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VISCOUS DISSIPATION, RADIATION, HEAT SOURCE (SINK), PRESSURE WORK AND MHD EFFECTS IN BOUNDARY LAYERS ON CONTINUOUS MOVING SURFACE ALONG WITH TEMPERATURE DEPENDENT VISCOSITY

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Abstract. This work is focused on steady, laminar, natural convective boundary layer flow over a continuous moving surface in the presence of MHD, pressure work, viscous dissipation, heat source (sink), radiation and temperature dependent viscosity effects. A suitable set of dimensionless variables is used and nonsimilar equations governing the problem are obtained. An adequate, implicit, tridiagonal infinite difference scheme known as quasilinearization is utilized for the mathematical solution of the obtained equations. The outcomes are found to be in astounding agreement for the above said parameters in terms of the velocity, temperature, local Nusselt number and the local skin-friction coefficient which is shown graphically.

1. Introduction

A few engineering applications include boundary layer flows on a continuously moving surface. Aerodynamic extrusion of a sheet of plastic, condensation processes, the cooling of an infinite metal plate in a cooling bath, and geophysical and industrial fields, are instances of useful applications. Since the spearheading work of Sakiadis [1, 2], different parts of the issue have been examined by numerous creators. The majority of studies have focused on maintaining fixed velocity and temperature surface (See Tsou et al. [3]), but in some real world applications, the surface is stretched and then cooled or heated, causing temperature and surface velocity variations. Some researchers [4-6] have examined the stretching problem with fixed temperature surface, while Soundalgekar et.al [7] explored the fixed velocity surface condition with a power-law temperature variance.
The fluid viscosity was thought to be constant in all of the studies described above. Anyway it is realized that this physical property is susceptible to temperature changes. To precisely anticipate the flow behavior, it is important to consider this variety of viscosity. In comparison to the fixed viscosity state, results [8, 9] show that when this effect is taken into account, the flow characteristics

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can be significantly altered.
There has been extraordinary enthusiasm for the investigation of MHD flow and heat transfer in any medium because of the impact of applied magnetic field on the control of the boundary layer flow and also on the output of several systems utilizing electrically conductive fluids. Such a stream has snatched the interest of several investigators [10-13] because of its applications in MHD generators, plasma studies, crystal growth and design for cooling of nuclear reactors.
Radiation is energy that comes from a source and travels through some material or through space. Light, heat and sound are types of radiation. Radiation is considered in this study due to the fact that thermal radiation effect might play a significant role in controlling heat transfer process in polymer processing industry. Many new engineering processes such as fossil fuel combustion energy processes, solar power technology, astrophysical flows, gas turbines and the various propulsion devises for aircraft, missiles, satellites, and space vehicle re-entry occur at high temperature. So knowledge of radiation plays a very important role and hence, its effect cannot be neglected. Also thermal radiation is of major importance in many processes in engineering areas which occur at a high temperature for the design of many advance energy conversion systems and pertinent equipment. In the energy equation, the radiative heat flux is represented using the Rosseland approximation. Radiation effects on heat and mass transfer over an unsteady stretching surface were concentrated by M. A. Seddeek [14]. Recently, Stanford Shateyi [15] and Kalidas Das [16] considered the impact of MHD and radiation over a moving surface.
The effect of viscous dissipation, pressure work and heat source (sink), is not considered in all the above published works. Even at with reasonable velocities, viscous dissipation has a major impact on heat transfer, particularly for deeply viscous flows. It changes the kinetic energy into internal energy (warming up the fluid) because of viscosity and subsequently builds the smooth movement. Considering this explanation, different gadgets are planned in streambeds to decrease the kinetic energy of streaming water to diminishing their erosive potential on banks and waterway bottoms. Due to this, the dimensionless parameter Eckert number is called the fluid motion controlling parameter. The conversation and investigation of free convection flows and pressure work impacts are for the most part disregarded yet here we have thought about the impact of pressure work on a natural convective flow along a continuous moving surface.
In addition, the impact of heat source (sink) in thermal convection is huge where more temperature variations exist in between fluid and the surface. It is additionally significant with regards to endothermic or exothermic chemical responses. Feasible heat generation additionally modifies the temperature dissemination; thus the element deposition rate in atomic reactors, semiconductor wafers and electronic chips.
The present work is to study the influence of viscous dissipation, pressure work, MHD, radiation and heat source (sink) on the heat transfer over a continuous moving surface with temperature dependent viscosity.

2. Mathematical formulation

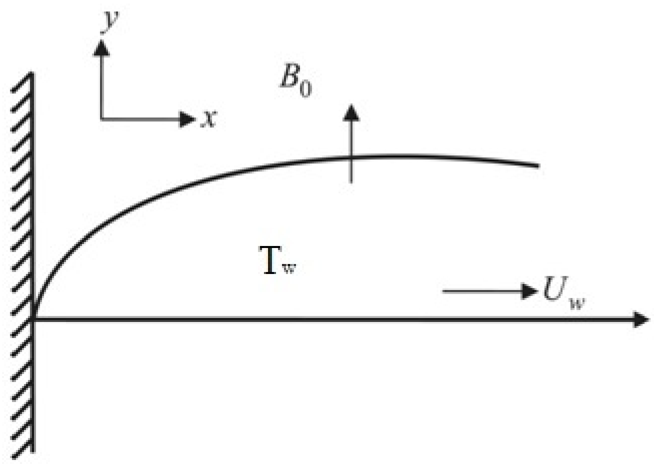


Fig. 1. Boundary layer on moving continuous surface.

Consider a two-dimensional laminar incompressible electrically conducting fluid flow traveling axially through a fixed fluid on a continuous, stretching surface with uniform surface temperatureT_w and velocity U_w .The coordinate system and flow model of the problem is shown in fig.1. The x-axis runs along the continuous surface in the direction of the motion and y-axis is perpendicular to it. The induced magnetic field (B_0) is assumed to be uniform and is in the positive direction normal to the surface. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field.

With the exception of fluid viscosity (μ), which is thought to be an inverse linear function of the temperature (T) [17], the fluid is accepted to have constant physical properties.

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \gamma(T - T_\infty)], \quad \frac{1}{\mu} = a(T - T_e)$$
$$here \quad a = \frac{\gamma}{\mu_\infty}, \quad T_e = T_\infty - \frac{1}{\gamma} \tag{2.1}$$

Where, both T_e and $'a'$ are constants and their values rely upon the reference state and fluid thermal property i.e., (γ). All in all, $a < 0$ for gases and $a > 0$ for liquids neglecting the impacts of transverse curvature, the boundary layer equations governing the flow are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0(2.2)$$
$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2 u}{\rho}(2.3)$$
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho_\infty C_p}\frac{\partial^2 T}{\partial y^2} - \frac{T\beta u}{\rho C_p}\frac{\partial p}{\partial x} + \frac{k}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + Q(T - T_\infty) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}(2.4)$$

The initial and boundary conditions are given by

$$\left.\begin{array}{c} y = 0, v = 0, u = U_w, T = T_w \\ y \to \infty, u = 0,\ T = T_\infty \end{array}\right\} \tag{2.5}$$

where $\frac{\partial p}{\partial x} = \rho g$ is the hydrostatic pressure.
The radiative heat flux is generalized by utilizing the Rosseland definition for radiation[18]

$$q_r = -\frac{4\sigma}{3\alpha^*}\frac{\partial T^4}{\partial y} \tag{2.6}$$

Accepting that the temperature inside the flows contrasts is one in which the term T^4 might be communicated as a temperature linear function. Thus, we get in a Taylor sequence regarding T_∞ expanding T^4 and disregarding higher level terms:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{2.7}$$

Utilizing eqn. (2.6) and (2.7), (2.4) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho_\infty C_p}\frac{\partial^2 T}{\partial y^2} - \frac{T\beta u}{\rho C_p}\frac{\partial p}{\partial x} + \frac{k}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + Q(T - T_\infty) - \frac{16\sigma T_\infty^3}{3\alpha^* \rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{2.8}$$

where u and v are the x and y -direction velocity components, T is the temperature within the boundary layer, μ is the viscosity, ρ_∞ is the density, k is the thermal conductivity, Q is the volumetric rate of heat generation, T_∞ is the free stream temperature and C_p is the specific heat at constant pressure .
Further,

$$\left.\begin{array}{c} u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x}, \\ \nu_\infty = \frac{\mu_\infty}{\rho_\infty},\ \eta = y\sqrt{\frac{U_w}{2\nu_\infty x}},\ T - T_\infty = (T_w - T_\infty)G, \\ \psi = \sqrt{2\nu_\infty U_w x}\, f(\eta), \\ f' = F = \frac{\partial f}{\partial \eta},\ \frac{\mu}{\mu_\infty} = \frac{G_e}{G_e - G},\ G = \frac{T - T_e}{T_w - T_\infty} + G_e \\ where\ G_e = \frac{T_e - T_\infty}{T_w - T_\infty} = \frac{-1}{\gamma(T_w - T_\infty)}; \end{array}\right\} \tag{2.9}$$

