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THE RESTRICTIVE SETS AND THEIR APPLICATIONS TO SOFT GROUP THEORY

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ABSTRACT. In this paper, firstly four different restrictions, $\alpha-including$, $\alpha-covered$, $\alpha-intersection$ and $\alpha-union$ sets of a soft set over an initial universe U obtained by using a non-empty subset α of U are defined and examined in detail. Then, an application of $\alpha-intersection$ sets over a soft intersection groups is given. Here, the novel concept of the soft int-subgroup generated by $\alpha \subseteq U$ of a group is introduced and some properties are studied. The generators of soft intersections and products of soft int-subgroups are given. Some related properties about generators of soft int-subgroups are investigated and shown by several examples.

1. INTRODUCTION

The complexity of inaccurate information modeling cannot be successfully solved by classical methods. In 1999, Molodtsov [21] defined the soft set theory by introducing a new approach to solve problems caused by uncertainties. This theory has been applied in many areas of uncertainty such as information systems, decision making problems, optimization theory, algebraic structures, probability, measurement theory. Practices related to some of these areas are shown in [21] using the theory of soft set. Studies for soft set theory are still ongoing. [2, 3] AktaÅ and Çağman gave some basic concepts in soft set theory and defined the concept of soft group. In [20, 22, 25], the authors theoretically defined various operations on soft sets and examined the algebraic properties of soft sets. Many authors worked on soft algebraic structures and soft operations [1, 11, 12, 13, 14, 18, 24] after the article [2] publication. In [10], the authors introduced new concepts on the soft sets, which are called soft quotient subgroup and quotient dual soft subgroup. They gave their algebraic properties of these structures and investigated the fundamental isomorphism theorems in soft subgroups analogous to the group theory. Çağman and Enginoğlu [8] initiated the soft matrix theory corresponding to soft sets. They defined soft matrices and some of their operations, then established a soft max-min decision making problem to solve problems containing uncertainty. The theory of soft set and soft

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matrices have been successfully applied to decision-making problems in articles [8, 15, 16, 17, 19, 23].
This paper is organized as follows: In Section 2, we recall the necessary definitions, notations and some fundamental properties. In section 3, we examine four different restriction of a soft set over U, using the subsets of the universal set U. In Section 3, we give some properties of the restrictive sets according to the soft operations. These restricted sets play an active role in the applications of algebraic structures and decision making. In section 4, the concept of set-generated soft intersection subgroup of a group G is given. Many properties of this concept are investigated. This section also includes our main theorems, where we examine the generator sets of soft groups obtained by the operations of soft intersection and product. We also give several examples. Section 5 is the conclusion part.

2. Preliminary

Definition 2.1. [7, 21] Let U be an initial universe set, E be a set of parameters, $P(U)$ be the power set of U and $A \subseteq E$. A soft set (K, A) or simply K_A on the universe U is defined by the ordered pairs

$$K_A = \{(x, k_A(x))|x \in E, k_A(x) \in P(U)\},$$

where $k_A : E \to P(U)$ such that $k_A(x) = \emptyset$ if $x \notin A$. Here k_A is called approximate function of the soft set K_A.

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition 2.2. [7] Let K_A and (K_B) be soft sets over U.

a) If $k_A(x) \subseteq k_B(x)$ for all $x \in E$, then K_A is a soft subset of K_B, denoted by $K_A \widetilde{\subseteq} (K_B)$.
b) If $K_A \widetilde{\subseteq} K_B$ and $K_B \widetilde{\subseteq} K_A$, then K_A and K_B is said to be soft equal and denoted by $K_A = K_B$.

Definition 2.3. [20] If K_A and L_B are soft sets over a common universe U, then "K_A AND L_B" denoted by $K_A \wedge L_B$ is defined by $K_A \wedge L_B = (H, A \times B)$, where $H(x, y) = K(x) \cap L(y)$ for all $(x, y) \in A \times B$.

Definition 2.4. [20] If K_A and L_B are soft sets over a common universe U, then "K_A OR L_B" denoted by $K_A \vee L_B$ is defined by $K_A \vee L_B = (H, A \times B)$, where $H(x, y) = K(x) \cup L(y)$ for all $(x, y) \in A \times B$.

