calculate the net difference in the weight (dwijp) for all assigned cells. Locate ${d_{w_{ij}}}^p \geq 0/\ {d_{w_{ij}}}^p \leq 0$ for the minimization or maximization problem. Continue this process until all assigned cells are resolved.

(4) If the condition is not satisfactory (${d_{w_{ij}}}^p \geq 0/\ {d_{w_{ij}}}^p \leq 0$) i.e., any assigned cell (${d_{w_{ij}}}^p$) value is negative or positive for the minimization or maximization problem, then select the lowest negative or highest positive value of ${d_{w_{ij}}}^p$ for the minimization or maximization problem and pairwise diagonally interchange the cells, i.e., interchange the assigned cells of i^{th} row and $(\mathrm{i+p})^{\text{th}}$ row. This is called the revised cost matrix.

Formula: The number of cycles for the assigned cell $= \sum_{i=1}^{n-1} i$, where n is the number of rows or columns in a given square matrix.

3. NUMERICAL EXAMPLES

Example 1 An advocate's company hires typist in terms of hourly remuneration for their regular typing jobs. There are five typists for their jobs, and their remuneration and typing speeds are different. As per the company's policy, only one job is to be given to one typist. Determine the minimum job allocation pattern for the provided statistics:

	P	Q	R	S	T
A	85	75	65	125	75
B	90	78	66	132	78
C	75	66	57	114	69
D	80	72	60	120	72
E	76	64	56	112	68

- <u>MAM Method</u>

Using the median approach method, specify the median against the corresponding row and column. Identify the largest median among them and assign the cell with the lowest cost. We got assignments for the cost matrix as follows:

	P	Q	R	S	T	Row Median	priorities
A	85	75	65	125	$\boxed{75}$	75	4
B	90	78	$\boxed{66}$	132	78	78	3
C	$\boxed{75}$	66	57	114	69	69	×
D	80	$\boxed{72}$	60	120	72	72	5
E	76	64	56	$\boxed{112}$	68	68	×
Column Median	80	72	60	120	72		
priorities	2	×	×	1	×		

Total Number of cycles for the assigned cell $= \sum_{i=1}^{5-1} i$ =10

We check the optimality,

$d_{15}{}^1$=75 − 65 + 66 − 78 =(-2), $d_{15}{}^2$=(-4), $d_{15}{}^3$=0, $d_{15}{}^4$=(-6), $d_{23}{}^1$= (-6), $d_{23}{}^2$=0, $d_{23}{}^3$=(-10), $d_{31}{}^1$=1, $d_{31}{}^2$=(-3), $d_{42}{}^1$=0

Here, $d_{31}{}^1$=1> 0, so we interchange assigned cells of the 3rd and 4th Row

	P	Q	R	S	T
A	85	75	65	125	$\boxed{75}$
B	90	78	$\boxed{66}$	132	78
C	75	$\boxed{66}$	57	114	69
D	$\boxed{80}$	72	60	120	72
E	76	64	56	$\boxed{112}$	68

$d_{15}{}^1$=(-2), $d_{15}{}^2$=(-3), $d_{15}{}^3$=(-2), $d_{15}{}^4$=(-6), $d_{23}{}^1$=(-3), $d_{23}{}^2$=(-4), $d_{23}{}^3$=(-10), $d_{32}{}^1$=(-1), $d_{32}{}^2$=0, $d_{41}{}^1$=(-4)

Since, $\forall d_{ij}{}^p \leqslant 0$ so, the optimality test is satisfied and the optimum assignment has been attained and is given as:

$$A\to T,\ B\to R,\ C\to Q,\ D\to P,\ E\to S$$

Thus, the least cost =75 + 66 + 66 + 80 + 112 = 399

- VAM Method

Using variable approach method, choose the minimum cost cell for each row and get the assigned cost matrix as given below,

	P	Q	R	S	T
A	85	75	$\boxed{65}$	125	75
B	90	78	66	132	$\boxed{78}$
C	75	$\boxed{66}$	57	114	69
D	$\boxed{80}$	72	60	120	72
E	76	64	56	$\boxed{112}$	68