By replacing the above transformations in (2.2),(2.3) and (2.8), we obtain:

$$F'' + \left(1 - \frac{G}{G_e}\right) fF' - \left(1 - \frac{G}{G_e}\right) Mf + \frac{G'}{(G_e - G)}F' = 0 \tag{2.10}$$

$$\left(1 + N_R\right) G'' + fG'Pr - \epsilon GFPr + Pr\lambda G + PrEcF'^2 = 0 \tag{2.11}$$

The transformed boundary conditions are:

$$\left.\begin{array}{lll} F = 1; & G = 1 & \text{at} \quad \eta = 0 \\ F = 0; & G = 0 & \text{as} \quad \eta \to \infty \end{array}\right\} \tag{2.12}$$

Here, $Pr = \rho_\infty \nu_\infty C_p/k$ is the Prandtl number, λ is the heat source or sink parameter, M is the Magnetic field parameter, N_R is the Radiation parameter, Ec is the Eckert number, ϵ is the Pressure work parameter, the primes($'$) denote differentiation with respect to η.

The skin friction coefficient can be expressed in form

$$C_f = \frac{2\tau_w}{\rho_\infty U_w^2} \tag{2.13}$$

$$C_f R_e^{\frac{1}{2}} = \frac{\sqrt{2}G_e}{G_e - 1} f''(0, G_e) \tag{2.14}$$

and the local heat transfer coefficient in the form of Nusselt number is given by

$$Nu = \frac{-x\frac{\partial T}{\partial y}}{(T_w - T_\infty)} \tag{2.15}$$

$$Nu R_e^{\frac{-1}{2}} = -(1 + N_R)G'(0, G_e) \tag{2.16}$$

3. Results and Discussion

The partial differential equations which are coupled nonlinear (2.10) and (2.11) are solved along with the boundary conditions (2.12) by utilizing an implicit finite difference method along with quasilinearization scheme. Since the technique is described in [19, 20], for the sake of brevity, the description of the same has been omitted here. The obtained results are presented through the graphs for skin friction coefficient, heat transfer coefficient, velocity and temperature distributions having different values for magnetic (M), Prandtl number (Pr), viscous dissipation (Ec), viscosity (Ge), radiation (N_R) , Pressure work (ϵ) and heat source(sink) (λ) parameters, which is as shown in figures 2-14.

Impacts of magnetic field parameter (M) on the velocity profiles are appeared in fig. 2(a). As expected, the velocity profiles decline with the expansion in M. This is because of the velocity profiles having lower peak values for higher M values. Fig. 2(b) addresses the temperature profiles for different estimations of M. The presence of a magnetic field tends to create a drag-like power called the Lorentz force which acts the other way of the smooth movement. This makes the fluid temperature to increment as the magnetic field parameter M increments.

The results incorporate the effects of radiation (N_R) on velocity (F) and temperature (G) profiles are presented in fig.3. This increase in fluid temperature causes more streams to enter the boundary layer, increasing the fluid velocity there. Furthermore, the momentum boundary layer thickness increments because of expanding F. As N_R builds, the temperature profile (G) also increases. This allows the fluid to transport heat energy from the flow locale and cool the device. This is true in light of the fact that the Rosseland calculation causes a temperature increase.

Fig. 4 portray velocity and temperature profiles for various estimations of Prandtl number (Pr).