Definition 2.5. [4] Let K_A and L_B be soft sets over a common universe U such that $A \cap B \neq \emptyset$. The restricted union of K_A and L_B is denoted by $K_A \cup_R L_B$, and is defined as $K_A \cup_R L_B = H_C$, where $C = A \cap B$ and for all $x \in C$, $H(x) = K(x) \cup L(x)$.

Definition 2.6. [4] Let K_A and L_B be soft sets over a common universe U such that $A \cap B \neq \emptyset$. The restricted intersection of K_A and L_B is denoted by $K_A \cap_R L_B$, and is defined as $K_A \cap L_B = H_C$, where $C = A \cap B$ and for all $x \in C$, $H(x) = K(x) \cap L(x)$.

Definition 2.7. [4] Let K_A and L_B be soft sets over a common universe U such that $A \cap B \neq \emptyset$. The restricted difference of K_A and L_B is denoted by $K_A \sim_R L_B$, and is defined as $K_A \sim_R L_B = H_C$, where $C = A \cap B$ and for all $x \in C, H(x) = K(x) \setminus L(x)$.

Definition 2.8. [4] The relative complement of a soft set K_A is denoted by K_A^c and is defined by $K_A^c = (F^c, A)$, where $F^c : A \to P(U)$ is a mapping given by $F^c(x) = U \setminus K(x)$, for all $x \in A$.

Definition 2.9. [20] A soft set K_A over U is said to be a null soft set denoted by Φ_A, if $K(x) = \emptyset$, for all $x \in A$.

Definition 2.10. [20] A soft set K_A over U is said to be an absolute soft set, if $K(x) = U$, for all $x \in A$.

Definition 2.11. [11] Let K_A be a soft set over U. Then the set

$$suppK_A = \{x \in A | K(x) \neq \emptyset\}$$

is called the support of the soft set K_A. A soft set K_A is called non-null if $suppK_A \neq \emptyset$.

3. The Restrictive Soft Sets

In this section, by using a subset α of U, we examined four different restriction, *upper α-inclusion*, *lower α-inclusion*, *α-intersection* and *α-union* of a soft set over a universal set U,

Definition 3.1. [19] Let K_A be soft set over U and $\alpha \subseteq U$. Then

a) *upper α-inclusion* of K_A is defined as
$K_A^{\supseteq\alpha} = \{x \in A \mid k_A(x) \supseteq \alpha\}$,

b) *lower α-inclusion* of K_A is defined as
$K_A^{\subseteq\alpha} = \{x \in A \mid k_A(x) \subseteq \alpha\}$.

Definition 3.2. [5] Let K_A be soft set over U and $\emptyset \neq \alpha \subseteq U$. Then

a) *α-intersection* of K_A is defined as
$K_A^{\cap\alpha} = \{x \in A \mid k_A(x) \cap \alpha \neq \emptyset\}$,

b) *α-union* of K_A is defined as
$K_A^{\cup\alpha} = \{x \in A \mid k_A(x) \cup \alpha = U\}$.

By this definition, it is easily seen that $K_A^{\cap\emptyset} = \emptyset$, $K_A^{\cap U} = SuppK_A$, $K_A^{\cup U} = A$. If $k_A(x) \neq U$ for all $x \in A$, then $K_A^{\cup\emptyset} = \emptyset$.
From now on, we investigate some useful properties of these four restrictive sets according to soft set operations. Firstly, we give some properties of the restrictive sets according to soft subset $\widetilde{\subseteq}$:

Theorem 3.3. *Let K_A, L_B be soft sets over U and let $\emptyset \neq \alpha \subseteq U$. If $K_A \widetilde{\subseteq} L_B$, then*

a) $K_A^{\cap\alpha} \subseteq L_B^{\cap\alpha}$.

b) i) $L_B^{\subseteq\alpha} \subseteq K_A^{\subseteq\alpha}$.

   ii) *If $K_A(x) \subseteq \alpha$ for all $x \in A$ and $L_B(x) \subseteq \alpha$ for all $x \in B$, then* $K_A^{\subseteq\alpha} \subseteq L_B^{\subseteq\alpha}$.

c) $K_A^{\supseteq\alpha} \subseteq L_B^{\supseteq\alpha}$.
d) $K_A^{\cup\alpha} \subseteq L_B^{\cup\alpha}$.