Check the optimality test,
$d_{13}{}^1$=2, $d_{13}{}^2$=(-1), $d_{13}{}^3$=0, $d_{13}{}^4$=(-4), $d_{25}{}^1$=(-3), $d_{25}{}^2$=(-4), $d_{25}{}^3$=(-10), $d_{32}{}^1$=(-1), $d_{32}{}^2$=0, $d_{41}{}^1$=(-4)

Here, $d_{13}{}^1$=2> 0, so we interchange assigned cells of the 1st and 2nd Row

	P	Q	R	S	T
A	85	75	65	125	$\boxed{75}$
B	90	78	$\boxed{66}$	132	78
C	75	$\boxed{66}$	57	114	69
D	$\boxed{80}$	72	60	120	72
E	76	64	56	$\boxed{112}$	68

$d_{15}{}^1$=(-2), $d_{15}{}^2$=(-3), $d_{15}{}^3$=(-2), $d_{15}{}^4$=(-6), $d_{23}{}^1$=(-3), $d_{23}{}^2$=(-4), $d_{23}{}^3$=(-10), $d_{32}{}^1$=(-1), $d_{32}{}^2$=0, $d_{41}{}^1$=(-4)

Since, $\forall d_{ij}{}^{p} \leqslant 0$ so, the optimality test is satisfied and the optimum assignment has been attained and is given as:

$$A \to T,\ B \to R,\ C \to Q,\ D \to P,\ E \to S$$

Thus, the least cost =75 + 66 + 66 + 80 + 112 = 399

• <u>WAM Method</u>

Using step-2 assign weights to the cost matrix and obtain the sum of weights S_{ij} given below

	P	Q	R	S	T
A	85_2^2	75_2^3	65_2^5	125_2^1	75_2^4
B	90_1^2	78_1^3	66_1^5	132_1^1	78_1^4
C	75_5^2	66_4^4	57_4^5	114_4^1	69_4^3
D	80_3^2	72_3^3	60_5^3	120_3^1	72_3^4
E	76_2^4	64_5^4	56_5^5	112_5^1	68_5^3

	P	Q	R	S	T
A	85^4	75^5	65^7	125^3	75^6
B	90^3	78^4	66^6	132^2	78^5
C	75^7	66^8	57^9	114^5	69^7
D	80^5	72^6	60^8	120^4	72^7
E	76^6	64^9	56^10	112^6	68^8

By step-3 identify the maximum S_{ij} of each row and assign cell, and get assigned cost matrix given below,

	P	Q	R	S	T
A	85^4	75^5	$\boxed{65^7}$	125^3	75^6
B	90^3	78^4	66^6	132^2	$\boxed{78^5}$
C	75^7	$\boxed{66^8}$	57^9	114^5	69^7
D	$\boxed{80^5}$	72^6	60^8	120^4	72^7
E	76^6	64^9	56^10	$\boxed{112^6}$	68^8

Total No. of cycles for assigned cell = $\sum_{i=1}^{5-1} i$ =10, we check the optimality,
$d_{w_{13}}{}^{1}$=0, $d_{w_{13}}{}^{2}$=1, $d_{w_{13}}{}^{3}$=0, $d_{w_{13}}{}^{4}$=0, $d_{w_{25}}{}^{1}$=2, $d_{w_{25}}{}^{2}$=0, $d_{w_{25}}{}^{3}$=1,$d_{w_{33}}{}^{1}$=0, $d_{w_{33}}{}^{2}$=0, $d_{w_{41}}{}^{1}$=1

Since, $\forall\ d_{w_{ij}}{}^{p} \geq 0$ so, the solution is optimal and the optimum assignment has been attained and is given as:

$$A \to R,\ B \to T,\ C \to Q,\ D \to P,\ E \to S$$

Thus, the least cost =65 + 78 + 66 + 80 + 112 = 401

Example 2 [Maximization problem] Five salespersons are required to be allotted to five particular regions. Forecast of sales revenue (in thousands) for each individual salesperson is provided in the following matrix. Find the assignment pattern that maximizes the sales revenue.

	P	Q	R	S	T
A	32	38	40	28	40
B	40	24	28	21	36
C	41	27	33	30	37
D	22	38	41	36	36
E	29	33	40	35	39

- <u>MAM Method</u>

With the help of MAM, we obtain the assigned cost matrix for maximization.