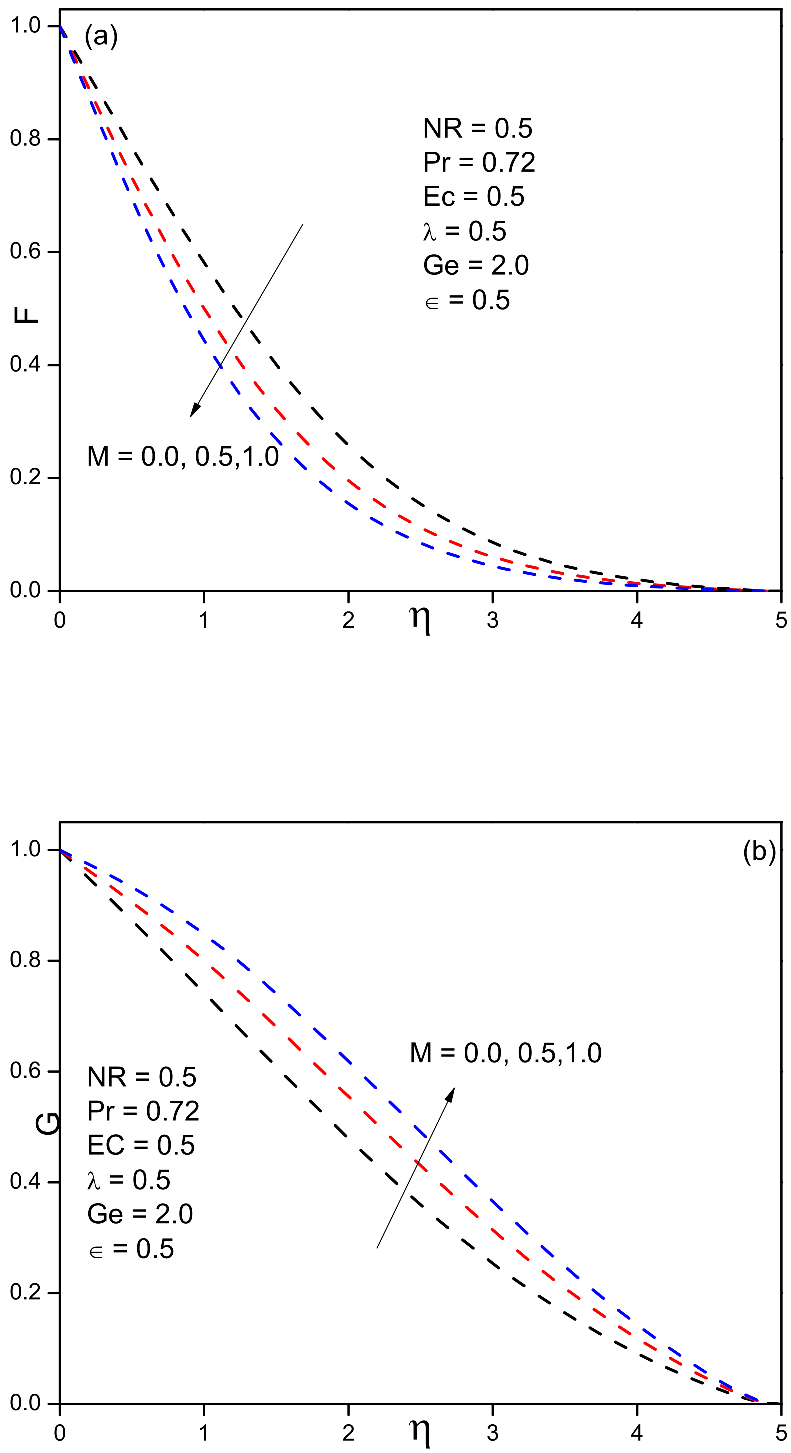


Figure 2.a) the velocity and b) temperature distributions for M = 0.0, 0.5, 1.0

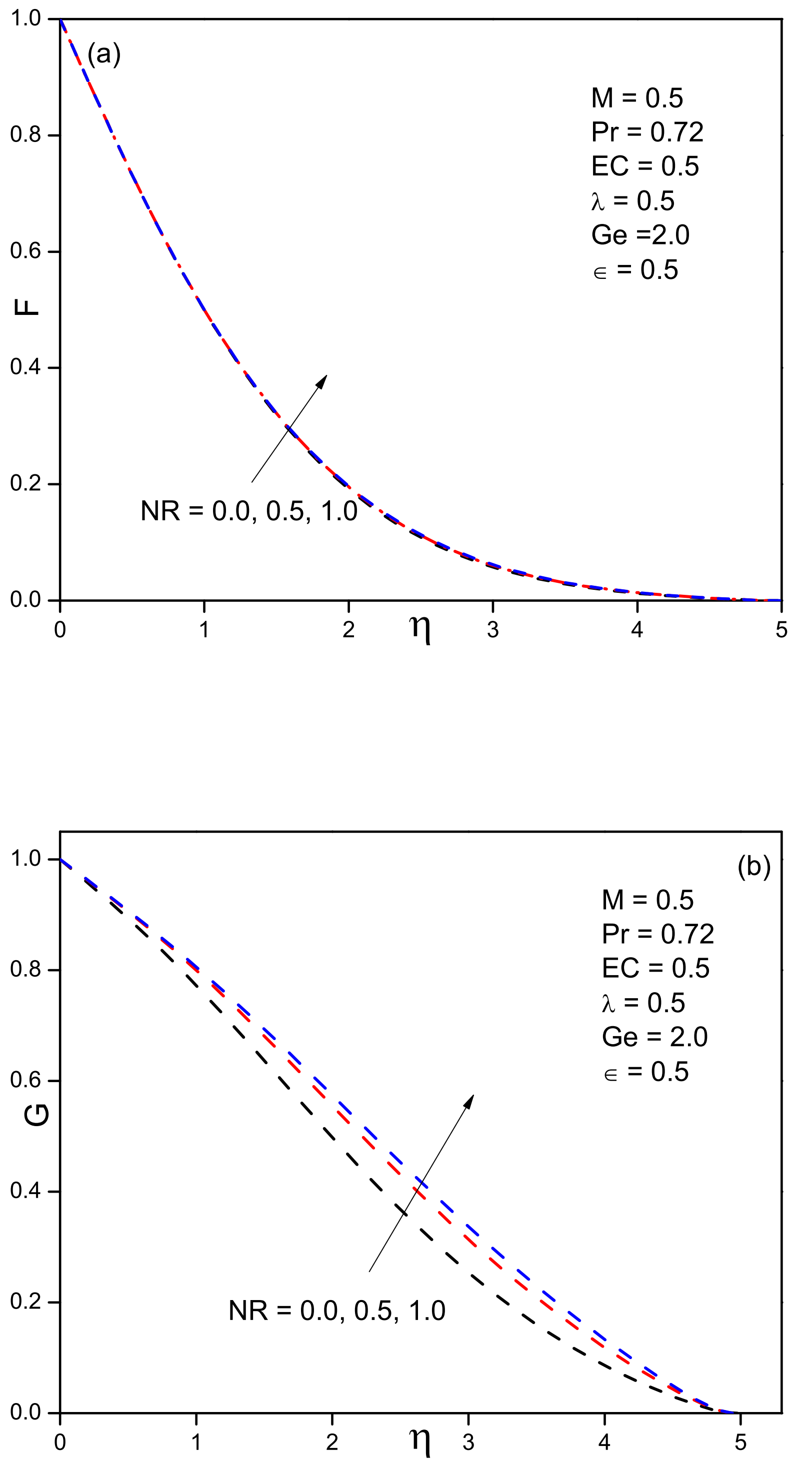


Figure 3.a) the velocity and b) temperature distributions for N_R 0.0, 0.5, 1.0

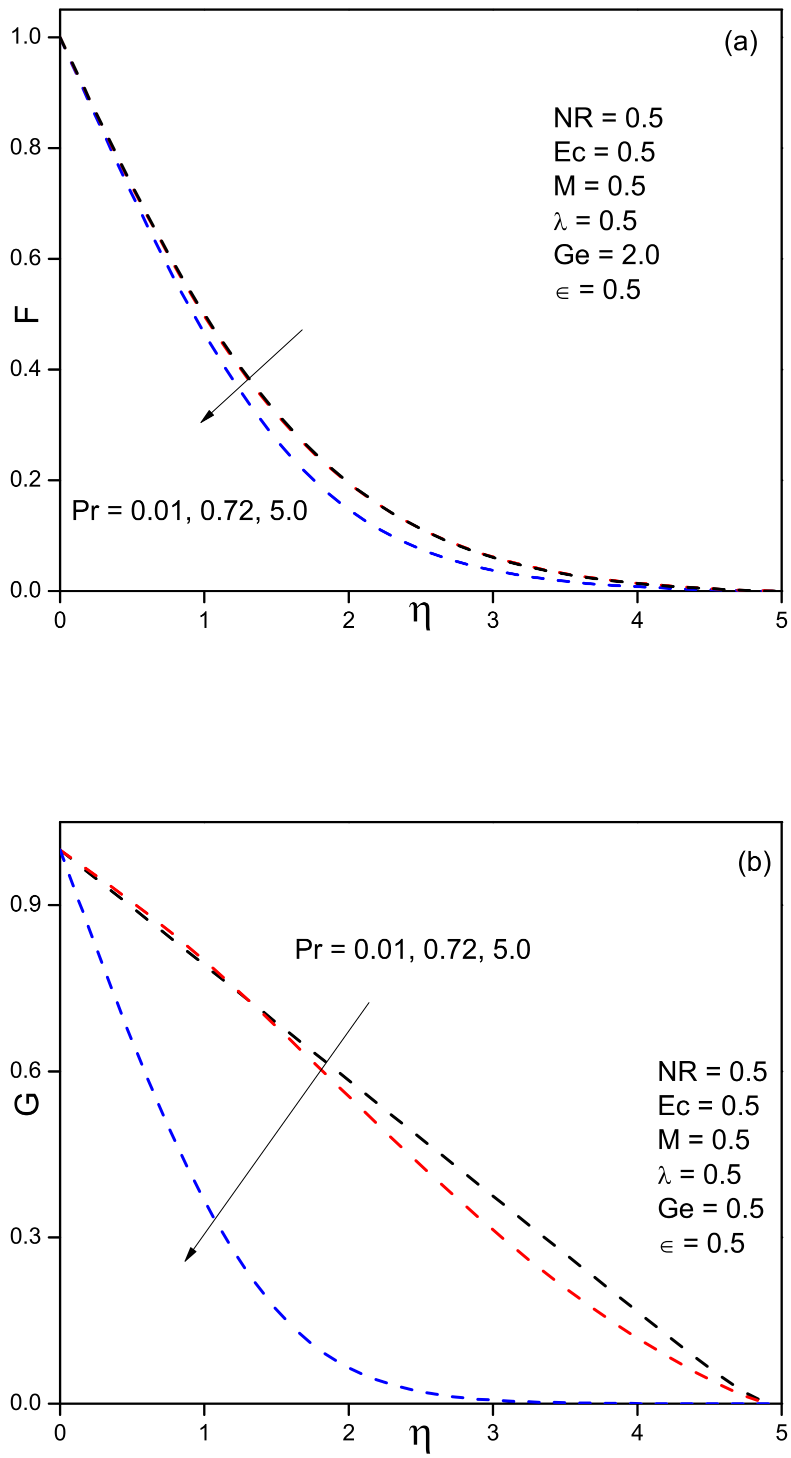


Figure 4.a) The velocity and b) temperature distributions for Pr = 0.01, 0.72, 5.0

