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ON SOME GENERALIZED GRAPHS RELATED TO CYCLE

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Abstract. The generalized graphs related to cycle are defined by Scaria et al.[43]. Motivated by this, we define three new classes namely $P(n,k,t)$, $Q(n,k,t)$ and $S(n,k)$, obtain their adjacency spectrum and thus adding them to the classes of graphs whose spectrum is completely known. Further, we compute (a,b)-KA indices of some generalized graphs related to cycle.

1. Introduction

The topological indices are graph invariants which are numerical values associated with molecular graphs. In mathematical chemistry, these are known as molecular descriptors. The topological indices play vital role in mathematical chemistry, especially quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) investigations. For chemical applications of topological indices refer [3, 4, 11, 12, 16, 17, 42, 44, 49, 52, 53].

In this paper, we are concerned with simple graphs, having no directed or weighted edges and no loops. Let G be a finite graph with n vertices and m edges is called (n,m) graph. We denote vertex set and edge set of graph G as $V(G)$ and $E(G)$, respectively. The degree of a vertex $d_G(v)$ is the number of edges incident to it in G. For undefined graph theoretic terminologies and notations refer [15].

The (a, b)-KA indices were introduced by Kulli [24] and elaborated in [25]. The first (a, b)-KA index $KA^1_{(a,b)}(G)$, the second (a, b)-KA index $KA^2_{(a,b)}(G)$ and the third (a, b)-KA index $KA^3_{(a,b)}(G)$ are defined as

$$KA^1_{(a,b)}(G) = \sum_{uv\in E(G)} [d_G(u)^a + d_G(v)^a]^b,$$

$$KA^2_{(a,b)}(G) = \sum_{uv\in E(G)} [d_G(u)^a \cdot d_G(v)^a]^b,$$

and

$$KA^3_{(a,b)}(G) = \sum_{uv\in E(G)} |d_G(u)^a - d_G(v)^a|^b,$$

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where a and b are real numbers.

The particular values of a and b in (a,b)-KA indices [24].

(1) $KA^1_{(1,1)}(G) = M_1(G)$, the first Zagreb index, [10].
(2) $KA^1_{(-1,1)}(G) = RM_1(G)$, the redefined first Zagreb index, [41].
(3) $KA^1_{(-1,-1)}(G) = RM_2(G)$, the redefined second Zagreb index, [41].
(4) $KA^1_{(2,1)}(G) = F_1(G)$, the first Forgotten index, [14].
(5) $KA^1_{(2,2)}(G) = HF_1(G)$, the first hyper Forgotten index, [9].
(6) $KA^1_{(1,2)}(G) = HM_1(G)$, the first hyper Zagreb index, [48].
(7) $KA^1_{(1,-\frac{1}{2})}(G) = \chi(G)$, the sum connectivity index, [54].
(8) $\frac{1}{2}KA^1_{(1,1)}(G) = SK(G)$, the SK-index, [47].
(9) $KA^1_{(2,\frac{1}{2})}(G) = SO(G)$, the Sombor index, [12].
(10) $KA^1_{(1,b)}(G) = M_1^b(G)$, the first general Zagreb index, [34].
(11) $KA^1_{(a,1)}(G) = M_\alpha(G)$, An edge α-Zagreb index, [38].
(12) $KA^1_{(3,1)}(G) = Y(G)$, the Y-index, [1].
(13) $KA^1_{(1,\frac{1}{2})}(G) = N(G)$, the Nirmala index, [26].
(14) $KA^1_{(3,\frac{1}{2})}(G) = D(G)$, the Dharwad index, [27].
(15) $KA^2_{(1,1)}(G) = M_2(G)$, the second Zagreb index, [10].
(16) $KA^2_{(2,1)}(G) = F_2(G)$, the second Forgotten index, [28].
(17) $KA^2_{(1,2)}(G) = HM_2(G)$, the second hyper Zagreb index, [51].
(18) $KA^2_{(1,-\frac{1}{2})}(G) = R(G)$, the Randic index, [40].
(19) $KA^2_{(1,b)}(G) = R^b(G)$, the general Randic index, [6].
(20) $KA^3_{(1,1)}(G) = Alb(G)$, the Albertson index, [2].
(21) $KA^3_{(-1,1)}(G) = M_{in}(G)$, the misbalance indeg index, [50].
(22) $KA^3_{(\frac{1}{2},1)}(G) = M_{ro}(G)$, the misbalance rodeg index, [50].
(23) $KA^3_{(-\frac{1}{2},1)}(G) = M_{ir}(G)$, the misbalance irdeg index, [50].
(24) $KA^3_{(-2,1)}(G) = M_s(G)$, the misbalance sdeg index, [50].
(25) $KA^3_{(1,2)}(G) = \sigma(G)$, the sigma index, [13].
(26) $KA^3_{(2,1)}(G) = MF(G)$, the minus F-index, [29].
(27) $KA^3_{(2,2)}(G) = \rho F(G)$, the square F-index, [29].
(28) $KA^3_{(2,\frac{1}{2})}(G) = RMF_c(G)$, the reciprocal minus F-index, [32].
(29) $KA^3_{(1,b)}(G) = M_i^b(G)$, the general minus index, [31].
(30) $KA^3_{(a,1)}(G) = M_i^a(G)$, the general misbalance deg index, [35].
(31) $KA^3_{(2,b)}(G) = MF^b(G)$, the general minus F-index, [32].