	P	Q	R	S	T	Row Median	priorities
A	32	$\boxed{38}$	40	28	40	38	×
B	$\boxed{40}$	24	28	21	36	28	1
C	41	27	33	30	37	33	4
D	22	38	41	36	36	36	×
E	29	33	40	35	39	35	5
Column Median	32	33	40	30	37		
priorities	×	3	×	2	×		

Way of optimality test ,
$d_{12}{}^1$=22, $d_{12}{}^2$=8, $d_{12}{}^3$=8, $d_{12}{}^4$=5, $d_{21}{}^1$=0, $d_{21}{}^2$=33, $d_{21}{}^3$=23, $d_{35}{}^1$=7, $d_{35}{}^2$=5, $d_{44}{}^1$=0

By, optimality test, since, $\forall\ d_{ij}{}^p \geq 0$, so, the solution is optimal and optimum assignment has been attained and is given as:

$$1\rightarrow \text{B},\ 2\rightarrow \text{A},\ 3\rightarrow \text{E},\ 4\rightarrow \text{C},\ 5\rightarrow \text{D}$$

Thus, the maximum sales revenue = 38 + 40 + 37 + 36 + 40 = 191 thousand rupees.

- <u>VAM Method</u>

With the help of VAM, we obtain the assigned cost matrix for maximization.

	P	Q	R	S	T
A	32	38	$\boxed{40}$	28	40
B	$\boxed{40}$	24	28	21	36
C	41	27	33	30	$\boxed{37}$
D	22	$\boxed{38}$	41	36	36
E	29	33	40	$\boxed{35}$	39

Using the optimality test,
$d_{13}{}^1$=20, $d_{13}{}^2$=4, $d_{13}{}^3$=(-1), $d_{13}{}^4$=7, $d_{21}{}^1$=0, $d_{21}{}^2$=32, $d_{21}{}^3$=25, $d_{35}{}^1$=12, $d_{35}{}^2$=3, $d_{42}{}^1$=4
Here, $d_{13}{}^3$=(-1)$<$ 0 so we interchange 1st and 4th row

	P	Q	R	S	T
A	32	$\boxed{38}$	40	28	40
B	$\boxed{40}$	24	28	21	36
C	41	27	33	30	$\boxed{37}$
D	22	38	$\boxed{41}$	36	36
E	29	33	40	$\boxed{35}$	39

Way of optimality test,
$d_{12}{}^1$=22, $d_{12}{}^2$=8, $d_{12}{}^3$=1, $d_{12}{}^4$=12, $d_{21}{}^1$=0, $d_{21}{}^2$=31, $d_{21}{}^3$=25, $d_{35}{}^1$=9, $d_{35}{}^2$=3, $d_{43}{}^1$=0
By, optimality test, since,$\forall$ $d_{ij}{}^p \geq 0$, so, the solution is optimal and optimum assignment has been attained and is given as:

$$1\rightarrow \text{B},\ 2\rightarrow \text{A},\ 3\rightarrow \text{E},\ 4\rightarrow \text{C},\ 5\rightarrow \text{D}$$

Thus, the maximum sales revenue = 38 + 40 + 37 + 36 + 40 = 191 thousand rupees.

- <u>WAM Method</u>

By, step-2 assign weights to the cost matrix and obtain the sum of weights S_{ij}.

	P	Q	R	S	T
A	32^4_3	38^3_1	40^1_2	28^5_4	40^2_1
B	40^1_2	24^4_5	28^3_5	21^5_5	36^2_4
C	41^1_1	27^5_4	33^3_4	30^4_3	37^2_3
D	22^5_5	38^2_2	41^1_1	36^3_1	36^4_5
E	29^5_4	33^4_5	40^1_3	35^3_2	39^2_2

	P	Q	R	S	T
A	32^7	38^4	40^3	28^9	40^3
B	40^3	24^9	28^8	$21^1$0	36^6
C	41^2	27^9	33^7	30^7	37^5
D	$22^1$0	38^4	41^2	36^4	36^9
E	29^9	33^9	40^4	35^5	39^4

