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SOME COMMON COUPLED FIXED POINT THEOREMS FOR CONTRACTIVE TYPE CONDITIONS IN S-METRIC SPACES

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ABSTRACT. In this paper, we prove some common coupled fixed point theorems for contractive type conditions in the setting of S-metric spaces and give some consequences of the main results. We have applied quadratic inequality to prove certain common fixed point results. We also give an example to support the result. The results obtained in this paper generalize, extend and unify several previously published results from the existing literature.

1. INTRODUCTION

Banach contraction principle [5] in metric spaces is one of the most important results in the theory of fixed points and non-linear analysis. This famous result states that every self mapping $\mathcal{Q}$ defined on a complete metric space (X, d) satisfying the condition:

$$d(\mathcal{Q}(x), \mathcal{Q}(y)) \leq b\, d(x, y), \tag{1.1}$$

for all $x, y \in X$, where $b \in (0, 1)$, has a unique fixed point and for every $x_0 \in X$ a sequence $\{\mathcal{Q}^n x_0\}_{n \geq 1}$ is convergent to the fixed point.

Remark 1.1. Inequality (1.1) also implies the continuity of the mapping $\mathcal{Q}$.

Fixed point problem for contractive mappings in metric spaces with a partial order have been studied by many authors. On the other hand, *Guo and Lakshmikantham* [11] introduced the notion of coupled fixed point. *Bhashkar and Lakshmikantham* in [6] reconsidered the concept of a coupled fixed point of the mapping $F: X \times X \to X$ and established some coupled fixed point theorems in partially ordered complete metric spaces. They also proved mixed monotone property for the first time and gave their classical coupled fixed point theorem for mapping which satisfy the mixed monotone property. As, an application, they studied the existence and uniqueness of the solution for a periodic boundary value problem associated with first order differential equation. Several other authors such as *Ciric and Lakshmikantham* [8], *Olaleru et al.* [21], *Luong and Thuan* [18]

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and *Nashine et al.* [19]) have proved some coupled fixed point theorems in metric spaces.

In literature, there are many generalizations of the metric spaces and generalized metric spaces are exist. One of such generalization is an S-metric space proposed by *Sedghi et al.* [23] as a generalization of G-metric and D^*-metric in 2012. They studied its some properties and they also stated that S-metric space is a generalization of G-metric space. But *Dung et al.* [9] in 2014 showed by an example that an S-metric space is not a generalization of G-metric space and conversely. Consequently, the class of S-metric spaces and the class of G-metric spaces are different. Many results which were proved earlier in metric space are valid in the framework of S-metric spaces.

Several authors have introduced various conditions, known as compatible conditions in order to establish the presence of common fixed points. If the two mappings commute (*Jungck* [13]), it is the simplest technique to find common fixed points. However, this condition is the strongest one, it is quite common to look for weaker one. In 1986, *Jungck* [14] proposed the property of compatibility between two mappings. After that, the idea of weak compatibility was first introduced by *Jungck and Rhoades* [15] in the year 1998. Recently, some researchers employed this notion to obtain some new fixed point results in various metric spaces (see, for example, [2], [4], [7], [17], [20], [27] and many others).

Recently, *Sedghi et al.* [25] proved some existence results of the unique common fixed point for a pair of weakly compatible self-mappings satisfying some Φ-type contractive conditions in the framework of an S-metric spaces and give an example to validate the results. The results presented in this paper extend and improve several results from the existing literature.

Aydi [3] proved some coupled fixed point theorems for various contractive type conditions in the setting of partial metric spaces and give some corollaries of the established results.

Quite recently, Kim et al. [17] proved some common coupled fixed point results satisfying some contractive condition for a pair of weakly compatible mappings in the framework of complete partial metric spaces.

Motivated by the results and concepts mentioned above, the purpose of this present paper is to improve results on fixed point theorems and prove some common coupled fixed point theorems for a pairwise weakly compatible (wc) mappings in the setting of S-metric spaces. The results presented in this paper extend and generalize several previous works from the existing literature.

2. Preliminaries

In this section, we need some definitions, lemmas and auxiliary results to prove our main results (see, [23]).

Definition 2.1. ([23]) Let X be a nonempty set and let $S\colon X^3 \to [0,\infty)$ be a function satisfying the following conditions for all x, y, z, $a \in X$:

$(S1)$ $0 < S(x,y,z)$ for all $x,y,z \in X$ with $x \neq y \neq z$;

$(S2)$ $S(x,y,z) = 0$ if and only if $x = y = z$;

$(S3)$ $S(x,y,z) \leq S(x,x,a) + S(y,y,a) + S(z,z,a)$.

Then the function S is called an S-metric on X and the pair (X, S) is called an S-metric space.

Example 2.2. ([23])

(1) Let $X = \mathbb{R}^n$ and $\|\cdot\|$ a norm on X, then $S(x, y, z) = \|y + z - 2x\| + \|y - z\|$ is an S-metric on X.

(2) Let $X = \mathbb{R}^n$ and $\|\cdot\|$ a norm on X, then $S(x, y, z) = \|x - z\| + \|y - z\|$ is an S-metric on X.

Example 2.3. ([24]) Let $X = \mathbb{R}$ be the real line. Then $S(x, y, z) = |x-z|+|y-z|$ for all $x, y, z \in \mathbb{R}$ is an S-metric on X. This S-metric on X is called the usual S-metric on X.

Example 2.4. ([16]) Let X be a non-empty set and d be an ordinary metric on X. Then $S(x, y, z) = d(x, z) + d(y, z)$ for all $x, y, z \in \mathbb{R}$ is an S-metric on X.

Example 2.5. ([26]) Let X be a non-empty set and d_1, d_2 be two ordinary metrics on X. Then $S(x, y, z) = d_1(x, z) + d_2(y, z)$ for all $x, y, z \in X$ is an S-metric on X.