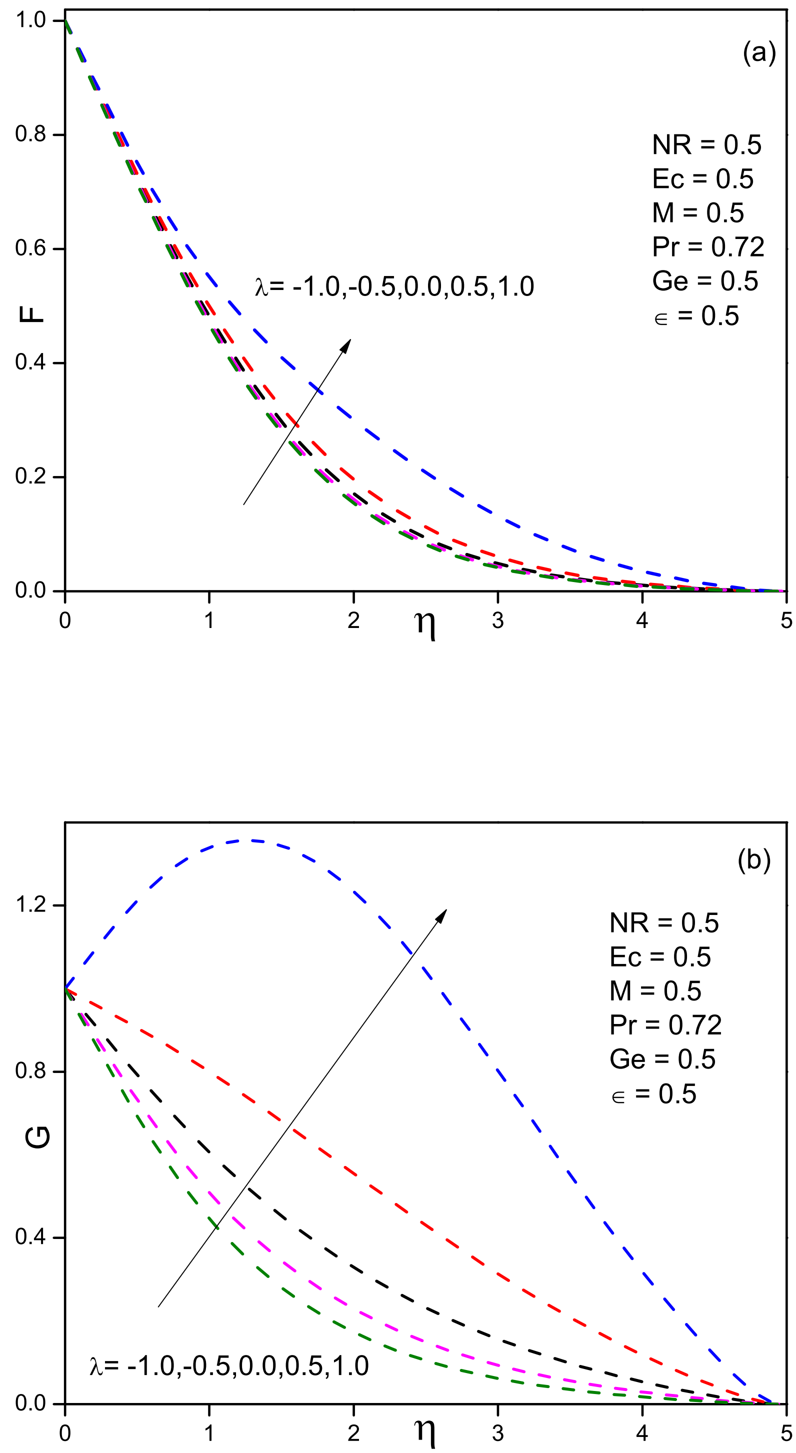


Figure 5.a) The velocity and b) temperature distributions for λ = -1.0,-0.5, 0.0, 0.5, 1.0.

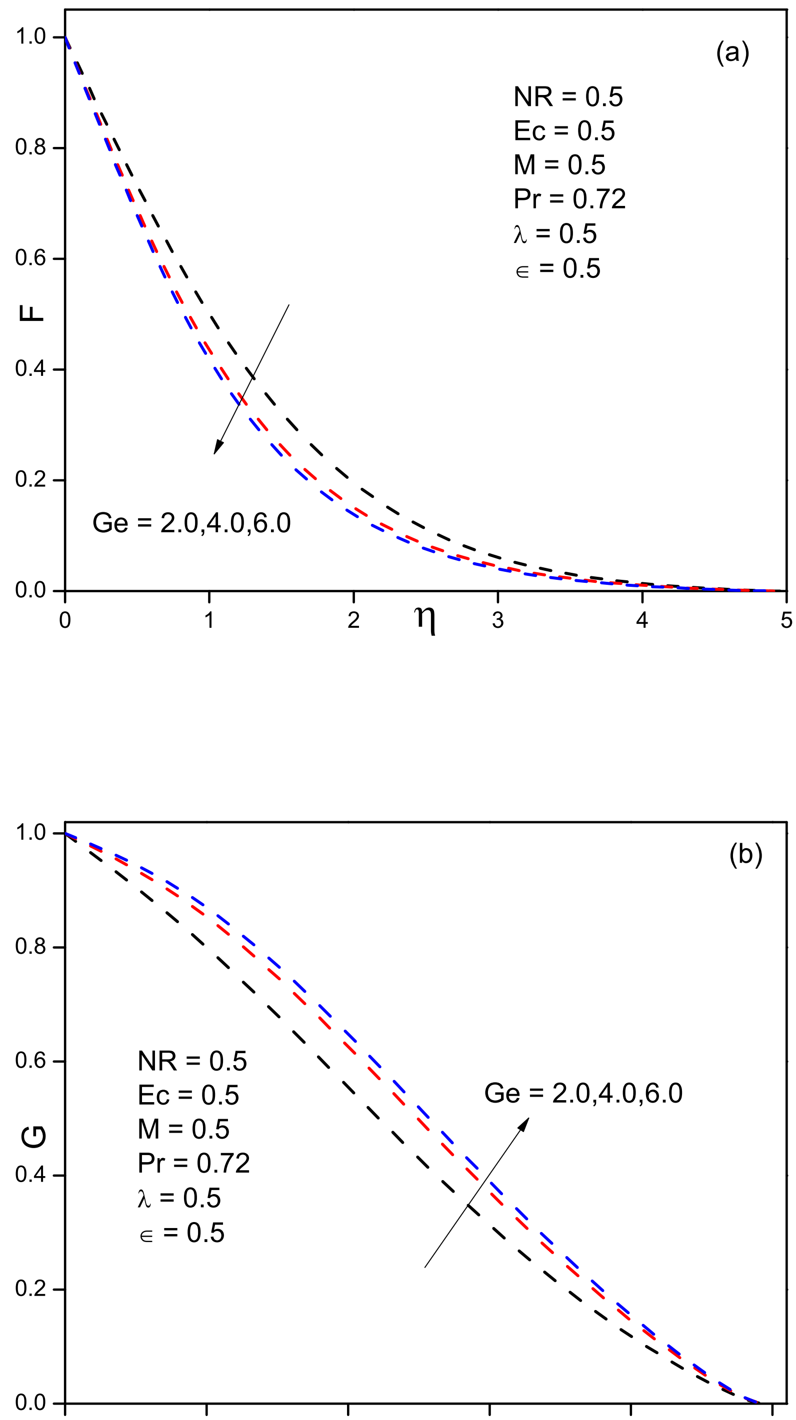
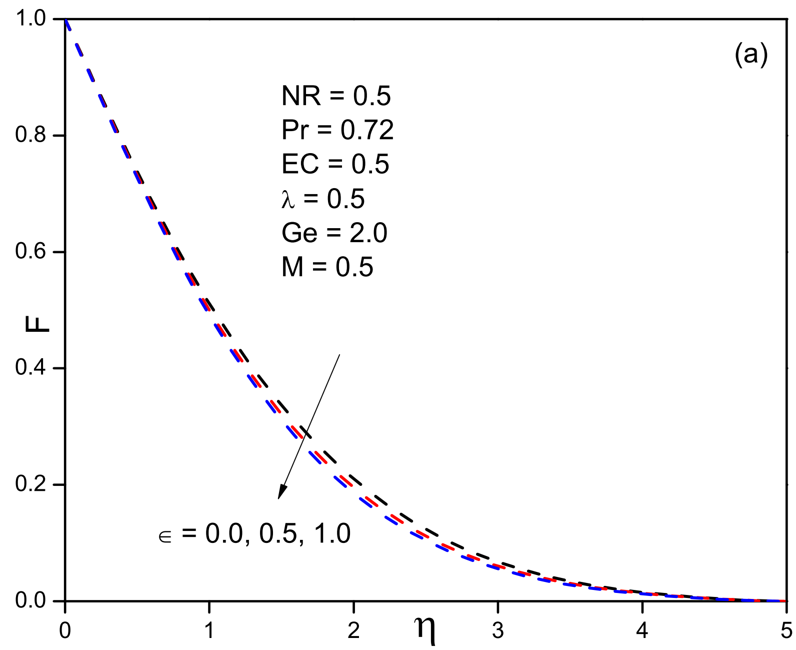