Let $G = (V(G), E(G))$ be a simple graph with an adjacency matrix A. The multiset of eigenvalues of A constitute the spectrum of G. Numerous studies have been conducted recently on the computation of various graph polynomials and the related spectra.

2. Generalized graphs related to cycle

Scaria et al.[43] defined some generalised graphs related to the cycle C_n namely $M(n,k)$, $T(n,k,t)$, $N(n,k,t)$, $D(n,k,t,m)$, $C(n,k,t)$, $PC(n,k,t)$ and $CC(n,k,t)$. The definitions of $N(n,k,t)$ and $D(n,k,t,m)$ contained some error. Basavanagoud et al. corrected these definitions in [5].

Definition 2.1. [43] The graph $M(n,k)$ has vertices, respectively, edges given by

$$V(M(n,k)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

$$E(M(n,k)) = \{u_iv_i, u_iv_{i+1}, v_iv_{i+k}/0 \leq i \leq n-1\},$$

where the subscripts are expressed as integers modulo $n > 4$ and $k < \frac{n}{2}$.

Definition 2.2. [43] The graph $T(n,k,t)$ has vertices, respectively, edges given by

$$V(T(n,k,t)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

$$E(T(n,k,t)) = \{u_iv_i, u_iv_{i+1}, u_iu_{i+t}, v_iv_{i+k}/0 \leq i \leq n-1\},$$

where the subscripts are expressed as integers modulo $n > 4$ and $t, k < \frac{n}{2}$.

Definition 2.3. [5] The graph $N(n,k,t)$ has vertices, respectively, edges given by

$$V(N(n,k,t)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

$$E(N(n,k,t)) = \{u_iv_{i+t}, v_iu_{i+t}, v_iv_{i+k}/0 \leq i \leq n-1\},$$

where the subscripts are expressed as integers modulo $n > 4$ and $t, k < \frac{n}{2}$.

Definition 2.4. [5] The graph $D(n,k,t,m)$ has vertices, respectively, edges given by

$$V(D(n,k,t,m)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

$$E(D(n,k,t,m)) = \{u_iv_{i+t}, v_iu_{i+t}, v_iv_{i+k}/0 \leq i \leq n-1\},$$

where the subscripts are expressed as integers modulo $n > 4$ and $t, m, k < \frac{n}{2}$.

Definition 2.5. [43] The graph $C(n,k,t)$ has vertices, respectively, edges given by

$$V(C(n,k,t)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

$$E(C(n,k,t)) = \{u_iu_{i+k}, u_iv_i, v_iv_{i+t}/0 \leq i \leq n-1\},$$

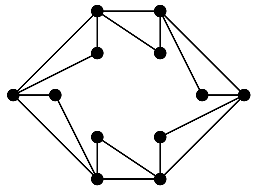
where the subscripts are expressed as integers modulo $n > 4$ and $t, k < \frac{n}{2}$.

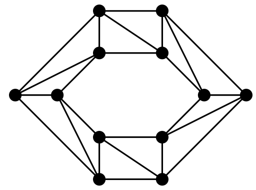
Definition 2.6. [43] The graph $PC(n,k,t)$ has vertices, respectively, edges given by

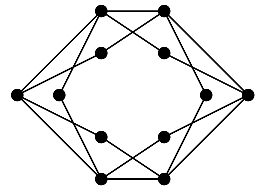
$$V(PC(n,k,t)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

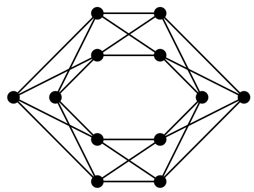
$$E(PC(n,k,t)) = \{u_iv_j, u_iv_i, v_iv_{i+k}/0 \leq i, j \leq n-1\},$$

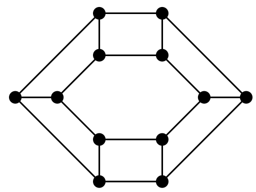
where $j \notin \{i+n-t, i+t\}$ and the subscripts are expressed as integers modulo $n > 4$ and $t, k < \frac{n}{2}$.

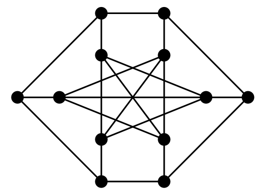

(a) $M(6,1)$


(b) $T(6,1,1)$


(c) $N(6,1,1)$


(d) $D(6,1,1,1)$


(e) $C(6,1,1)$


(f) $CC(6,1,1)$

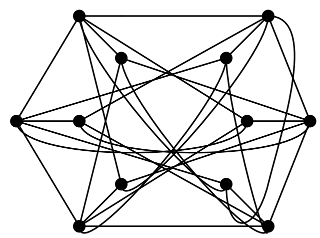

(g) $PC(6,1,1)$

FIGURE 1. Generalized graphs related to cycle.

Definition 2.7. [43] The graph $CC(n,k,t)$ has vertices, respectively, edges given by

$$V(CC(n,k,t)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

$$E(CC(n,k,t)) = \{u_iu_j, u_iv_i, v_iv_{i+k}/0 \leq i,j \leq n-1\},$$

where $j \notin \{i+n-t, i+t\}$ and the subscripts are expressed as integers modulo $n > 4$ and $t, k < \frac{n}{2}$.