Example 2.6. ([23]) Let $X = \mathbb{R}^2$ and d an ordinary metric on X. Put $S(x, y, z) = d(x, y) + d(x, z) + d(y, z)$ for all $x, y, z \in \mathbb{R}^2$, that is, S is the perimeter of the triangle given x, y, z. Then S is an S-metric on X.

Definition 2.7. Let (X, S) be an S-metric space. For $r > 0$ and $x \in X$ we define the open ball $\mathcal{B}_S(x, r)$ and closed ball $\mathcal{B}_S[x, r]$ with center x and radius r as follows, respectively:

$$\mathcal{B}_S(x, r) = \{y \in X : S(y, y, x) < r\},$$

$$\mathcal{B}_S[x, r] = \{y \in X : S(y, y, x) \leq r\}.$$

Example 2.8. ([24]) Let $X = \mathbb{R}$. Denote by $S(x, y, z) = |y + z - 2x| + |y - z|$ for all $x, y, z \in \mathbb{R}$. Then

$$\begin{aligned} \mathcal{B}_S(1, 2) &= \{y \in \mathbb{R} : S(y, y, 1) < 2\} = \{y \in \mathbb{R} : |y - 1| < 1\} \\ &= \{y \in \mathbb{R} : 0 < y < 2\} = (0, 2), \end{aligned}$$

and

$$\begin{aligned} \mathcal{B}_S[2, 4] &= \{y \in \mathbb{R} : S(y, y, 2) \leq 4\} = \{y \in \mathbb{R} : |y - 2| \leq 2\} \\ &= \{y \in \mathbb{R} : 0 \leq y \leq 4\} = [0, 4]. \end{aligned}$$

Definition 2.9. ([23], [24]) Let (X, S) be an S-metric space and $A \subset X$.

(Δ_1) The subset A is said to be an open subset of X, if for every $x \in A$ there exists $r > 0$ such that $\mathcal{B}_S(x, r) \subset A$.

(Δ_2) A sequence $\{y_n\}$ in X converges to $y \in X$ if $S(y_n, y_n, y) \to 0$ as $n \to \infty$, that is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ we have $S(y_n, y_n, y) < \varepsilon$. We denote this by $\lim_{n\to\infty} y_n = y$ or $y_n \to y$ as $n \to \infty$.

(Δ_3) A sequence $\{y_n\}$ in X is called a Cauchy sequence if $S(y_n, y_n, y_m) \to 0$ as $n, m \to \infty$, that is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n, m \geq n_0$ we have $S(y_n, y_n, y_m) < \varepsilon$.

(Δ_4) The S-metric space (X, S) is called complete if every Cauchy sequence in X is convergent in X.

(Δ_5) Let τ be the set of all $A \subset X$ with the property that for each $x \in A$ and there exists $r > 0$ such that $\mathcal{B}_S(x, r) \subset A$. Then τ is a topology on X (induced by the S-metric space).

(Δ_6) A nonempty subset A of X is S-closed if closure of A coincides with A.

Definition 2.10. ([23]) Let (X, S) be an S-metric space. A mapping $\mathcal{T} \colon X \to X$ is said to be a contraction if there exists a constant $0 \leq h < 1$ such that

$$S(\mathcal{T}x, \mathcal{T}y, \mathcal{T}z) \leq h\, S(x, y, z), \tag{2.1}$$

for all $x, y, z \in X$.

Remark 2.11. If the S-metric space (X, S) is complete then the mapping defined as above has a unique fixed point (see [23], Theorem 3.1).

Definition 2.12. ([23]) Let (X, S) and (Y, S') be two S-metric spaces. A function $f \colon X \to Y$ is said to be continuous at a point $x_0 \in X$ if for every sequence $\{x_n\}$ in X with $S(x_n, x_n, x_0) \to 0$, $S'(f(x_n), f(x_n), f(x_0)) \to 0$ as $n \to \infty$. We say that f is continuous on X if f is continuous at every point $x_0 \in X$.

Definition 2.13. Let X be a non-empty set and let $f, g \colon X \to X$ be two self mappings of X. Then a point $u \in X$ is called a

- fixed point of operator f if $f(u) = u$;
- common fixed point of f and g if $f(u) = g(u) = u$.

Definition 2.14. ([1]) Let F and G be single valued self-mappings on a set X. If $z = Fv = Gv$ for some $v \in X$, then v is called a coincidence point of F and G, and z is called a point of coincidence of F and G. We denote the coincidence point of F and G by $C(F, G)$, that is, $C(F, G) = \{v \in X : Fv = Gv\}$.

Definition 2.15. ([14]) Let F and G be single valued self-mappings on a set X. Mappings F and G are said to be commuting if $FGz = GFz$ for all $z \in X$.

Example 2.16. Let $X = [0, \frac{3}{4}]$ and define $F, G \colon X \to X$ defined by $F(x) = \frac{x^3}{4}$ and $G(x) = x^4$ for all $x, y \in X$. Then the mappings F and G have two coincidence points 0 and $\frac{1}{4}$. Clearly, they commute at 0 but not at $\frac{1}{4}$.

Definition 2.17. ([3]) An element $(x, y) \in X \times X$ is said to be a coupled fixed point of the mapping $F \colon X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Example 2.18. Let $X = [0, +\infty)$ and $F \colon X \times X \to X$ defined by $F(x, y) = \frac{x+y}{3}$ for all $x, y \in X$. One can easily see that F has a unique coupled fixed point $(0, 0)$.

Example 2.19. Let $X = [0, +\infty)$ and $F \colon X \times X \to X$ be defined by $F(x, y) = \frac{x+y}{2}$ for all $x, y \in X$. Then we see that F has two coupled fixed point $(0, 0)$ and $(1, 1)$, that is, the coupled fixed point is not unique.