Figure 6.a) The velocity and b) temperature distributions for Ge = 2.0, 4.0, 6.0



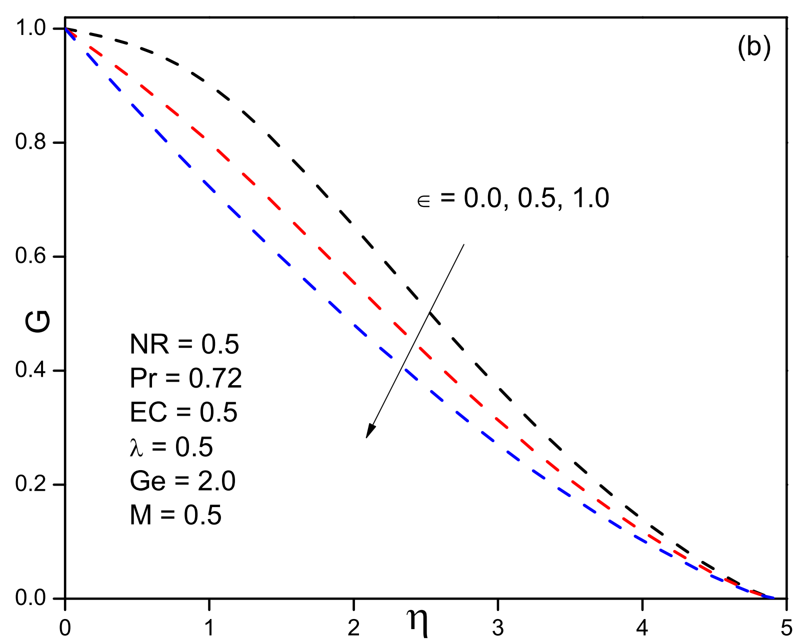


Figure 7.a) The velocity and b) temperature distributions for $\epsilon = 0.0,\ 0.5,\ 1.0$

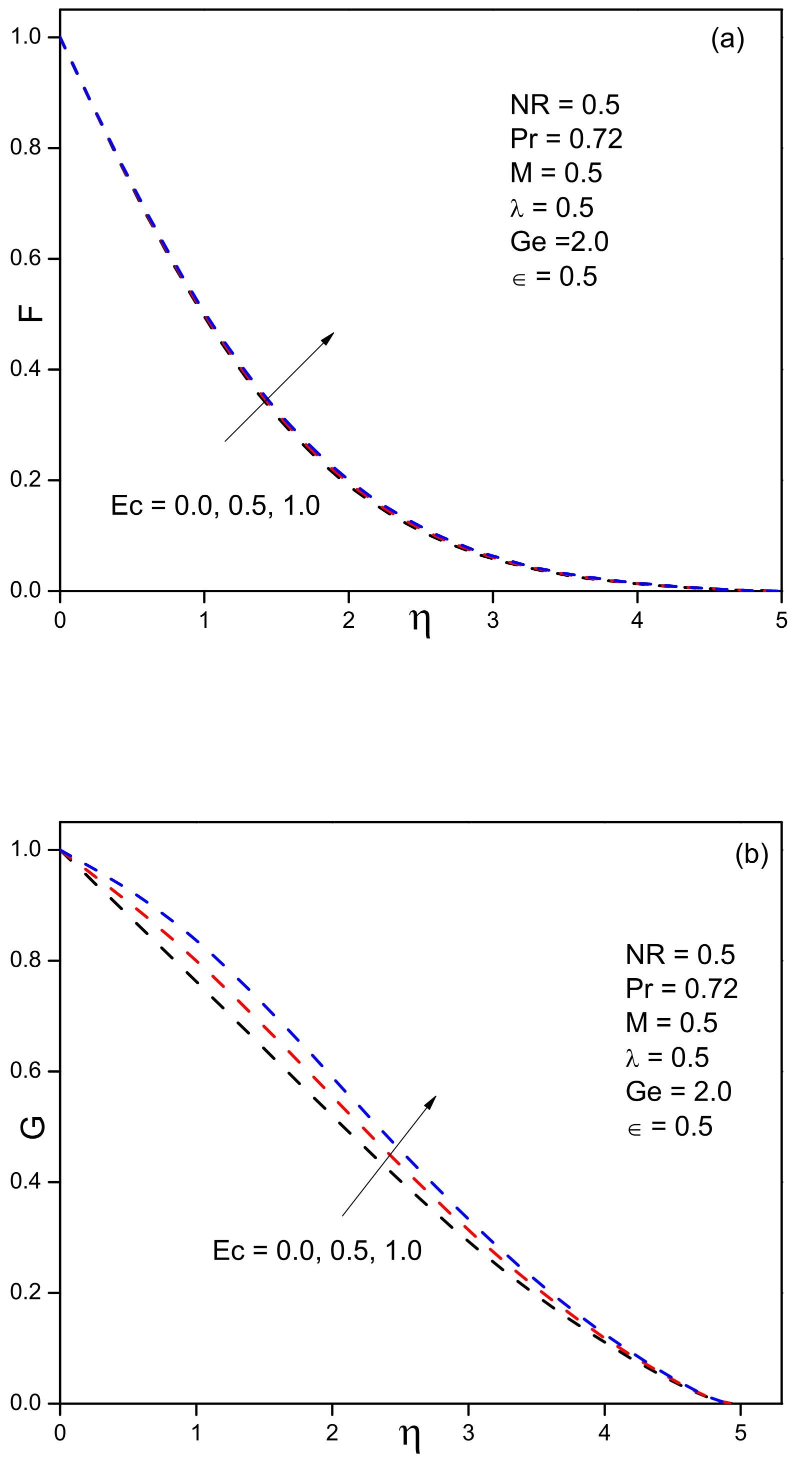


Figure 8.a) The velocity and b) temperature distributions for Ec = 0.0, 0.5, 1.0

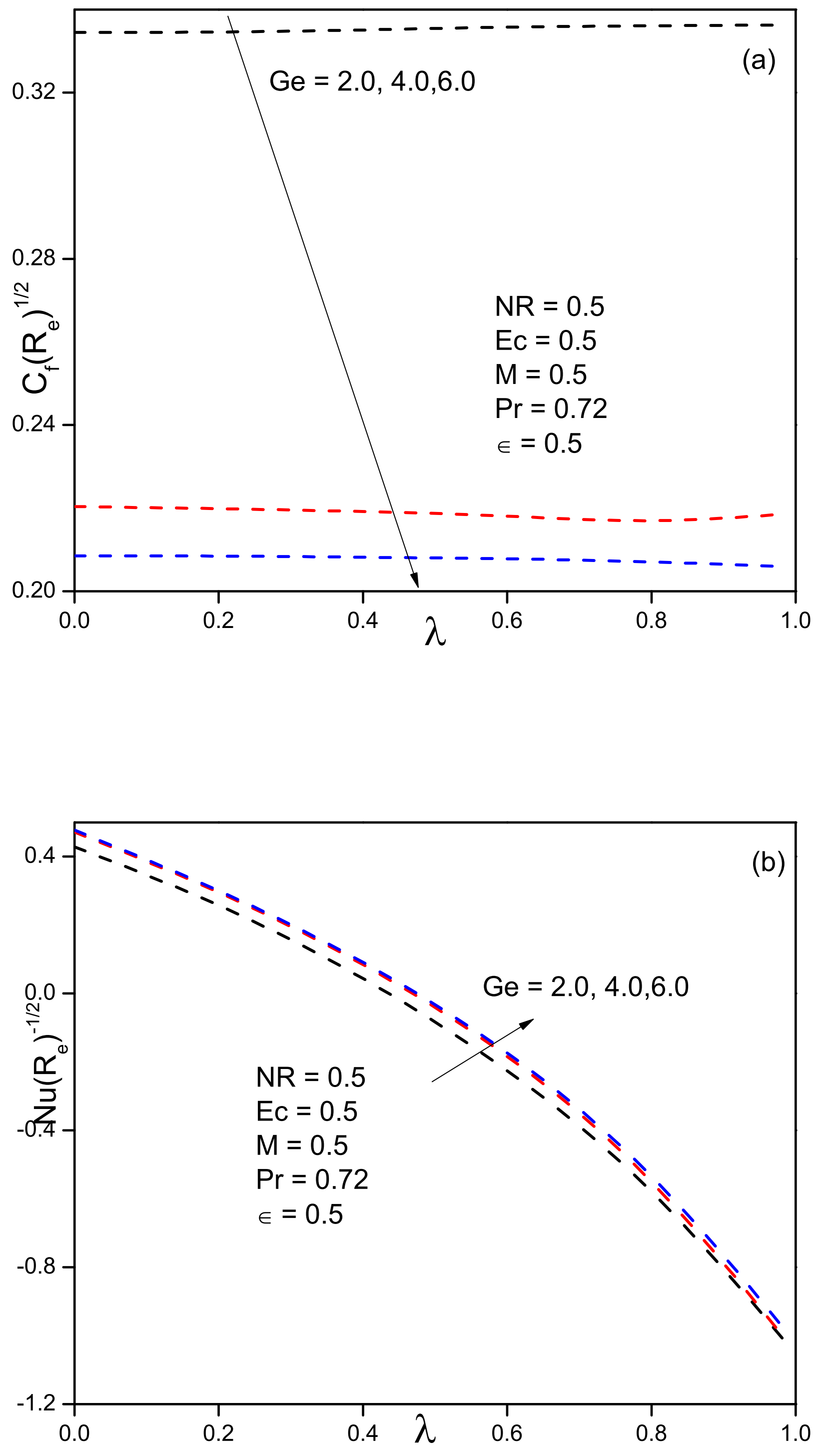


Figure 9.a) Skin friction coefficient and b) heat transfer coefficient for various values of Ge with λ