Proof. Let $K_A \widetilde{\subseteq} L_B$. Then, by Definition 2.2, $A \subseteq B$ and $K_A(x) \subseteq L_B(x)$ for all $x \in A$

a) $x \in K_A^{\cap\alpha} \Rightarrow K_A(x) \cap \alpha \neq \emptyset \Rightarrow L_B(x) \cap \alpha \neq \emptyset$ since $L_B(x) \supseteq K_A(x)$ for all $x \in A$. Therefore, $x \in L_B^{\cap\alpha}$.
b) (i) $x \in L_B^{\subseteq\alpha} \Rightarrow L_B(x) \subseteq \alpha \Rightarrow K_A(x) \subseteq L_B(x) \subseteq \alpha \Rightarrow K_A(x) \subseteq \alpha \Rightarrow x \in K_A^{\subseteq\alpha}$. (ii) Let $K_A(x) \subseteq \alpha$ for all $x \in A$ and $L_B(x) \subseteq \alpha$ for all $x \in B$. Since $A \subseteq B$ and $A = K_A^{\subseteq\alpha}, B = L_B^{\subseteq\alpha}$, then $K_A^{\subseteq\alpha} \subseteq L_B^{\subseteq\alpha}$.
c) $x \in K_A^{\supseteq\alpha} \Rightarrow K_A(x) \supseteq \alpha \Rightarrow L_B(x) \supseteq K_A(x) \supseteq \alpha$. Therefore, $x \in L_B^{\supseteq\alpha}$.
d) $x \in K_A^{\cup\alpha} \Rightarrow K_A(x) \cup \alpha = U \Rightarrow L_B(x) \cup \alpha = U$ since $K_A(x) \subseteq L_B(x)$. Therefore, $x \in L_B^{\cup\alpha}$.

□

Some properties of the restrictive sets according to the restricted union operation "$\cup_R$" are given as follows:

Theorem 3.4. *Let K_A, L_B be soft sets over U and $\alpha \subseteq U$. Then,*

$$(K_A \cup_R L_B)^{\subseteq\alpha} \subseteq K_A^{\subseteq\alpha} \cup L_B^{\subseteq\alpha}.$$

Proof. Let $C = A \cap B, H_C = K_A \cup_R L_B$. $x \in H_C^{\subseteq\alpha} \Leftrightarrow (K_A \cup_R L_B)(x) \subseteq \alpha \Leftrightarrow (K_A(x) \cup L_B(x)) \subseteq \alpha \Leftrightarrow K_A(x) \subseteq \alpha$ and $L_B(x) \subseteq \alpha \Rightarrow x \in K_A^{\subseteq\alpha} \cap L_B^{\subseteq\alpha} \subseteq K_A^{\subseteq\alpha} \cup L_B^{\subseteq\alpha}$. □

We have the following example:

Example 3.5. Let E be the universe set of parameters and let $A, B \subseteq E$ such that

$$E = \{e_1, e_2, e_3, e_4, e_5\}, A = \{e_1, e_2, e_3\}, B = \{e_1, e_2, e_3\}.$$

Let K_A and L_B be soft sets over $U = \{u_1, u_2, u_3, u_4\}$ such that

$$K_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\}), (e_3, \{u_1\})\},$$
$$L_B = \{(e_1, \{u_3, u_4\}), (e_2, \{u_1, u_2\}), (e_3, \{u_2\})\}.$$

Let $H_C = K_A \cup_R L_B$, where $C = A \cap B$, $H(x) = K_A(x) \cup L_B(x)$ for all $x \in C$. Then, $H_C = \{(e_1, \{u_1, u_2, u_3, u_4\}), (e_2, \{u_1, u_2, u_3\}), (e_3, \{u_1, u_2\})\}$. Let $\alpha = \{u_1, u_2\}$. Then, we have restricted sets as follows:

$$H_C^{\subseteq\alpha} = \{e_3\},\ K_A^{\subseteq\alpha} = \{e_1, e_3\},\ L_B^{\subseteq\alpha} = \{e_2, e_3\}.$$

Hence, we have $(K_A \cup_R L_B)^{\subseteq\alpha} \subseteq K_A^{\subseteq\alpha} \cup L_B^{\subseteq\alpha}$.