Definition 2.20. ([8]) Let X be a nonempty set. Then we say that the mappings $F \colon X \times X \to X$ and $A \colon X \to X$ are commutative if $A(F(x, y)) = F(Ax, Ay)$.

Definition 2.21. ([8]) An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings $F: X \times X \to X$ and $A: X \to X$ if $F(x, y) = Ax$ and $F(y, x) = Ay$.

Definition 2.22. ([2]) The mappings $F: X \times X \to X$ and $A: X \to X$ are called weakly compatible if $A(F(x, y)) = F(Ax, Ay)$ and $A(F(y, x)) = F(Ay, Ax)$ for all $x, y \in X$, whenever $A(x) = F(x, y)$ and $A(y) = F(y, x)$.

Proposition 2.23. *([1]) Let F and G be weakly compatible self mappings on a set X. If F and G have a unique point of coincidence $w = Fx = Gx$, then w is the unique common fixed point of F and G.*

Lemma 2.24. *([23], Lemma 2.5) Let (X, S) be an S-metric space. Then, we have $S(x, x, y) = S(y, y, x)$ for all $x, y \in X$.*

Lemma 2.25. *([23], Lemma 2.12) Let (X, S) be an S-metric space. If $x_n \to x$ and $y_n \to y$ as $n \to \infty$ then $S(x_n, x_n, y_n) \to S(x, x, y)$ as $n \to \infty$.*

Lemma 2.26. *([10], Lemma 8) Let (X, S) be an S-metric space and A is a nonempty subset of X. Then A is said to be S-closed if and only if for any sequence $\{x_n\}$ in A such that $x_n \to x$ as $n \to \infty$, then $x \in A$.*

Lemma 2.27. *([23]) Let (X, S) be an S-metric space. If $r > 0$ and $x \in X$, then the ball $\mathcal{B}_S(x, r)$ is an open subset of X.*

Lemma 2.28. *([24]) The limit of a sequence $\{x_n\}$ in an S-metric space (X, S) is unique.*

Lemma 2.29. *([23]) Let (X, S) be an S-metric space. Then any convergent sequence $\{x_n\}$ in X is Cauchy.*

In the following lemma we see the relationship between a metric and S-metric.

Lemma 2.30. *([12]) Let (X, d) be a metric space. Then the following properties are satisfied:*

(i) $S_d(x, y, z) = d(x, z) + d(y, z)$ for all $x, y, z \in X$ is an S-metric on X.
(ii) $x_n \to x$ in (X, d) if and only if $x_n \to x$ in (X, S_d).
(iii) $\{x_n\}$ is Cauchy in (X, d) if and only if $\{x_n\}$ is Cauchy in (X, S_d).
(iv) (X, d) is complete if and only if (X, S_d) is complete.

We call the function S_d defined in Lemma 2.30 (i) as the S-metric generated by the metric d. It can be found an example of an S-metric which is not generated by any metric in [12, 22].

Example 2.31. ([12]) Let $X = \mathbb{R}$ and the function $S: X^3 \to [0, \infty)$ be defined as

$$S(x, y, z) = |x - z| + |x + z - 2y|,$$

for all $x, y, z \in \mathbb{R}$. Then the function S is an S-metric on X and (X, S) is an S-metric space. Now, we prove that there does not exists any metric d such that $S = S_d$. On the contrary, suppose that there exists a metric d such that

$$S(x, y, z) = d(x, z) + d(y, z),$$

for all $x, y, z \in \mathbb{R}$. Hence, we obtain

$$S(x, x, z) = 2d(x, z) = 2|x - z|,$$

and

$$d(x, z) = |x - z|.$$

Similarly, we get

$$S(y, y, z) = 2d(y, z) = 2|y - z|,$$

and

$$d(y, z) = |y - z|,$$

for all $x, y, z \in \mathbb{R}$. Hence, we have

$$|x - z| + |x + z - 2y| = |x - z| + |y - z|,$$

which is a contradiction. Therefore, $S \neq S_d$ and $(\mathbb{R}, S)$ is a complete S-metric space.

3. Main results

In this section, we prove some unique common coupled fixed point theorems for contractive type mappings in the setting of S-metric spaces.

Theorem 3.1. *Let (X, S) be a complete S-metric space. Let $F\colon X \times X \to X$ and $H\colon X \to X$ be two mappings such that*

$$\begin{aligned}[S(F(x,y), F(u,v), F(z,w))]^2 \leq\ & a_1\,[S(Hx, Hu, Hz)]^2 + a_2\,[S(Hy, Hv, Hw)]^2 \\ & + a_3\,[S(F(x,y), F(x,y), Hx)]^2 \\ & + a_4\,[S(F(u,v), F(u,v), Hu)]^2 \\ & + a_5\,[S(F(z,w), F(z,w), Hz)]^2, \end{aligned} \tag{3.1}$$

for all $x, y, u, v, z, w \in X$, where a_1, a_2, a_3, a_4, a_5 are nonnegative constants such that $a_1 + a_2 + a_3 + a_4 + a_5 < 1$. Assume that F and H satisfy the following conditions:

(i) $F(X \times X) \subseteq H(X)$,

(ii) $H(X)$ is complete, and

(iii) H is continuous and commute with F.

Then F and H have a coupled coincidence point in X. Moreover, if F and H are weakly compatible, then F and H have a unique common coupled fixed point.