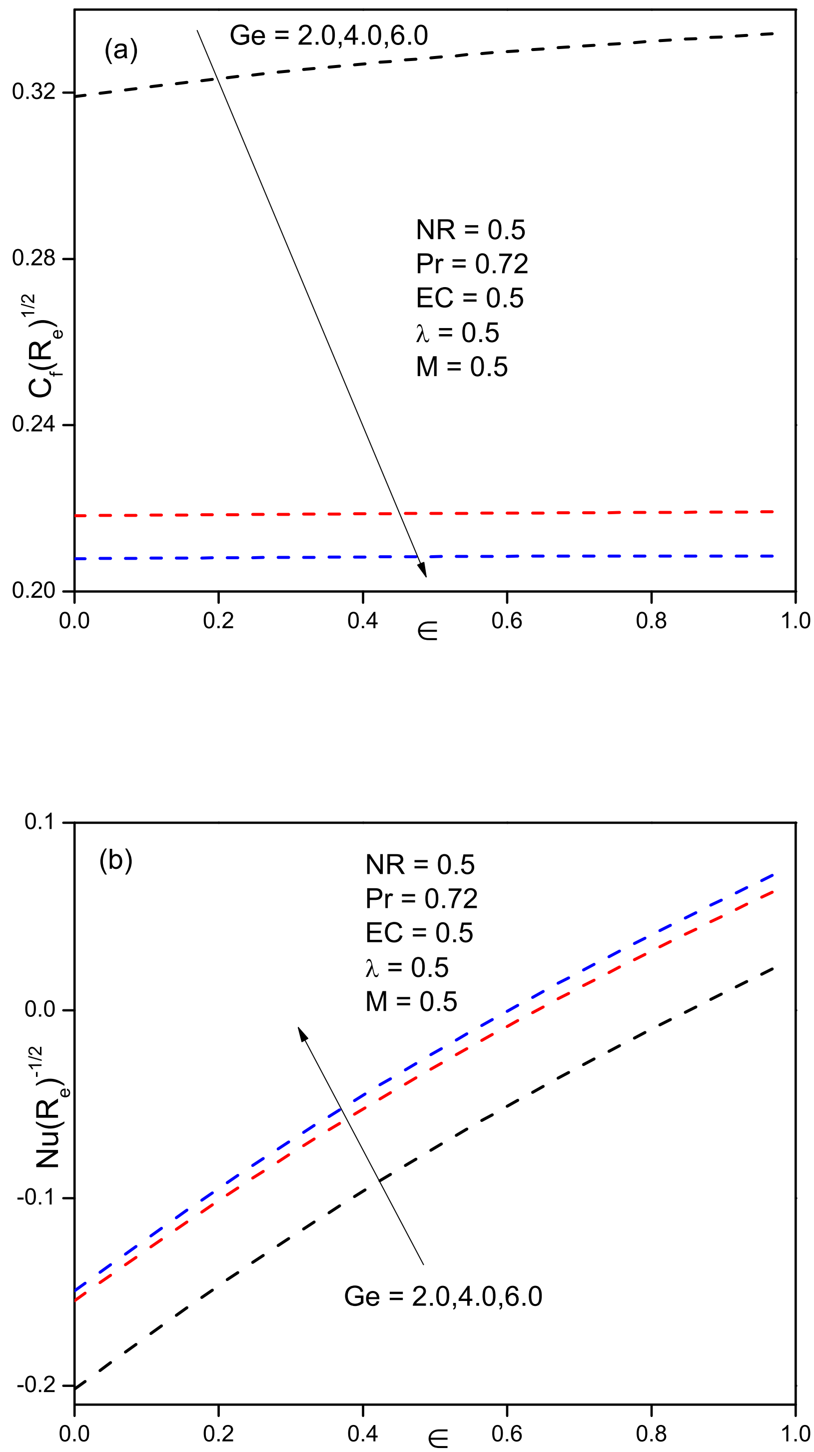


Figure 10.a) Skin friction coefficient and b) heat transfer coefficient for various values of Ge with ϵ

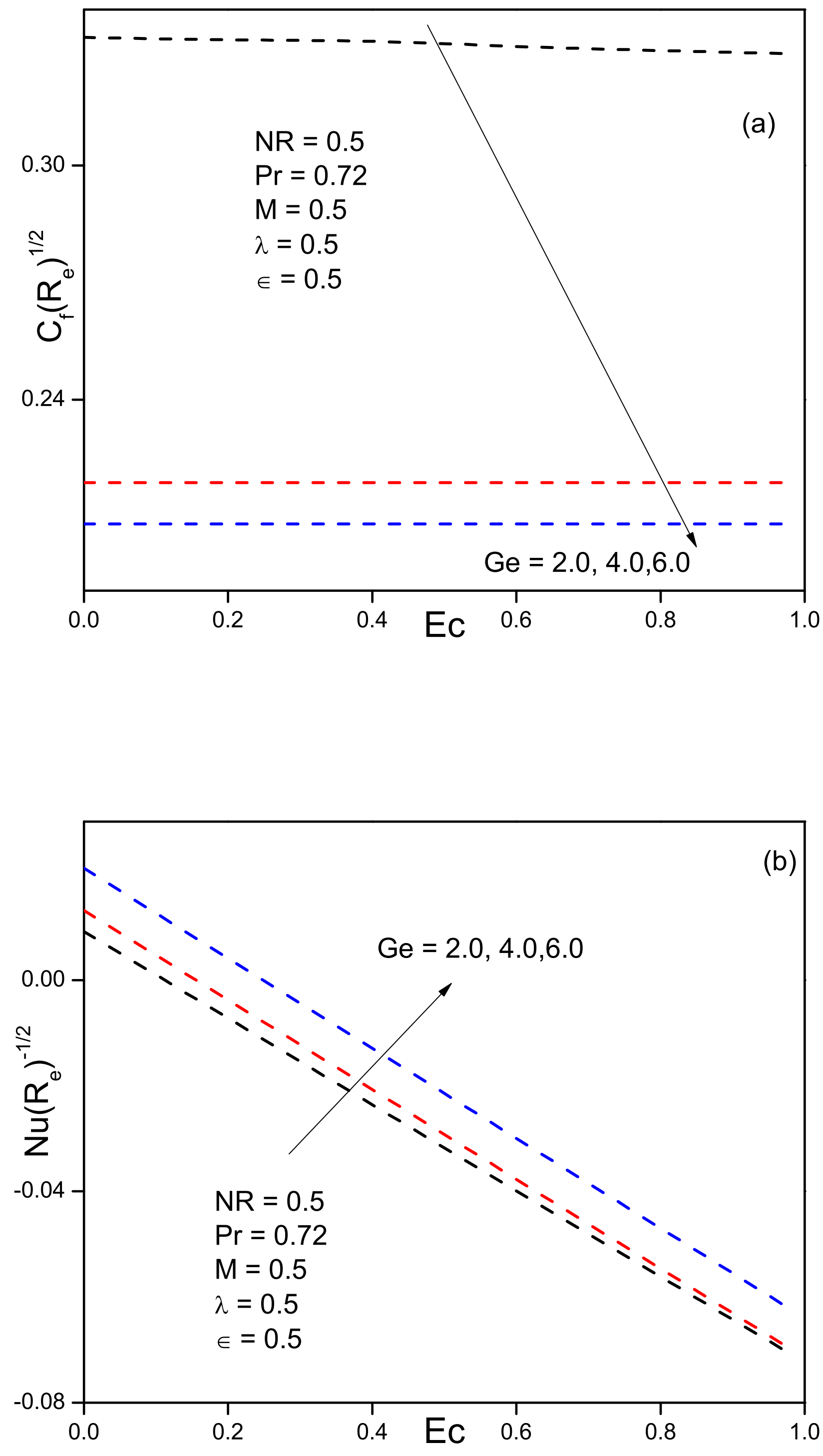


Figure 11.a) Skin friction coefficient and b) heat transfer coefficient for various values of Ge with Ec