Theorem 3.6. *Let K_A, L_B be soft sets over U and $\alpha \subseteq U$. Then*

a) $(K_A \cup_R L_B)^{\supseteq\alpha} \supseteq K_A^{\supseteq\alpha} \cup L_B^{\supseteq\alpha}$.
b) $(K_A \cup_R L_B)^{\cap\alpha} = K_A^{\cap\alpha} \cup L_B^{\cap\alpha}$.
c) $(K_A \cup_R L_B)^{\cup\alpha} \supseteq K_A^{\cup\alpha} \cup L_B^{\cup\alpha}$.

Proof. a) $x \in K_A^{\supseteq\alpha} \cup L_B^{\supseteq\alpha} \Rightarrow x \in K_A^{\supseteq\alpha}$ or $x \in L_B^{\supseteq\alpha} \Rightarrow K_A(x) \supseteq \alpha$ or $L_B(x) \supseteq \alpha \Rightarrow (K_A(x) \cup L_B(x)) \supseteq \alpha \Rightarrow x \in (K_A \cup_R L_B)^{\supseteq\alpha}$.

b) Let $H_C = K_A \cup_R L_B$, where $C = A \cap B$. $x \in H_C^{\cap\alpha} \Leftrightarrow H_C(x) \cap \alpha \neq \emptyset \Leftrightarrow (K_A(x) \cup L_B(x)) \cap \alpha \neq \emptyset \Leftrightarrow K_A(x) \cap \alpha \neq \emptyset$ or $L_B(x) \cap \alpha \neq \emptyset \Leftrightarrow x \in K_A^{\cap\alpha}$ or $x \in L_B^{\cap\alpha} \Leftrightarrow x \in K_A^{\cap\alpha} \cup L_B^{\cap\alpha}$.

c) $x \in K_A^{\cup\alpha} \cup L_B^{\cup\alpha} \Rightarrow x \in K_A^{\cup\alpha}$ or $x \in L_B^{\cup\alpha} \Rightarrow K_A(x) \cup \alpha = U$ or $L_B(x) \cup \alpha = U \Rightarrow (K_A(x) \cup L_B(x)) \cup \alpha = U \Rightarrow x \in (K_A \tilde{\cup} L_B)^{\cup\alpha}$.

□

The following example shows Theorem 3.6:

Example 3.7. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$, $A = \{e_1, e_2, e_3, e_4\}$, $B = \{e_1, e_2, e_3, e_4\}$ and let K_A and L_B be following soft sets over $U = \{u_1, u_2, u_3, u_4, u_5\}$

$$K_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\}), (e_3, \{u_1\}), (e_4, \{u_3\})\},$$
$$L_B = \{(e_1, \{u_3, u_4\}), (e_2, \{u_1, u_2\}), (e_3, \{u_2\}), (e_4, \{u_4\})\}.$$

Let $H_C = K_A \cup_R L_B$, where $C = A \cap B$, $H(x) = K_A(x) \cup L_B(x)$ for all $x \in C$.

$$H_C = \{(e_1, \{u_1, u_2, u_3, u_4\}), (e_2, \{u_1, u_2, u_3\}), (e_3, \{u_1, u_2\}), (e_4, \{u_3, u_4\})\}.$$

Let $\alpha = \{u_1, u_2\}$ and $\beta = \{u_1, u_4, u_5\}$. Then, we have the restricted sets as follows:

$H_C^{\supseteq\alpha} = \{e_1, e_2, e_3\}$, $K_A^{\supseteq\alpha} = \{e_1\}$, $L_B^{\supseteq\alpha} = \{e_2\}$ and hence $(K_A \tilde{\cup} L_B)^{\supseteq\alpha} \supseteq K_A^{\supseteq\alpha} \cup L_B^{\supseteq\alpha}$.

$H_C^{\cap\alpha} = \{e_1, e_2, e_3\}$, $K_A^{\cap\alpha} = \{e_1, e_2, e_3\}$, $L_B^{\cap\alpha} = \{e_2, e_3\}$, it is seen that $(K_A \tilde{\cup} L_B)^{\cap\alpha} = K_A^{\cap\alpha} \cup L_B^{\cap\alpha}$.