Proof. Let $x_0, y_0 \in X$. Since $F(X \times X) \subseteq H(X)$, we can choose $x_1, y_1 \in X$ such that $Hx_1 = F(x_0, y_0)$ and $Hy_1 = F(y_0, x_0)$. Again since $F(X \times X) \subseteq H(X)$, we can choose $x_2, y_2 \in X$ such that $Hx_2 = F(x_1, y_1)$ and $Hy_2 = F(y_1, x_1)$. Continuing this process, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X

such that $Hx_{n+1} = F(x_n, y_n)$ and $Hy_{n+1} = F(y_n, x_n)$. For $n \in \mathbb{N}$, by equation (3.1), and using Lemma 2.24, we have

$$\begin{aligned}
[S(Hx_n, Hx_n, Hx_{n+1})]^2 &= [S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), F(x_n, y_n))]^2 \\
&\leq a_1 [S(Hx_{n-1}, Hx_{n-1}, Hx_n)]^2 + a_2 [S(Hy_{n-1}, Hy_{n-1}, Hy_n)]^2 \\
&\quad + a_3 [S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), Hx_{n-1})]^2 \\
&\quad + a_4 [S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), Hx_{n-1})]^2 \\
&\quad + a_5 [S(F(x_n, y_n), F(x_n, y_n), Hx_n)]^2 \\
&= a_1 [S(Hx_{n-1}, Hx_{n-1}, Hx_n)]^2 + a_2 [S(Hy_{n-1}, Hy_{n-1}, Hy_n)]^2 \\
&\quad + a_3 [S(Hx_n, Hx_n, Hx_{n-1})]^2 + a_4 [S(Hx_n, Hx_n, Hx_{n-1})]^2 \\
&\quad + a_5 [S(Hx_{n+1}, Hx_{n+1}, Hx_n)]^2 \\
&= (a_1 + a_3 + a_4) [S(Hx_{n-1}, Hx_{n-1}, Hx_n)]^2 \\
&\quad + a_2 [S(Hy_{n-1}, Hy_{n-1}, Hy_n)]^2 \\
&\quad + a_5 [S(Hx_n, Hx_n, Hx_{n+1})]^2. \qquad (3.2)
\end{aligned}$$

Similarly by equation (3.1), we obtain

$$\begin{aligned}
[S(Hy_n, Hy_n, Hy_{n+1})]^2 &= [S(F(y_{n-1}, x_{n-1}), F(y_{n-1}, x_{n-1}), F(y_n, x_n))]^2 \\
&\leq (a_1 + a_3 + a_4) [S(Hy_{n-1}, Hy_{n-1}, Hy_n)]^2 \\
&\quad + a_2 [S(Hx_{n-1}, Hx_{n-1}, Hx_n)]^2 \\
&\quad + a_5 [S(Hy_n, Hy_n, Hy_{n+1})]^2. \qquad (3.3)
\end{aligned}$$

Set

$$\mathcal{M}_n = [S(Hx_n, Hx_n, Hx_{n+1})]^2 \quad \text{and} \quad \mathcal{N}_n = [S(Hy_n, Hy_n, Hy_{n+1})]^2, \qquad (3.4)$$

and

$$\mathcal{R}_n = \mathcal{M}_n + \mathcal{N}_n. \qquad (3.5)$$

Hence, from equations (3.2)-(3.5), we obtain

$$\begin{aligned}
\mathcal{R}_n &= [S(Hx_n, Hx_n, Hx_{n+1})]^2 + [S(Hy_n, Hy_n, Hy_{n+1})]^2 \\
&= \mathcal{M}_n + \mathcal{N}_n \\
&\leq (a_1 + a_3 + a_4) \mathcal{M}_{n-1} + a_2 \mathcal{N}_{n-1} + a_5 \mathcal{M}_n \\
&\quad + (a_1 + a_3 + a_4) \mathcal{N}_{n-1} + a_2 \mathcal{M}_{n-1} + a_5 \mathcal{N}_n \\
&= (a_1 + a_2 + a_3 + a_4) [\mathcal{M}_{n-1} + \mathcal{N}_{n-1}] \\
&\quad + a_5 [\mathcal{M}_n + \mathcal{N}_n] \\
&= (a_1 + a_2 + a_3 + a_4) \mathcal{R}_{n-1} + a_5 \mathcal{R}_n. \qquad (3.6)
\end{aligned}$$

This implies that

$$\mathcal{R}_n \leq \Big(\frac{a_1 + a_2 + a_3 + a_4}{1 - a_5}\Big) \mathcal{R}_{n-1} = \delta \, \mathcal{R}_{n-1}, \qquad (3.7)$$

where $\delta = \Big(\frac{a_1+a_2+a_3+a_4}{1-a_5}\Big) < 1$, since $a_1 + a_2 + a_3 + a_4 + a_5 < 1$.

Consequently, for each $n \in \mathbb{N}$, we obtain

$$\mathcal{R}_n \leq \delta \, \mathcal{R}_{n-1} \leq \delta^2 \, \mathcal{R}_{n-2} \leq \cdots \leq \delta^n \, \mathcal{R}_0. \qquad (3.8)$$

If $\mathcal{R}_0 = 0$, then $[S(Hx_0, Hx_0, Hx_1)]^2 + [S(Hy_0, Hy_0, Hy_1)]^2 = 0$ and so $S(Hx_0, Hx_0, Hx_1) = 0$ and $S(Hy_0, Hy_0, Hy_1) = 0$. Hence, by condition $(S2)$, we get $Hx_0 = Hx_1 = F(x_0, y_0)$ and $Hy_0 = Hy_1 = F(y_0, x_0)$. Thus, (Hx_0, Hy_0) is a coupled fixed point of F and H. Now, we assume that $\mathcal{R}_0 > 0$. For each $m > n$, where $n, m \in \mathbb{N}$, and using $(S3)$, we have