Gera et al. described the spectrum of the generalized petersen graph [8]. The work is motivated from a work on the spectrum of the generalized graphs related to cycle [43] in which the authors describe the spectrum of the generalized graphs related to cycle in terms of the eigenvalues of a cycle.

We propose the following definitions of graphs which are all new in literature.

Definition 2.8. The graph $P(n,k,t)$ has vertices, respectively, edges given by

$$V(P(n,k,t)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

$$E(P(n,k,t)) = \{u_iu_j, u_iv_i, u_iv_{i+1}, v_iv_{i+k}/0 \leq i,j \leq n-1\},$$

where $j \notin \{i+n-t, i+t\}$ and the subscripts are expressed as integers modulo $n > 4$ and $k < \frac{n}{2}$.

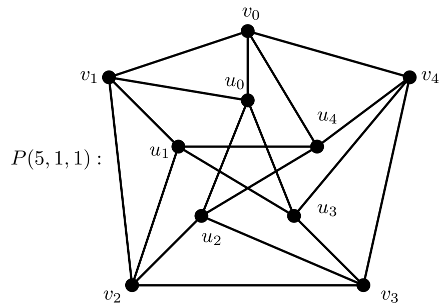


FIGURE 2. The graph $P(5, 1, 1)$.

Definition 2.9. The graph $Q(n, k, t)$ has vertices, respectively, edges given by

$$V(Q(n, k, t)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

$$E(Q(n, k, t)) = \{u_i u_j, u_i v_i, u_i v_{i+1}, v_i v_{i+k}/0 \leq i, j \leq n-1\},$$

where $j \in \{i+1, i+2, ..., n-1\}$ and the subscripts are expressed as integers modulo $n > 4$ and $k < \frac{n}{2}$.

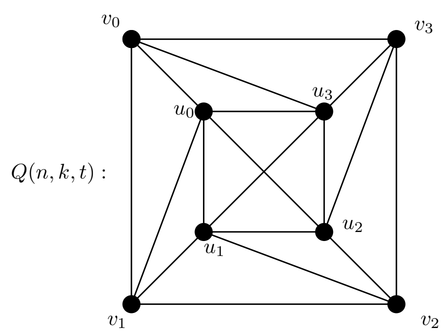


FIGURE 3. The graph $Q(4, 1, 1)$.

Definition 2.10. The graph $S(n, k)$ has vertices, respectively, edges given by

$$V(S(n, k)) = \{u_i, v_i/0 \leq i \leq n-1\},$$

$$E(S(n, k)) = \{u_i u_j, u_i v_i, u_i v_{i+1}/0 \leq i \leq n-1\},$$

where $j \in \{i+n-k, i+k\}$ and the subscripts are expressed as integers modulo $n > 4$ and $k < \frac{n}{2}$.

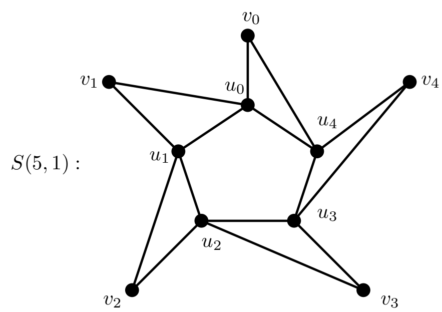


FIGURE 4. The graph $S(5,1)$.

We require the following definitions for further results:

The *quasi-total graph* $P(G)$ [45] of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if and only if they correspond to two nonadjacent vertices of G or to two adjacent edges of G or one is a vertex and other is an edge incident with it in G.

The *quasivertex-total graph* $T_{qv}(G)$ [33] of graph G is the graph whose point set is the union of the set of points and the set of lines of G in which two points are adjacent if and only if they correspond to two non adjacent points of G or to two adjacent lines of G or to two adjacent points of G or to a point and a line incident to it in G.

The *semitotal point graph* $T_2(G)$ of a graph G [46] is defined as a graph with vertex set $V(G) \cup E(G)$ where two vertices of $T_2(G)$ are adjacent if and only if they correspond to two adjacent vertices of G or one is a vertex of G and another is an edge G incident with it in G.

Now, we define the following definitions:

Definition 2.11. The quasi-total line complement graph $QP(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if and only if they correspond to two non adjacent vertices of G or one is a vertex and other is an edge incident with it in G.

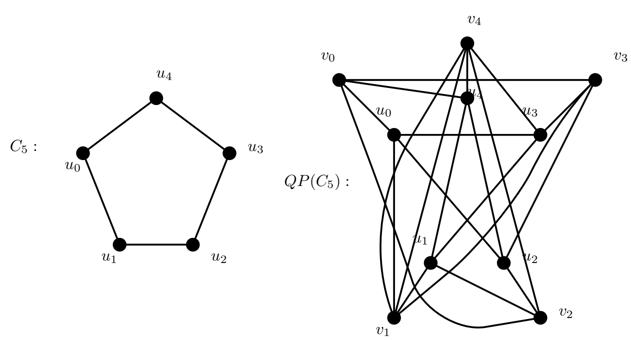


FIGURE 5. The grph C_5 and $QP(C_5)$.