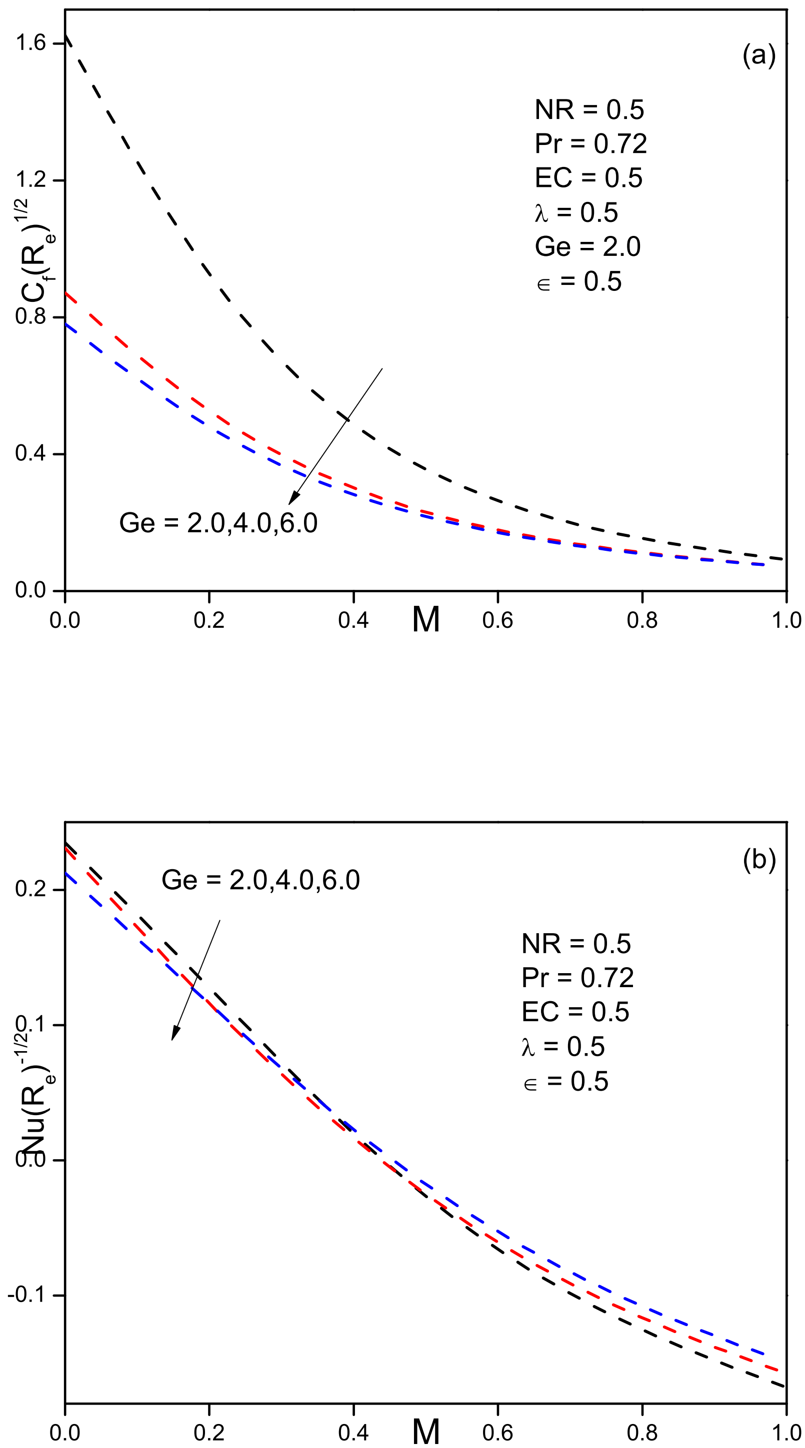
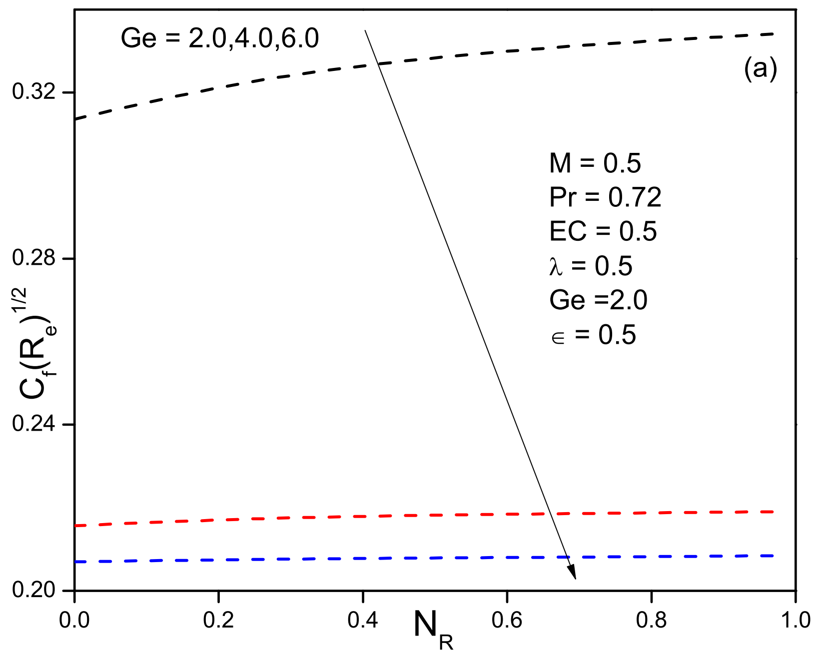


Figure 12.a) Skin friction coefficient and b) heat transfer coefficient for various values of Ge with M



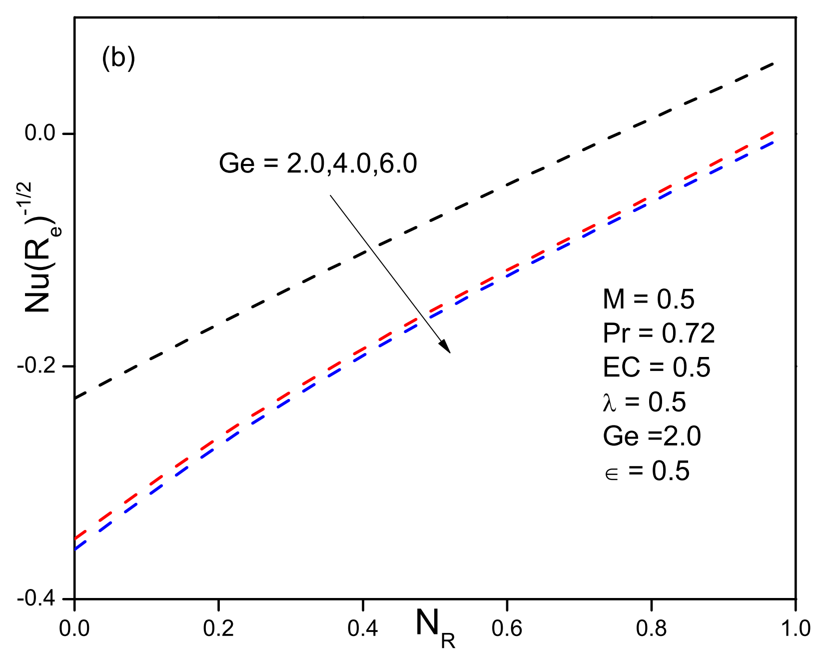


Figure 13.a) Skin friction coefficient and b) heat transfer coefficient for various values of Ge with N_R