$$
\begin{aligned}
& S(Hx_n, Hx_n, Hx_m) + S(Hy_n, Hy_n, Hy_m) \\
\leq\ & 2S(Hx_n, Hx_n, Hx_{n+1}) + S(Hx_m, Hx_m, Hx_{n+1}) \\
& +2S(Hy_n, Hy_n, Hy_{n+1}) + S(Hy_m, Hy_m, Hy_{n+1}) \\
=\ & 2(S(Hx_n, Hx_n, Hx_{n+1}) + S(Hy_n, Hy_n, Hy_{n+1})) \\
& +S(Hx_m, Hx_m, Hx_{n+1}) + S(Hy_m, Hy_m, Hy_{n+1}) \\
\leq\ & \dots \\
\leq\ & 2\Big(\mathcal{R}_n + \mathcal{R}_{n+1} + \cdots + \mathcal{R}_{m-1} + \mathcal{R}_m\Big) \\
\leq\ & 2(\delta^n + \delta^{n+1} + \cdots + \delta^{m-1} + \delta^m)\mathcal{R}_0 \\
\leq\ & 2\delta^n(1 + \delta + \delta^2 + \dots)\mathcal{R}_0 \\
\leq\ & \Big(\frac{2\delta^n}{1-\delta}\Big)\mathcal{R}_0 \\
\to\ & 0 \text{ as } n \to \infty,
\end{aligned}
$$

since $0 < \delta < 1$. Thus, $\{Hx_n\}$ and $\{Hy_n\}$ are S-Cauchy sequence in $H(X)$. Since $H(X)$ is complete, we get $\{Hx_n\}$ and $\{Hy_n\}$ are S-convergent to some $r \in X$ and $t \in X$ respectively. Since H is continuous, we have $\{HHx_n\}$ is S-convergent to Hr and $\{HHy_n\}$ is S-convergent to Ht. Also, since H and F are commute, we have

$$HHx_{n+1} = H(F(x_n, y_n)) = F(Hx_n, Hy_n),$$

and

$$HHy_{n+1} = H(F(y_n, x_n)) = F(Hy_n, Hx_n).$$

Therefore,

$$
\begin{aligned}
[S(HHx_{n+1}, HHx_{n+1}, F(r,t))]^2 &= [S(F(Hx_n, Hy_n), F(Hx_n, Hy_n), F(r,t))]^2 \\
&\leq a_1\,[S(HHx_n, HHx_n, Hr)]^2 + a_2\,[S(HHy_n, HHy_n, Ht)]^2 \\
&\quad +a_3\,[S(F(Hx_n, Hy_n), F(Hx_n, Hy_n), HHx_n)]^2 \\
&\quad +a_4\,[S(F(Hx_n, Hy_n), F(Hx_n, Hy_n), HHx_n)]^2 \\
&\quad +a_5\,[S(F(r,t), F(r,t), Hr)]^2 \\
&= a_1\,[S(HHx_n, HHx_n, Hr)]^2 + a_2\,[S(HHy_n, HHy_n, Ht)]^2 \\
&\quad +a_3\,[S(HHx_{n+1}, HHx_{n+1}, HHx_n)]^2 \\
&\quad +a_4\,[S(HHx_{n+1}, HHx_{n+1}, HHx_n)]^2 \\
&\quad +a_5\,[S(F(r,t), F(r,t), Hr)]^2.
\end{aligned}
$$

Passing to the limit as $n \to \infty$, using Lemma 2.24, Lemma 2.25 and the condition $(S2)$, we obtain that

$$\begin{aligned}
[S(Hr, Hr, F(r,t))]^2 &\leq a_5\,[S(Hr, Hr, F(r,t))]^2 \\
&\leq (a_1 + a_2 + a_3 + a_4 + a_5)\,[S(Hr, Hr, F(r,t))]^2 \\
&< [S(Hr, Hr, F(r,t))]^2,
\end{aligned}$$

which is a contradiction, since $a_1 + a_2 + a_3 + a_4 + a_5 < 1$. Hence, we get $S(Hr, Hr, F(r,t)) = 0$, that is, $Hr = F(r,t)$. Similarly, we can show that $Ht = F(t,r)$. Thus (Hr, Ht) is a coupled coincidence point of the mappings F and H. Since the pair (F, H) is weakly compatible, so by weak compatibility of the mappings F and H, we have

$$H(F(r,t)) = F(Hr, Ht) \text{ and } H(F(t,r)) = F(Ht, Hr).$$

Hence (Hr, Ht) is a common coupled fixed point of F and H.

Now, we show the uniqueness of the common coupled fixed point of F and H. Assume that (Hr_1, Ht_1) is another common coupled fixed point of F and H with $Hr \neq Hr_1$ and $Ht \neq Ht_1$, that is, $(Hr, Ht) \neq (Hr_1, Ht_1)$. Then by using equation (3.1), using Lemma 2.24 and the condition $(S2)$, we have

$$\begin{aligned}
[S(Hr, Hr, Hr_1)]^2 &= [S(F(r,t), F(r,t), F(r_1,t_1))]^2 \\
&\leq a_1\,[S(Hr, Hr, Hr_1)]^2 + a_2\,[S(Ht, Ht, Ht_1)]^2 \\
&\quad + a_3\,[S(F(r,t), F(r,t), Hr)]^2 \\
&\quad + a_4\,[S(F(r,t), F(r,t), Hr)]^2 \\
&\quad + a_5\,[S(F(r_1,t_1), F(r_1,t_1), Hr_1)]^2 \\
&= a_1\,[S(Hr, Hr, Hr_1)]^2 + a_2\,[S(Ht, Ht, Ht_1)]^2 \\
&\quad + a_3\,[S(Hr, Hr, Hr)]^2 + a_4\,[S(Hr, Hr, Hr)]^2 \\
&\quad + a_5\,[S(Hr_1, Hr_1, Hr_1)]^2 \\
&= a_1\,[S(Hr, Hr, Hr_1)]^2 + a_2\,[S(Ht, Ht, Ht_1)]^2.
\end{aligned} \tag{3.9}$$

Similarly, one can obtain

$$[S(Ht, Ht, Ht_1)]^2 \leq a_1\,[S(Ht, Ht, Ht_1)]^2 + a_2\,[S(Hr, Hr, Hr_1)]^2. \tag{3.10}$$