Definition 2.12. The quasi-vertex total line complement graph $QT_{qv}(G)$ of a graph is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if and only if they correspond to two non adjacent vertices of G or to two non adjacent edges of G or one is a vertex and other is an edge incident with it in G.

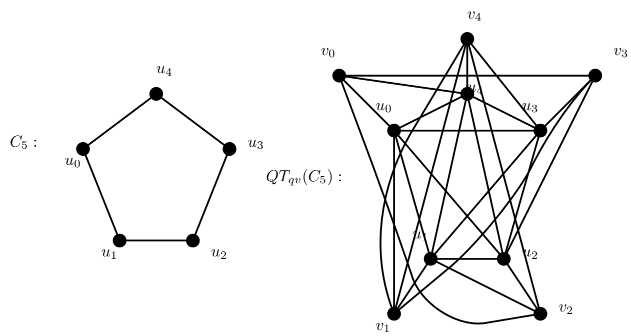


FIGURE 6. The grph C_5 and $QT_{qv}(C_5)$.

Definition 2.13. The quasi-semi total point graph of a graph $QT_2(G)$ is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if and only if they correspond to two non adjacent vertices of G or one is a vertex and other is an edge incident with its in G.

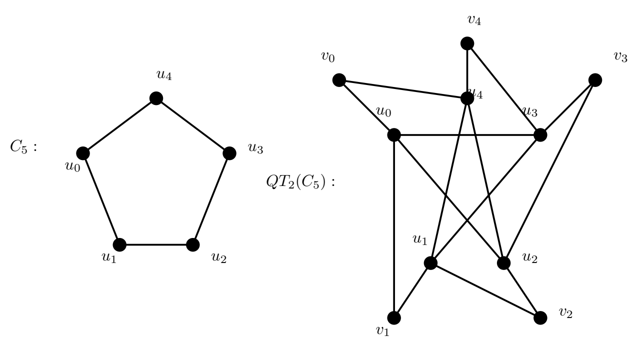


FIGURE 7. The grph C_5 and $QT_2(C_5)$.

Note that when $k = 1$ and $k = 2$, $P(n, k, t)$ is a quasi-total graph of C_n and quasi-total line complement graph of C_n, respectively, when $k = 1$ and $k = 2$, $Q(n, k, t)$ is a quasivertex-total graph of C_n and quasivertex-total line complement graph of C_n, respectively, when $k = 1$ and $k = 2$, $S(n, k)$ is a semitotal-point graph of C_n and quasi-semi total point graph of C_n, respectively.

Definition 2.14. An $n \times n$ matrix is called circulant if it can be written in the form

$$\begin{bmatrix} b_1 & b_2 & b_3 & \cdots & b_n \\ b_n & b_1 & b_2 & \cdots & b_{n-1} \\ b_{n-1} & b_n & b_1 & \cdots & b_{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_2 & b_3 & b_4 & \cdots & b_1 \end{bmatrix}$$

and is denoted by $circ(b_1, b_2, ..., b_n)$.

In the above definitions let $A(n,k)$ and $B(n,k)$ respectively denote the subgraphs induced by the edges $A = \{u_iu_{i+1}, i = 0,1,2,...,n-1\}$ and $B = \{v_iv_{i+k}, i = 0,1,2,...,n-1\}$. Then the adjacency matrix of $A(n,k)$ is the adjacency matrix of the cycle C_n which is the circulant C_n with the first row [0 1 0...0 0 1] and that of $B(n,k)$ is the circulant C_n^k with the first row [0.....0 1 0.....0 1 0.....0].

We require this result to prove our main results.

Lemma 2.15. [7] *Let G be an $r-$regular graph with an adjacency matrix A and an incidence matrix R. Then $RR^T = A + rI$ so that for any eigen vector v of A with an eigenvalue λ, $RR^Tv = (\lambda + 2)v$.*

Theorem 2.16. [24] *Let G be an r-regular graph with $n \geq 2$ vertices. Then*

(i) $KA^1_{(a,b)}(G) = 2^{b-1}nr^{ab+1}$,
(ii) $KA^2_{(a,b)}(G) = \frac{1}{2}nr^{2ab+1}$,
(iii) $KA^3_{(a,b)}(G) = 0$.

3. Main Results

3.1. Spectrum of new generalized graphs related to cycle in terms of the eigenvalues of a cycle.

Theorem 3.1. *The eigenvalues of $P(n,k,t)$ are all roots of the quadratic equation*

$$\delta^2 - \delta[-\gamma_j + \mu_j + (J - I)] + \mu_j(J - I - \mu_j) - (\lambda_j + 2) \;\;=\;\; 0,$$

where $\lambda_j = 2cos\left(\frac{2\pi j}{n}\right)$, $\mu_j = 2\ cos\left(\frac{2\pi jk}{n}\right)$ and $\gamma_j = 2\ cos\left(\frac{2\pi jt}{n}\right)$, $0 \leq i \leq n-1$ are the eigenvalues of C_n, C_n^k and C_n^t, respectively.