$H_C^{\cup\beta} = \{e_1, e_2\}$, $K_A^{\cup\beta} = \{e_2\}$, $L_B^{\cup\beta} = \emptyset$, then we have $(K_A \tilde{\cup} L_B)^{\cup\beta} \supseteq K_A^{\cup\beta} \cup L_B^{\cup\beta}$
.

Some properties of these restrictive sets according to the operation soft restricted union "$\cup_R$" are given as follows:

Theorem 3.8. *Let K_A, L_B be soft sets over U and $\alpha \subseteq U$. Then*

a) $(K_A \cup_R L_B)^{\subseteq\alpha} \subseteq K_A^{\subseteq\alpha} \cup L_B^{\subseteq\alpha}$.
b) $(K_A \cup_R L_B)^{\supseteq\alpha} \supseteq K_A^{\supseteq\alpha} \cap L_B^{\supseteq\alpha}$.
c) $(K_A \cup_R L_B)^{\cap\alpha} = K_A^{\cap\alpha} \cup L_B^{\cap\alpha}$.
d) $(K_A \cup_R L_B)^{\cup\alpha} \supseteq K_A^{\cup\alpha} \cap L_B^{\cup\alpha}$.

Proof. Let $K_A \cup_R L_B = H_C$, where $H_C(x) = K_A(x) \cup L_B(x)$ for all $x \in C = A \cap B$.

a) Let $x \in C$. Then $x \in H_C^{\subseteq\alpha} \Rightarrow H_C(x) \subseteq \alpha \Rightarrow (K_A \cup L_B)(x) = K_A(x) \cup L_B(x) \subseteq \alpha \Rightarrow K_A(x) \subseteq \alpha$ and $L_B(x) \subseteq \alpha \Rightarrow x \in K_A^{\subseteq\alpha}$ and $x \in L_B^{\subseteq\alpha} \Rightarrow x \in K_A^{\subseteq\alpha} \cap L_B^{\subseteq\alpha} \subseteq K_A^{\subseteq\alpha} \cup L_B^{\subseteq\alpha}$. $\Rightarrow (K_A \cup_R L_B)^{\subseteq\alpha} \subseteq K_A^{\subseteq\alpha} \cup L_B^{\subseteq\alpha}$.

b) $x \in K_A^{\supseteq\alpha} \cap L_B^{\supseteq\alpha} \Rightarrow x \in A$, $K_A(x) \supseteq \alpha$ and $x \in B, L_B(x) \supseteq \alpha \Rightarrow x \in A \cap B, K_A(x) \cap L_B(x) \supseteq \alpha \Rightarrow x \in (K_A \tilde{\cap} L_B)^{\supseteq\alpha} \subseteq (K_A \cup_R L_B)^{\supseteq\alpha} \Rightarrow x \in (K_A \cup_R L_B)^{\supseteq\alpha}$.

c) $x \in H_C^{\cap\alpha} \Leftrightarrow H_C(x) \cap \alpha \neq \emptyset \Leftrightarrow (K_A(x) \cup L_B(x)) \cap \alpha \neq \emptyset \Leftrightarrow (K_A(x) \cap \alpha) \cup (L_B(x) \cap \alpha) \neq \emptyset \Leftrightarrow (K_A(x) \cap \alpha) \neq \emptyset$ or $(L_B(x) \cap \alpha) \neq \emptyset \Leftrightarrow x \in K_A^{\cap\alpha} \cup L_B^{\cap\alpha} \Leftrightarrow (K_A \cup_R L_B)^{\cap\alpha} = K_A^{\cap\alpha} \cup L_B^{\cap\alpha}$.

d) $x \in K_A^{\cup\alpha} \cap L_B^{\cup\alpha} \Rightarrow x \in K_A^{\cup\alpha}$ and $x \in L_B^{\cup\alpha} \Rightarrow x \in A, K_A(x) \cup \alpha = U$ and $x \in B, L_B(x) \cup \alpha = U \Rightarrow x \in A \cap B, (K_A(x) \cup L_B(x)) \cup \alpha = U \Rightarrow x \in (K_A \cup_R L_B)^{\cup\alpha}$.