Again, set

$$\mathcal{D} = [S(Hr, Hr, Hr_1)]^2 \quad \text{and} \quad \mathcal{E} = [S(Ht, Ht, Ht_1)]^2, \tag{3.11}$$

and

$$\mathcal{F} = \mathcal{D} + \mathcal{E}. \tag{3.12}$$

Hence from equations (3.9) and (3.12), we obtain

$$\begin{aligned}
\mathcal{F} &= [S(Hr, Hr, Hr_1)]^2 + [S(Ht, Ht, Ht_1)]^2 \\
&= \mathcal{D} + \mathcal{E} \\
&\leq a_1\,\mathcal{D} + a_2\,\mathcal{E} + a_1\,\mathcal{E} + a_2\,\mathcal{D} \\
&= (a_1 + a_2)\,\mathcal{D} + (a_1 + a_2)\,\mathcal{E} \\
&= (a_1 + a_2)\,(\mathcal{D} + \mathcal{E}) = (a_1 + a_2)\,\mathcal{F} \\
&\leq (a_1 + a_2 + a_3 + a_4 + a_5)\,\mathcal{F} \\
&< \mathcal{F},
\end{aligned}$$

which is a contradiction, since $a_1 + a_2 + a_3 + a_4 + a_5 < 1$. Hence, we conclude that $\mathcal{F} = 0$, that is, $[S(Hr, Hr, Hr_1)]^2 + [S(Ht, Ht, Ht_1)]^2 = 0$ and so $S(Hr, Hr, Hr_1) = 0$ and $S(Ht, Ht, Ht_1) = 0$. Thus, $Hr = Hr_1$ and $Ht = Ht_1$. This shows that F and H have a unique common coupled fixed point. This completes the proof. □

From Theorem 3.1, we obtain the following result.

Corollary 3.2. *Let (X, S) be a complete S-metric space. Let $F\colon X \times X \to X$ and $H\colon X \to X$ be two mappings such that*

$$\begin{aligned}
[S(F(x,y), F(u,v), F(z,w))]^2 \;\leq\; \lambda \max\Big\{&[S(Hx, Hu, Hz)]^2, [S(Hy, Hv, Hw)]^2, \\
&[S(F(x,y), F(x,y), Hx)]^2, \\
&[S(F(u,v), F(u,v), Hu)]^2, \\
&[S(F(z,w), F(z,w), Hz)]^2\Big\},
\end{aligned}$$

for all $x, y, u, v, z, w \in X$, where $\lambda \in [0, 1)$ is a constant. Assume that F and H satisfy the following conditions:

(i) $F(X \times X) \subseteq H(X)$,

(ii) $H(X)$ is complete, and

(iii) H is continuous and commute with F.

Then F and H have a coupled coincidence point in X. Moreover, if F and H are weakly compatible, then F and H have a unique common coupled fixed point.

Proof. Follows from Theorem 3.1 by noting that

$$\begin{aligned}
&a_1\,[S(Hx, Hu, Hz)]^2 + a_2\,[S(Hy, Hv, Hw)]^2 + a_3\,[S(F(x,y), F(x,y), Hx)]^2 \\
&\qquad + a_4\,[S(F(u,v), F(u,v), Hu)]^2 + a_5\,[S(F(z,w), F(z,w), Hz)]^2 \\
\leq\;& (a_1 + a_2 + a_3 + a_4 + a_5) \max\Big\{[S(Hx, Hu, Hz)]^2, [S(Hy, Hv, Hw)]^2, \\
&\qquad [S(F(x,y), F(x,y), Hx)]^2, [S(F(u,v), F(u,v), Hu)]^2, \\
&\qquad [S(F(z,w), F(z,w), Hz)]^2\Big\} \\
=\;& \lambda \max\Big\{[S(Hx, Hu, Hz)]^2, [S(Hy, Hv, Hw)]^2, [S(F(x,y), F(x,y), Hx)]^2, \\
&\qquad [S(F(u,v), F(u,v), Hu)]^2, [S(F(z,w), F(z,w), Hz)]^2\Big\},
\end{aligned}$$

where $\lambda = a_1 + a_2 + a_3 + a_4 + a_5 < 1$. $\square$

Remark 3.3. Theorem 3.1 is an extension and a generalization of the results of Aydi [3] from partial metric space to the setting of S-metric space.

Theorem 3.4. *Let (X, S) be a complete S-metric space. Let $F\colon X \times X \to X$ and $H\colon X \to X$ be two mappings such that*

$$\begin{aligned}
[S(F(x,y), F(u,v), F(z,w))]^2 \ \leq\ & b_1 \max\Big\{[S(Hx, Hu, Hz)]^2, [S(F(x,y), F(x,y), Hx)]^2, \\
& \qquad [S(F(x,y), F(x,y), Hu)]^2\Big\} \\
& + b_2 \max\Big\{[S(F(u,v), F(u,v), Hz)]^2, \\
& \qquad [S(F(z,w), F(z,w), Hz)]^2\Big\} \\
& + b_3\, S(F(x,y), F(x,y), Hz) S(F(z,w), F(z,w), Hu),
\end{aligned} \tag{3.13}$$

for all $x, y, u, v, z, w \in X$, where b_1, b_2, b_3 are nonnegative constants with $b_1 + b_2 < 1$. Assume that F and H satisfy the following conditions:

(i) $F(X \times X) \subseteq H(X)$,

(ii) $H(X)$ is complete, and

(iii) H is continuous and commute with F.

Then F and H have a coupled coincidence point in X. Moreover, if F and H are weakly compatible, then F and H have a unique common coupled fixed point.