Proof. By a proper ordering of vertices of $P(n,k,t)$, an adjacency matrix of it $A(P(n,k,t))$ can be written as

$$A(P(n,k,t)) = \begin{bmatrix} C_n^k & R \\ R^T & J - I - C_n^t \end{bmatrix}$$

where R is the vertex-edge incidence matrix of C_n. Let X_j be the eigen vector of C_n with an eigenvalue $\lambda_j = 2\ cos\left(\frac{2\pi j}{n}\right)$. Then as per the above note, X_j is an eigen vector of C_n^k with an eigenvalue $\mu_j = 2\ cos\left(\frac{2\pi jk}{n}\right)$. Now we get the desired result. □

Theorem 3.2. *The eigenvalues of $Q(n,k,t)$ are all roots of the quadratic equation*

$$\delta^2 - \delta(J - I) + \mu_j(J - I) - (\lambda_j + 2) \;\;=\;\; 0,$$

where $\lambda_j = 2\ cos\left(\frac{2\pi j}{n}\right)$, $\mu_j = 2\ cos\left(\frac{2\pi jk}{n}\right)$ and $\gamma_j = 2\ cos\left(\frac{2\pi jt}{n}\right)$, $0 \leq i \leq n-1$ are the eigenvalues of C_n, C_n^k and C_n^t, respectively.

Proof. By a proper ordering of vertices of $Q(n,k,t)$, an adjacency matrix of it $Q(P(n,k,t))$ can be written as

$$A(Q(n,k,t)) = \begin{bmatrix} C_n^k & R \\ R^T & J - I \end{bmatrix}$$

where R is the vertex-edge incidence matrix of C_n. Let X_j be the eigen vector of C_n with an eigenvalue $\lambda_j = 2\ cos\left(\frac{2\pi j}{n}\right)$. Then as per the above note, X_j is an eigen vector of C_n^k with an eigenvalue $\mu_j = 2\ cos\left(\frac{2\pi jk}{n}\right)$. Now we get the desired result. □

Theorem 3.3. *The eigenvalues of $S(n, k)$ are all roots of the quadratic equation*

$$\delta^2 - \delta\mu_j - (\lambda_j + 2) \quad = \quad 0,$$

where $\lambda_j = 2\ cos\left(\frac{2\pi j}{n}\right)$ and $\mu_j = 2\ cos\left(\frac{2\pi jk}{n}\right)$, $0 \leq i \leq n-1$ are the eigenvalues of C_n and C_n^k, respectively.

Proof. By a proper ordering of vertices of $S(n, k)$, an adjacency matrix of it $S(n, k)$ can be written as

$$A(S(n,k)) = \begin{bmatrix} 0 & R \\ R^T & C_n^k \end{bmatrix}$$

where R is the vertex-edge incidence matrix of C_n. Let X_j be the eigen vector of C_n with an eigenvalue $\lambda_j = 2\ cos\left(\frac{2\pi j}{n}\right)$. Then as per the above note, X_j is an eigen vector of C_n^k with an eigenvalue $\mu_j = 2\ cos\left(\frac{2\pi jk}{n}\right)$. Now we get the desired result. □

3.2. **(a, b)-KA indices of some generalized graphs related to cycle.**

Theorem 3.4. *If G is a regular graph with n vertices. Then*

(i) $KA^1_{(a,b)}(G) = \frac{nr}{2} x^{[r^a + r^a]^b}$,
(ii) $KA^2_{(a,b)}(G) = \frac{nr}{2} x^{[r^{2ab}]}$,
(iii) $KA^3_{(a,b)}(G) = \frac{nr}{2}$.

Proof. Let G be a regular graph. We obtain two partition of edge sets of G as follows:
$E_1 = \{uv \in E(G)/d_G(u) = r, d_G(v) = r\}$, $|E_1| = \frac{nr}{2}$,
By the definitions of (a, b)-KA indices and edge partitions of G, we have
(i) The first (a, b)-KA polynomial is

$$\begin{aligned} KA^1_{(a,b)}(G) &= \sum_{uv \in E(G)} x^{[d_G(u)^a + d_G(v)^a]^b} \\ &= \frac{nr}{2} x^{[r^a + r^a]^b}. \end{aligned}$$

(ii) The second (a, b)-KA polynomial is

$$\begin{aligned} KA^2_{(a,b)}(G) &= \sum_{uv \in E(G)} x^{[d_G(u)^a \cdot d_G(v)^a]^b} \\ &= \frac{nr}{2} x^{[r^a \cdot r^a]^b} \\ &= \frac{nr}{2} x^{[r^{2ab}]}. \end{aligned}$$

(iii) The third (a, b)-KA polynomial is

$$\begin{aligned} KA^3_{(a,b)}(G) &= \sum_{uv\in E(G)} x^{|d_G(u)^a - d_G(v)^a|^b} \\ &= \frac{nr}{2} x^{|r^a - r^a|^b} \\ &= \frac{nr}{2}. \end{aligned}$$

□

$(d_G(u), d_G(v))$ **where** $uv \in E(G)$	**Number of edges**
$(2, 4)$	$2n$
$(4, 4)$	n

TABLE 1. Edge partition of $M(n, k)$ graph.

Theorem 3.5. *If G is $M(n, k)$ graph with $2n$ vertices. Then*
(i) $KA^1_{(a,b)}(G) = 2n[2^a + 4^a]^b + n2^b4^{ab}$,
(ii) $KA^2_{(a,b)}(G) = n[2^{3ab+1} + 4^{2ab}]$,
(iii) $KA^3_{(a,b)}(G) = 2n|2^a - 4^a|^b$.