We obtained the assigned cost matrix by WAM for maximization.
By, optimality test,
$d_{w_{13}}{}^1$=(-9), $d_{w_{13}}{}^2$=(-2), $d_{w_{13}}{}^3$=1, $d_{w_{13}}{}^4$=(-5), $d_{w_{21}}{}^1$=0, $d_{w_{21}}{}^2$=(-12), $d_{w_{21}}{}^3$=(-11), $d_{w_{35}}{}^1$=(-9), $d_{w_{35}}{}^2$=(-1), $d_{w_{42}}{}^1$=(-2)
Here, $d_{w_{13}}{}^3$=1 $\geq$ 0, so we interchange assigned cells of the 1st and 4th Row

	P	Q	R	S	T
A	32^7	38^4	$\boxed{40^3}$	28^9	40^3
B	$\boxed{40^3}$	24^9	28^8	21^{10}	36^6
C	41^2	27^9	33^7	30^7	$\boxed{37^5}$
D	22^{10}	$\boxed{38^4}$	41^2	36^4	36^9
E	29^9	33^9	40^4	$\boxed{35^5}$	39^4

	P	Q	R	S	T
A	32^7	$\boxed{38^4}$	40^3	28^9	40^3
B	$\boxed{40^3}$	24^9	28^8	21^{10}	36^6
C	41^2	27^9	33^7	30^7	$\boxed{37^5}$
D	22^{10}	38^4	$\boxed{41^2}$	36^4	36^9
E	29^9	33^9	40^4	$\boxed{35^5}$	39^4

Methods of optimality test,
${d_{w_{12}}}^1$=(-9), ${d_{w_{12}}}^2$=(-3), ${d_{w_{12}}}^3$=(-1), ${d_{w_{12}}}^4$=(-7), ${d_{w_{21}}}^1$=0, ${d_{w_{21}}}^2$=(-13), ${d_{w_{21}}}^3$=(-11), ${d_{w_{35}}}^1$=(-9), ${d_{w_{35}}}^2$=(-1), ${d_{w_{43}}}^1$=(-1)

By, Optimality test, since, $\forall\ {d_{w_{ij}}}^p \leq 0$ so, the solution is optimal and optimum assignment has been attained and is given as:

$$A\rightarrow Q,\ B\rightarrow P,\ C\rightarrow T,\ D\rightarrow R,\ E\rightarrow S$$

Thus, the maximum sales revenue = 38 + 40 + 37 + 36 + 40 = 191 thousand rupees.

4. Conclusion

The present paper discusses the three distinctive approaches by means of the statistical tools as well as the variables themselves. The methodology here provides viable results for the optimal solution to an assignment mix. Hither, the procedure focuses on the possibility of diagonally interchanging to pair of cells from the cycle of an assigned cell. This paper produces an initial basic feasible solution through the enumeration methods along with the discussion of the optimality test to acquire the best (or optimal) solution to the minimization and maximization assignment mixes.

References

1. Sahu, Anshuman and Tapadar, Rudrajit *Solving the Assignment Problem using Genetic Algorithm and Simulated Annealing*, IMECS, Citeseer **38** (2006), 762–765..
2. Bertsekas, Dimitri P, *A new algorithm for the assignment problem*, Mathematical Programming, Springer **21**1 (1981), 152–171.
3. Hadley, George, *Linear programming*, (1962).
4. Kuhn, Harold W *The Hungarian method for the assignment problem*, Naval research logistics quarterly, Wiley Online Library,. **2**,(1-2), (1955), 83–97.
5. Khalid, M and Sultana, Mariam and Zaidi, Faheem *A New Diagonal optimal approach for assignment problem*, Applied Mathematical Sciences, **8**, (160), (2014), 7979–7986.

6. Sharma, JK *Operations research: theory and applications*, Trinity Press, an impint of Laxmi Publications Pvt. LimitedTrinity Press, an impint of Laxmi Publications Pvt. Limited, **(2016)**.
7. C. Leung and C. Ng, *Property (T) and strong property (T) for unital C^*-algebras*. Journal of Functional Analysis **256** (2009), 3055-3070.
8. Taha, Hamdy A *Operations research: an introduction*, Pearson Education India, **(2013)**.

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