Proof. Let $x_0, y_0 \in X$. Since $F(X \times X) \subseteq H(X)$, we can choose $x_1, y_1 \in X$ such that $Hx_1 = F(x_0, y_0)$ and $Hy_1 = F(y_0, x_0)$. Again since $F(X \times X) \subseteq H(X)$, we can choose $x_2, y_2 \in X$ such that $Hx_2 = F(x_1, y_1)$ and $Hy_2 = F(y_1, x_1)$. Continuing this process, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $Hx_{n+1} = F(x_n, y_n)$ and $Hy_{n+1} = F(y_n, x_n)$. Set $\mathcal{M}_n = [S(Hx_n, Hx_n, Hx_{n+1})]^2$ and $\mathcal{N}_n = [S(Hy_n, Hy_n, Hy_{n+1})]^2$. For $n \in \mathbb{N}$, by equation (3.13), using Lemma

2.24 and $(S2)$, we have

$$\begin{aligned}
\mathcal{M}_n &= [S(Hx_n, Hx_n, Hx_{n+1})]^2 \\
&= [S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), F(x_n, y_n))]^2 \\
&\leq b_1 \max\Big\{[S(Hx_{n-1}, Hx_{n-1}, Hx_n)]^2, [S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), Hx_{n-1})]^2, \\
&\qquad\qquad [S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), Hx_{n-1})]^2\Big\} \\
&\quad + b_2 \max\Big\{[S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), Hx_n)]^2, \\
&\qquad\qquad [S(F(x_n, y_n), F(x_n, y_n), Hx_n)]^2\Big\} \\
&\quad + b_3\, S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), Hx_n) S(F(x_n, y_n), F(x_n, y_n), Hx_{n-1}) \\
&= b_1 \max\Big\{[S(Hx_{n-1}, Hx_{n-1}, Hx_n)]^2, [S(Hx_n, Hx_n, Hx_{n-1})]^2, \\
&\qquad\qquad [S(Hx_n, Hx_n, Hx_{n-1})]^2\Big\} \\
&\quad + b_2 \max\Big\{[S(Hx_n, Hx_n, Hx_n)]^2, [S(Hx_{n+1}, Hx_{n+1}, Hx_n)]^2\Big\} \\
&\quad + b_3\, S(Hx_n, Hx_n, Hx_n) S(Hx_{n+1}, Hx_{n+1}, Hx_{n-1}) \\
&= b_1 \max\Big\{[S(Hx_{n-1}, Hx_{n-1}, Hx_n)]^2, [S(Hx_{n-1}, Hx_{n-1}, Hx_n)]^2, \\
&\qquad\qquad [S(Hx_{n-1}, Hx_{n-1}, Hx_n)]^2\Big\} \\
&\quad + b_2 \max\Big\{0, [S(Hx_n, Hx_n, Hx_{n+1})]^2\Big\} + b_3\, .0 \\
&= b_1 \max\Big\{\mathcal{M}_{n-1}, \mathcal{M}_{n-1}, \mathcal{M}_{n-1}\Big\} + b_2 \max\Big\{0, \mathcal{M}_n\Big\} \\
&= b_1\, \mathcal{M}_{n-1} + b_2\, \mathcal{M}_n. \qquad (3.14)
\end{aligned}$$

Similarly by equation (3.13), we obtain

$$\begin{aligned}
\mathcal{N}_n &= [S(Hy_n, Hy_n, Hy_{n+1})]^2 \\
&= [S(F(y_{n-1}, x_{n-1}), F(y_{n-1}, x_{n-1}), F(y_n, x_n))]^2 \\
&\leq b_1\, \mathcal{N}_{n-1} + b_2\, \mathcal{N}_n. \qquad (3.15)
\end{aligned}$$

Again, we set

$$\mathcal{V}_n = \mathcal{M}_n + \mathcal{N}_n. \qquad (3.16)$$

Now, from equations (3.14)-(3.16), we obtain

$$\begin{aligned}
\mathcal{V}_n &= \mathcal{M}_n + \mathcal{N}_n \\
&\leq b_1\, \mathcal{M}_{n-1} + b_2\, \mathcal{M}_n + b_1\, \mathcal{N}_{n-1} + b_2\, \mathcal{N}_n \\
&= b_1\, (\mathcal{M}_{n-1} + \mathcal{N}_{n-1}) + b_2\, (\mathcal{M}_n + \mathcal{N}_n) \\
&= b_1\, \mathcal{V}_{n-1} + b_2\, \mathcal{V}_n.
\end{aligned}$$

This implies that

$$\mathcal{V}_n \leq \Big(\frac{b_1}{1-b_2}\Big)\mathcal{V}_{n-1} = \beta\, \mathcal{V}_{n-1}, \qquad (3.17)$$

where $\beta = \left(\frac{b_1}{1-b_2}\right) < 1$, since $b_1 + b_2 < 1$.

Consequently, for each $n \in \mathbb{N}$, we obtain

$$\mathcal{V}_n \leq \beta\, \mathcal{V}_{n-1} \leq \beta^2\, \mathcal{V}_{n-2} \leq \cdots \leq \beta^n\, \mathcal{V}_0. \tag{3.18}$$

If $\mathcal{V}_0 = 0$, then $[S(Hx_0, Hx_0, Hx_1)]^2 + [S(Hy_0, Hy_0, Hy_1)]^2 = 0$ and so $S(Hx_0, Hx_0, Hx_1) = 0$ and $S(Hy_0, Hy_0, Hy_1) = 0$. Hence, by condition $(S2)$, we get $Hx_0 = Hx_1 = F(x_0, y_0)$ and $Hy_0 = Hy_1 = F(y_0, x_0)$. Thus, (Hx_0, Hy_0) is a coupled fixed point of F and H. Now, we assume that $\mathcal{V}_0 > 0$. The rest of the proof follows from Theorem 3.1. This completes the proof.

□

If we take $b_2 = b_3 = 0$ and $b_1 = k$ in Theorem 3.4, then we obtain the following result.