Proof. Let $G = M(n, k)$. By the definitions of (a, b)-KA indices and Table 1, we have
(i) The first (a, b)-KA index is

$$\begin{aligned} KA^1_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a + d_G(v)^a]^b \\ &= 2n[2^a + 4^a]^b + n[4^a + 4^a]^b \\ &= 2n[2^a + 4^a]^b + n2^b4^{ab}. \end{aligned}$$

(ii) The second (a, b)-KA index is

$$\begin{aligned} KA^2_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a \cdot d_G(v)^a]^b \\ &= 2n[2^a \cdot 4^a]^b + n[4^a \cdot 4^a]^b \\ &= n[2^{3ab+1} + 4^{2ab}]. \end{aligned}$$

(iii) The third (a, b)-KA index is

$$\begin{aligned} KA^3_{(a,b)}(G) &= \sum_{uv\in E(G)} |d_G(u)^a - d_G(v)^a|^b \\ &= 2n|2^a - 4^a|^b + n|4^a - 4^a|^b \\ &= 2n|2^a - 4^a|^b. \end{aligned}$$

□

Theorem 3.6. *If G is $T(n, k, t)$ graph with $2n$ vertices. Then*
(i) $KA^1_{(a,b)}(G) = n2^{b+2(ab+1)}$,

(ii) $KA^{2}_{(a,b)}(G) = n4^{2ab+1}$,
(iii) $KA^{3}_{(a,b)}(G) = 0$.

Proof. Let $G = T(n,k,t)$ is a 4-regular graph. By using Theorem 2.16, we get the required result. □

Theorem 3.7. *If G is $T(n,k,t)$ graph with $2n$ vertices. Then*
(i) $KA^{1}_{(a,b)}(G,x) = nx^{2^{b+2(ab+1)}}$,
(ii) $KA^{2}_{(a,b)}(G,x) = nx^{4^{2ab+1}}$,
(iii) $KA^{3}_{(a,b)}(G,x) = 4n$.

Proof. Let $G = T(n,k,t)$ is a 4-regular graph. By using Theorem 3.4, we get the required result. □

$(d_G(u), d_G(v))$ **where** $uv \in E(G)$	**Number of edges**
$(1,3)$	n
$(3,3)$	n

TABLE 2. Edge partition of $N(n,k,t)$ graph.

Theorem 3.8. *If G is $N(n,k,t)$ graph with $2n$ vertices. Then*
(i) $KA^{1}_{(a,b)}(G) = n[1+3^a]^b + n2^b3^{ab}$,
(ii) $KA^{2}_{(a,b)}(G) = n[3^{ab} + 3^{2ab}]$,
(iii) $KA^{3}_{(a,b)}(G) = n|1-3^a|^b$.

Proof. Let $G = N(n,k,t)$. By the definitions of (a, b)-KA indices and Table 2, we have
(i) The first (a, b)-KA index is

$$\begin{aligned} KA^{1}_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a + d_G(v)^a]^b \\ &= n[1^a+3^a]^b + n[3^a+3^a]^b \\ &= n[1+3^a]^b + n2^b3^{ab}. \end{aligned}$$

(ii) The second (a, b)-KA index is

$$\begin{aligned} KA^{2}_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a \cdot d_G(v)^a]^b \\ &= n[1^a\cdot 3^a]^b + n[3^a\cdot 3^a]^b \\ &= n[3^{ab} + 3^{2ab}]. \end{aligned}$$

(iii) The third (a, b)-KA index is

$$\begin{aligned} KA^{3}_{(a,b)}(G) &= \sum_{uv\in E(G)} |d_G(u)^a - d_G(v)^a|^b \\ &= n|1^a-3^a|^b + n|3^a-3^a|^b \\ &= n|1-3^a|^b. \end{aligned}$$

□

Theorem 3.9. *If G is $C(n,k,t)$ graph with $2n$ vertices. Then*

(i) $KA^1_{(a,b)}(G) = n2^b3^{ab+1}$,
(ii) $KA^2_{(a,b)}(G) = n[3^{2ab+1}]$,
(iii) $KA^3_{(a,b)}(G) = 0$.

Proof. Let $G = C(n,k,t)$ is a 3-regular graph. By using Theorem 2.16, we get the required result. □

$(d_G(u), d_G(v))$ **where** $uv \in E(G)$	**Number of edges**
$(n, n-2)$	$n^2 - 2n$
(n, n)	n

TABLE 3. Edge partition of $PC(n,k,t)$ graph.

Theorem 3.10. *If G is $PC(n,k,t)$ graph with $2n$ vertices. Then*

(i) $KA^1_{(a,b)}(G) = n[1+3^a]^b + n2^b3^{ab}$,
(ii) $KA^2_{(a,b)}(G) = n[3^{ab} + 3^{2ab}]$,
(iii) $KA^3_{(a,b)}(G) = n|1-3^a|^b$.