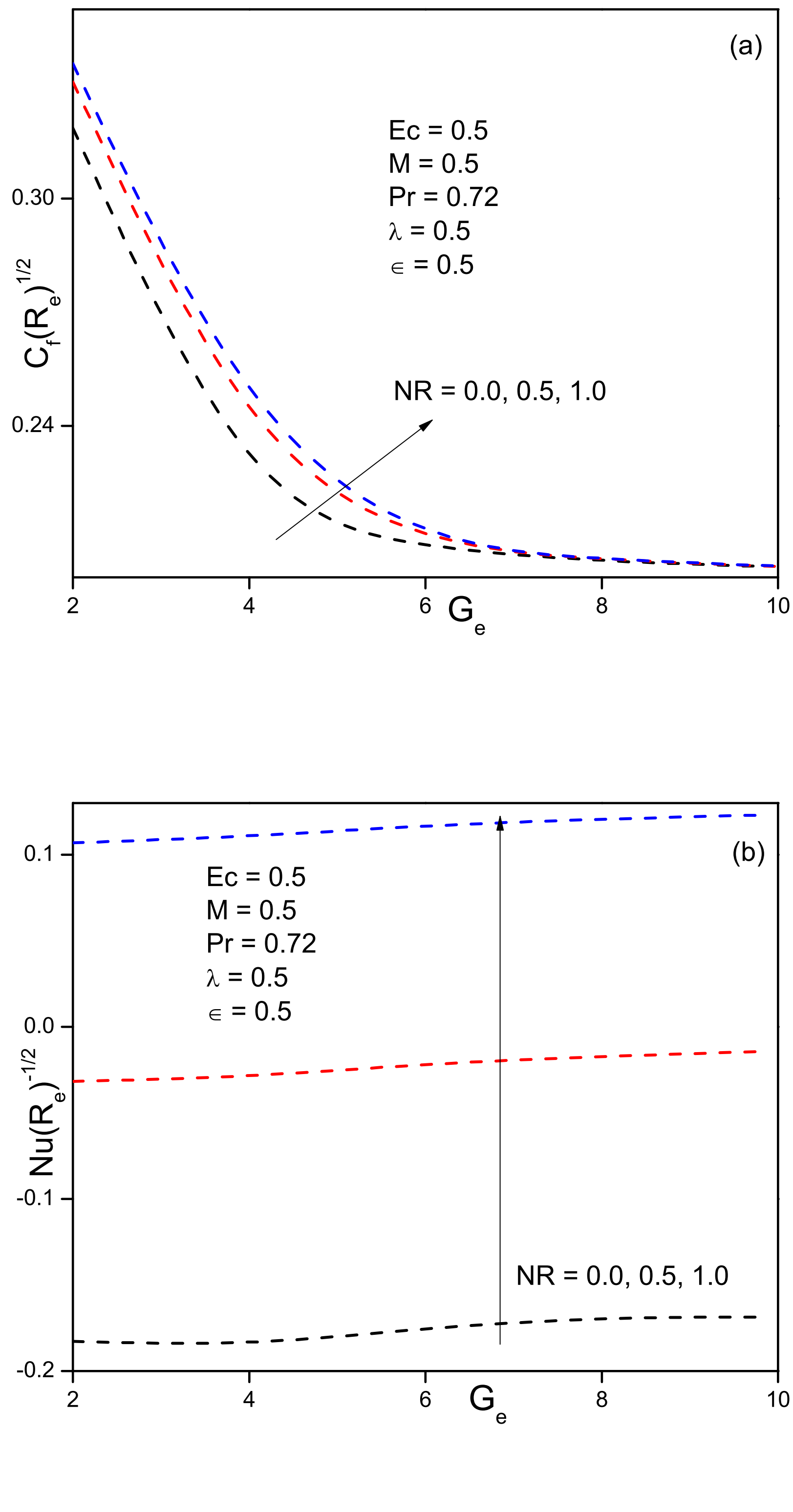


Figure 14.a) Skin friction coefficient and b) heat transfer coefficient for various values of N_R With Ge.

It characterizes the proportion of momentum diffusivity to thermal diffusivity. As the flow decelerates, the velocity profile decreases, followed by an increase in the Prandtl number.Therefore fluids with higher Prandtl number would have higher viscosities (and lower thermal conductivities), meaning that such fluids flow slower than fluids with lower Prandtl number. An increase in Pr causes a decrease in the thickness of the thermal boundary layer and, as a result, a decrease in the normal temperature within the boundary layer. The explanation for this is that lower Pr values expand thermal conductivities, allowing heat to disperse away from the surface more easily than higher Pr estimates. As a result of the lower Pr and the thicker boundary layers, the rate of heat transfer is diminished. Therefore the resistance to heat transfer is reduced.
It tends to be seen from fig. 5, the temperature and velocity of the fluid increment with the extension of heat source (sink) parameter (λ). This is normal as expanding λ basically remembers heat for the component that raises the fluid flow and finally temperature and velocity of the fluid increases. It is seen that axial velocity increments with time hence does thermal and momentum boundary layer. Consequently it is investigated that the thickness of the thermal and momentum boundary layers also increases slightly due to the rise in λ.
Fig.6 addresses the velocity and temperature profiles for various estimations of fluid viscosity parameter (Ge). The velocity diminishes with expanding the variable viscosity parameter, but the temperature increments with expanding this parameter. Further it is seen that the decrease in the velocity with Ge is not very remarkable near the boundary in this case. This effect is much noticeable little away from moving surface. Also, It has been found that thermal radiation assumes a critical part in the net heat transfer process.
Fig.7 showed the velocity and temperature distribution for different values of Pressure work parameter (ϵ). It is seen that both distributions diminishes as ϵ increments. Both profiles are suppressed and compressed towards the moving surface. Henceforth, it shows that both the thicknesses of thermal and momentum boundary layers are found to be diminishing.
Fig.8 represents the temperature and velocity distributions for various estimations of Eckert number (Ec). At the point when the estimations of Ec is expanded, it accelerates both velocity and heat transfer distributions. Ec expresses the relationship between enthalpy and kinetic energy in a fluid flow. It describes how work against viscous fluid stresses converts kinetic energy into internal energy. The temperature profile rises as viscous dissipative heat increases. This is due to energy dissipation caused by elastic deformation and viscosity, which keeps the energy in the fluid field.
The values of heat transfer coefficient $\left[Nu(Re)^{-1/2}\right]$ and skin friction coefficient $\left[C_f(Re)^{1/2}\right]$ are plotted vs λ,ϵ,Ec,M,N_R through fig.9,10,11 show that $C_f(Re)^{1/2}$ decreases and $Nu(Re)^{-1/2}$ increases with increasing values of Ge. The level of decrease of skin friction is about 14.69 % , 11.87% and 12.4% , and heat transfer is around 4.7 % ,5.12% and 1.07% for an increase of Ge, alongside a stream wise location λ,ϵ,Ec respectively. But from fig.12 and 13, it is observed that both $C_f(Re)^{1/2}$ and $Nu(Re)^{-1/2}$ decreases with increasing values of Ge. The level of

decrease of skin friction is about 18.38% and 11.89% , and heat transfer is around 4.1% and 8.76% alongside a stream wise location M and N_R respectively. In the fig.14, both $C_f(Re)^{1/2}$ and $Nu(Re)^{-1/2}$ increases with increasing values of N_R along with Ge at the level of 1.71% and 28.96% .

4. Conclusion

The effect of viscous dissipation, pressure work, MHD, radiation and heat source (sink) on heat transfer over a continuous moving surface with temperature dependent viscosity has been presented with more accuracy. The results can be summarized as follows:

1. Both temperature and velocity distributions increases for different estimations of N_R, λ and Ec respectively but decreases for different values of ϵ.
2. As the values of Ge and M are increased, the velocity profiles are reduced and the temperature profile is increased.
3. As the values of Pr are increased, the velocity profile expands and the temperature profile declines.
4. $C_f(Re)^{1/2}$ decreases and $Nu(Re)^{-1/2}$ increases with increasing values of Ge alongside a stream wise location λ ,ϵ, Ec respectively.
5. Both the coefficients of skin friction and heat transfer decreases with increasing values of Ge alongside a stream wise location M and N_R respectively. Whereas Both $C_f(Re)^{1/2}$ and $Nu(Re)^{-1/2}$ increases with increasing values of N_R along with Ge.

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