Corollary 3.5. *Let (X, S) be a complete S-metric space. Let $F\colon X \times X \to X$ and $H\colon X \to X$ be two mappings such that*

$$[S(F(x,y), F(u,v), F(z,w))]^2 \leq k \max\Big\{[S(Hx, Hu, Hz)]^2, [S(F(x,y), F(x,y), Hx)]^2, [S(F(x,y), F(x,y), Hu)]^2\Big\},$$

for all $x, y, u, v, z, w \in X$, where $k \in [0,1)$ is a constant. Assume that F and H satisfy the following conditions:

(i) $F(X \times X) \subseteq H(X)$,
(ii) $H(X)$ is complete, and
(iii) H is continuous and commute with F.

Then F and H have a coupled coincidence point in X. Moreover, if F and H are weakly compatible, then F and H have a unique common coupled fixed point.

If we take $b_1 = b_3 = 0$ and $b_2 = h$ in Theorem 3.4, then we obtain the following result.

Corollary 3.6. *Let (X, S) be a complete S-metric space. Let $F\colon X \times X \to X$ and $H\colon X \to X$ be two mappings such that*

$$[S(F(x,y), F(u,v), F(z,w))]^2 \leq h \max\Big\{[S(F(u,v), F(u,v), Hz)]^2, [S(F(z,w), F(z,w), Hz)]^2\Big\},$$

for all $x, y, u, v, z, w \in X$, where $h \in [0,1)$ is a constant. Assume that F and H satisfy the following conditions:

(i) $F(X \times X) \subseteq H(X)$,
(ii) $H(X)$ is complete, and
(iii) H is continuous and commute with F.

Then F and H have a coupled coincidence point in X. Moreover, if F and H are weakly compatible, then F and H have a unique common coupled fixed point.

Remark 3.7. Our results extend and generalize the corresponding results of Aydi [3] from partial metric spaces to the setting of S-metric spaces.

Now, we give an example to justify the result.

Example 3.8. Let $X = [0, 1]$ and the function $S\colon X^3 \to [0, \infty)$ be defined as

$$S(x, y, z) = |y - z| + |y + z - 2x|,$$

for all $x, y, z \in X$. Then the function S is an S-metric on X and (X, S) is an S-metric space. Define a map $F\colon X \times X \to X$ by $F(x, y) = \frac{x+y}{25}$ for $x, y \in X$. Also, define $H\colon X \to X$ by $H(x) = \frac{x}{5}$. We have

(1)

$$\begin{aligned}
[S(F(x,y), F(u,v), F(z,w))]^2 &= [|F(u,v) + F(z,w) - 2F(x,y)| \\
&\quad +|F(u,v) - F(z,w)|]^2 \\
&= \Big[\Big|\frac{u+v}{25} + \frac{z+w}{25} - \frac{2(x+y)}{25}\Big| \\
&\quad +\Big|\frac{u+v}{25} - \frac{z+w}{25}\Big|\Big]^2 \\
&= \Big[\frac{1}{25}|u+z-2x| + \frac{1}{25}|v+w-2y| \\
&\quad +\frac{1}{25}|u-z| + \frac{1}{25}|v-w|\Big]^2 \\
&= \Big[\frac{1}{25}\big(|u+z-2x| + |u-z|\big) \\
&\quad +\frac{1}{25}\big(|v+w-2y| + |v-w|\big)\Big]^2 \\
&= \Big[\frac{1}{5}\Big(\Big|\frac{u}{5} + \frac{z}{5} - \frac{2x}{5}\Big| + \Big|\frac{u}{5} - \frac{z}{5}\Big|\Big) \\
&\quad +\frac{1}{5}\Big(\Big|\frac{v}{5} + \frac{w}{5} - \frac{2y}{5}\Big| + \Big|\frac{v}{5} - \frac{w}{5}\Big|\Big)\Big]^2 \\
&= \frac{1}{25}\big[S(Hx, Hu, Hz) + S(Hy, Hv, Hw)\big]^2 \\
&\leq \frac{1}{12}\Big([(S(Hx, Hu, Hz)]^2 + [S(Hy, Hv, Hw)]^2\Big) \\
&\leq \frac{1}{12}\Big([S(Hx, Hu, Hz)]^2 + [S(Hy, Hv, Hw)]^2 \\
&\qquad +[S(F(x,y), F(x,y), Hx)]^2 \\
&\qquad +[S(F(u,v), F(u,v), Hu)]^2 \\
&\qquad +[S(F(z,w), F(z,w), Hz)]^2\Big),
\end{aligned}$$

holds for all $x, y, z, u, v, w \in X$. It is easy to check that inequality (3.1) of Theorem 3.1 is satisfied for $a_1 = a_2 = a_3 = a_4 = a_5 = \frac{1}{12}$ with $a_1 + a_2 + a_3 + a_4 + a_5 = \frac{5}{12} < 1$.

(2) $H(x) = \frac{x}{5}$ is continuous and $H(X)$ is a complete subspace of X.

(3) Now, we show that H is commute with F. Here, we have

$$H(F(x,y)) = H\Big(\frac{x+y}{25}\Big) = \frac{x+y}{125},$$

and

$$F(Hx, Hy) = F\Big(\frac{x}{5}, \frac{y}{5}\Big) = \frac{x+y}{125}.$$

Thus

$$H(F(x,y)) = F(Hx, Hy).$$

This shows that H is commute with F. Hence F and H satisfy all the conditions of Theorem 3.1 and so F and H have a unique common coupled fixed point, namely $F(0,0) = 0$ and $H(0) = 0$.

4. Conclusion

In this paper, we prove some unique common coupled fixed point theorems in the setting of S-metric spaces for a pair of weakly compatible mappings. Our results extend and generalize several previously published results from the existing literature.

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