Proof. Let $G = PC(n,k,t)$. By the definitions of (a, b)-KA indices and Table 3, we have
(i) The first (a, b)-KA index is

$$\begin{aligned} KA^1_{(a,b)}(G) &= \sum_{uv \in E(G)} [d_G(u)^a + d_G(v)^a]^b \\ &= (n^2-2n)[n^a + (n-2)^a]^b + n[n^a + n^a]^b \\ &= (n^2-2n)[n^a + (n-2)^a]^b + n[2^b n^{ab}]. \end{aligned}$$

(ii) The second (a, b)-KA index is

$$\begin{aligned} KA^2_{(a,b)}(G) &= \sum_{uv \in E(G)} [d_G(u)^a \cdot d_G(v)^a]^b \\ &= n[1^a \cdot 3^a]^b + n[3^a \cdot 3^a]^b \\ &= n[3^{ab} + 3^{2ab}]. \end{aligned}$$

(iii) The third (a, b)-KA index is

$$\begin{aligned} KA^3_{(a,b)}(G) &= \sum_{uv \in E(G)} |d_G(u)^a - d_G(v)^a|^b \\ &= n|1^a - 4^a|^b + n|3^a - 3^a|^b \\ &= n|1 - 4^a|^b. \end{aligned}$$

□

$(d_G(u), d_G(v))$ **where** $uv \in E(G)$	**Number of edges**
$(3,3)$	n
$(3, n-2)$	n
$(n-2, n-2)$	$\frac{n^2-3n}{2}$

TABLE 4. Edge partition of $CC(n,k,t)$ graph.

Theorem 3.11. *If G is $CC(n,k,t)$ graph with $2n$ vertices. Then*
(i) $KA^1_{(a,b)}(G) = n[3^a + 3^a]^b + [3^a + (n-2)^a]^b + \frac{n^2-3n}{2}[(n-2)^a + (n-2)^a]$,
(ii) $KA^2_{(a,b)}(G) = n3^{ab}[3^{ab} + n(n-2)]^{ab} + \frac{n^2-3n}{2}(n-2)^{2ab}$,
(iii) $KA^3_{(a,b)}(G) = n|3^a - (n-2)^a|^b$.

Proof. Let $G = CC(n,k,t)$. By the definitions of (a, b)-KA indices and Table 4, we have
(i) The first (a, b)-KA index is

$$\begin{aligned} KA^1_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a + d_G(v)^a]^b \\ &= n[3^a + 3^a]^b + n[3^a + (n-2)^a]^b + \frac{n^2-3n}{2}[(n-2)^a + (n-2)^a] \\ &= n[3^a + 3^a]^b + [3^a + (n-2)^a]^b + \frac{n^2-3n}{2}[(n-2)^a + (n-2)^a]. \end{aligned}$$

(ii) The second (a, b)-KA index is

$$\begin{aligned} KA^2_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a \cdot d_G(v)^a]^b \\ &= n[3^a \cdot 3^a]^b + n[3^a \cdot (n-2)^a]^b + \frac{n^2-3n}{2}[(n-2)^a \cdot (n-2)^a]^b \\ &= n3^{ab}[3^{ab} + n(n-2)]^{ab} + \frac{n^2-3n}{2}(n-2)^{2ab}. \end{aligned}$$

(iii) The third (a, b)-KA index is

$$\begin{aligned} KA^3_{(a,b)}(G) &= \sum_{uv\in E(G)} |d_G(u)^a - d_G(v)^a|^b \\ &= n|3^a - 3^a|^b + n|3^a - (n-2)^a|^b + \frac{n^2-3n}{2}|(n-2)^a - (n-2)^a|^b \\ &= n|3^a - (n-2)^a|^b. \end{aligned}$$

□

$(d_G(u), d_G(v))$ **where** $uv \in E(G)$	**Number of edges**
$(4,4)$	n
$(4, n-1)$	$2n$
$(n-1, n-1)$	$\frac{n^2-3n}{2}$

TABLE 5. Edge partition of $P(n,k,t)$ graph.

Theorem 3.12. *If G is $P(n,k,t)$ graph with $2n$ vertices. Then*
(i) $KA^1_{(a,b)}(G) = n2^b4^{ab} + 2n[4^a + (n-1)^a]^b + 2^{b-1}(n^2 - 3n)(n-1)^{ab}$,
(ii) $KA^2_{(a,b)}(G) = n4^{2ab} + 2n[4^{ab} \cdot (n-1)]^{ab} + \frac{n^2-3n}{2}(n-1)^{2ab}$,
(iii) $KA^3_{(a,b)}(G) = 2n|4^a - (n-1)^a|^b$.

Proof. Let $G = P(n,k,t)$. By the definitions of (a, b)-KA indices and Table 5, we have
(i) The first (a, b)-KA index is

$$\begin{aligned}
KA^1_{(a,b)}(G) &= \sum_{uv \in E(G)} [d_G(u)^a + d_G(v)^a]^b \\
&= n[4^a + 4^a]^b + 2n[4^a + (n-1)^a]^b + \frac{n^2-3n}{2}[(n-1)^a + (n-1)^a]^b \\
&= n[2(4^a)]^b + [4^a + (n-1)^a]^b + \frac{n^2-3n}{2}[2(n-1)^a]^b.
\end{aligned}$$

(ii) The second (a, b)-KA index is

$$\begin{aligned}
KA^2_{(a,b)}(G) &= \sum_{uv \in E(G)} [d_G(u)^a \cdot d_G(v)^a]^b \\
&= n[4^a \cdot 4^a]^b + 2n[4^a \cdot (n-1)^a]^b + \frac{n^2-3n}{2}[(n-1)^a \cdot (n-1)^a]^b \\
&= n4^{2ab} + 2n[4(n-1)]^{ab} + \frac{n^2-3n}{2}(n-1)^{2ab}.
\end{aligned}$$

(iii) The third (a, b)-KA index is

$$\begin{aligned}
KA^3_{(a,b)}(G) &= \sum_{uv \in E(G)} |d_G(u)^a - d_G(v)^a|^b \\
&= n|4^a - 4^a|^b + 2n|4^a - (n-1)^a|^b + \frac{n^2-3n}{2}|(n-1)^a - (n-1)^a|^b \\
&= 2n|4^a - (n-1)^a|^b.
\end{aligned}$$

□

$(d_G(u), d_G(v))$ **where** $uv \in E(G)$	**Number of edges**
$(4,4)$	n
$(4, n+1)$	$2n$
$(n+1, n+1)$	$\frac{n(n-1)}{2}$

TABLE 6. Edge partition of $Q(n,k,t)$ graph.

Theorem 3.13. *If G is $Q(n,k,t)$ graph with $2n$ vertices. Then*
(i) $KA^1_{(a,b)}(G) = n2^b4^{ab} + 2n[4^a + (n+1)^a]^b + \frac{n(n-1)}{2}(n+1)^{ab}$,
(ii) $KA^2_{(a,b)}(G) = n4^{2ab} + 2n[4^{ab} \cdot (n+1)]^{ab} + \frac{n(n+1)}{2}(n-1)^{2ab}$,
(iii) $KA^3_{(a,b)}(G) = 2n|4^a - (n+1)^a|^b$.

Proof. Let $G = Q(n,k,t)$. By the definitions of (a, b)-KA indices and Table 6, we have
(i) The first (a, b)-KA index is

$$\begin{aligned} KA^1_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a + d_G(v)^a]^b \\ &= n[4^a + 4^a]^b + 2n[4^a + (n+1)^a]^b + \frac{n(n-1)}{2}[(n+1)^a + (n+1)^a]^b \\ &= n[2(4^a)]^b + [4^a + (n+1)^a]^b + \frac{n(n-1)}{2}[2(n+1)^a]^b. \end{aligned}$$

(ii) The second (a, b)-KA index is

$$\begin{aligned} KA^2_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a \cdot d_G(v)^a]^b \\ &= n[4^a \cdot 4^a]^b + 2n[4^a \cdot (n+1)^a]^b + \frac{n(n-1)}{2}[(n+1)^a \cdot (n+1)^a]^b \\ &= n4^{2ab} + 2n[4(n+1)]^{ab} + \frac{n(n-1)}{2}(n+1)^{2ab}. \end{aligned}$$

(iii) The third (a, b)-KA index is

$$\begin{aligned} KA^3_{(a,b)}(G) &= \sum_{uv\in E(G)} |d_G(u)^a - d_G(v)^a|^b \\ &= n|4^a - 4^a|^b + 2n|4^a - (n+1)^a|^b + \frac{n(n-1)}{2}|(n+1)^a - (n+1)^a|^b \\ &= 2n|4^a - (n+1)^a|^b. \end{aligned}$$

□

$(d_G(u), d_G(v))$ **where** $uv \in E(G)$	**Number of edges**
$(2,4)$	$2n$
$(4,4)$	n

TABLE 7. Edge partition of $Q(n,k,t)$ graph.

Theorem 3.14. *If G is $S(n,k)$ graph with $2n$ vertices. Then*

(i) $KA^1_{(a,b)}(G) = 2n[2^a + 4^a]^b + n[4^a + 4^a]^b$,
(ii) $KA^2_{(a,b)}(G) = 2n2^{3ab} + n4^{2ab}$,
(iii) $KA^3_{(a,b)}(G) = 2n|2^a - 4^a|^b$.

Proof. Let $G = S(n,k)$. By the definitions of (a, b)-KA indices and Table 6, we have

(i) The first (a, b)-KA index is

$$\begin{aligned} KA^1_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a + d_G(v)^a]^b \\ &= 2n[2^a + 4^a]^b + n[4^a + 4^a]^b \\ &= 2n[2^a + 4^a]^b + n[4^a + 4^a]^b. \end{aligned}$$

(ii) The second (a, b)-KA index is

$$\begin{aligned} KA^2_{(a,b)}(G) &= \sum_{uv\in E(G)} [d_G(u)^a \cdot d_G(v)^a]^b \\ &= 2n[2^a \cdot 4^a]^b + n[4^a \cdot 4^a]^b \\ &= 2n2^{3ab} + n4^{2ab}. \end{aligned}$$

(iii) The third (a, b)-KA index is

$$\begin{aligned} KA^3_{(a,b)}(G) &= \sum_{uv\in E(G)} |d_G(u)^a - d_G(v)^a|^b \\ &= 2n|2^a - 4^a|^b + n|4^a - 4^a|^b \\ &= 2n[2^a + 4^a]^b + n[4^a + 4^a]^b. \end{aligned}$$

□

4. Conclusion

In this paper, we have defined three new classes namely $P(n, k, t)$, $Q(n, k, t)$ and $S(n, k)$, obtained their adjacency spectrum and thus adding them to the classes of graphs whose spectrum is completely known. Further, we have computed (a, b)-KA indices of some generalized graphs related to cycle.

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