□

The following example shows Theorem 3.8:

Example 3.9. Let $E = \{e_1, e_2, e_3, e_4, e_5\}, A = \{e_1, e_2, e_3, e_4\}, B = \{e_1, e_2, e_3\}$ and let K_A and L_B be following soft sets over $U = \{u_1, u_2, u_3, u_4, u_5\}$;

$$K_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\}), (e_3, \{u_1\}), (e_4, \{u_3\})\},$$
$$L_B = \{(e_1, \{u_3, u_4\}), (e_2, \{u_1, u_2\}), (e_3, \{u_1, u_2\})\}.$$

Let $H_C = K_A \cup_R L_B$, where $H(x) = K_A(x) \cup L_B(x)$ for all $x \in C = A \cap B$. Then,

$$H_C = \{(e_1, \{u_1, u_2, u_3, u_4\}), (e_2, \{u_1, u_2, u_3\}), (e_3, \{u_1, u_2\})\}.$$

Let $\alpha = \{u_1, u_2\}$ and $\beta = \{u_1, u_4, u_5\}$. Then, we have the restricted sets as follows:

$H_C^{\subseteq\alpha} = \{e_3\}$, $K_A^{\subseteq\alpha} = \{e_1, e_3\}$, $L_B^{\subseteq\alpha} = \{e_2, e_3\}$ and hence $(K_A \cup_R L_B)^{\subseteq\alpha} \subseteq K_A^{\subseteq\alpha} \cup L_B^{\subseteq\alpha}$.

$H_C^{\supseteq\alpha} = \{e_1, e_2, e_3\}$, $K_A^{\supseteq\alpha} = \{e_1\}$, $L_B^{\supseteq\alpha} = \{e_2, e_3\}$ and hence $(K_A \cup_R L_B)^{\supseteq\alpha} \supseteq K_A^{\supseteq\alpha} \cap L_B^{\supseteq\alpha}$.

$H_C^{\cap\alpha} = \{e_1, e_2, e_3\}$, $K_A^{\cap\alpha} = \{e_1, e_2, e_3\}$, $L_B^{\cap\alpha} = \{e_2, e_3\}$ and hence $(K_A \cup_R L_B)^{\cap\alpha} = K_A^{\cap\alpha} \cup L_B^{\cap\alpha}$.

$H_C^{\cup\beta} = \{e_1, e_2\}$, $K_A^{\cup\beta} = \{e_2\}$, $L_B^{\cup\beta} = \emptyset$ and hence $(K_A \cup_R L_B)^{\cup\beta} \supseteq K_A^{\cup\beta} \cap L_B^{\cup\beta}$.

Some properties of the restrictive sets according to the operation soft restricted intersection "$\cap_R$" are given as follows:

Theorem 3.10. *Let K_A, L_B be soft sets over U and $\alpha \subseteq U$. Then*

a) $(K_A \cap_R L_B)^{\subseteq\alpha} \supseteq K_A^{\subseteq\alpha} \cap L_B^{\cap\alpha}$.
b) $(K_A \cap_R L_B)^{\supseteq\alpha} = K_A^{\supseteq\alpha} \cap L_B^{\supseteq\alpha}$.
c) $(K_A \cap_R L_B)^{\cup\alpha} = K_A^{\cup\alpha} \cap L_B^{\cup\alpha}$.
d) $(K_A \cap_R L_B)^{\cap\alpha} \subseteq K_A^{\cap\alpha} \cap L_B^{\cap\alpha}$.

Proof. Let $K_A \cap_R L_B = H_C$, where $H_C(x) = K_A(x) \cap L_B(x)$ for all $x \in C = A \cap B$.

a) $x \in K_A^{\subseteq\alpha} \cap L_B^{\subseteq\alpha} \Rightarrow x \in K_A^{\subseteq\alpha}$ and $x \in L_B^{\subseteq\alpha} \Rightarrow x \in A$, $K_A(x) \subseteq \alpha$ and $x \in B$, $L_B(x) \subseteq \alpha \Rightarrow x \in A \cap B$ and $(K_A \cap_R L_B)(x) \subseteq \alpha \Rightarrow x \in (K_A \cap_R L_B)^{\subseteq\alpha}$.
b) $x \in K_A^{\supseteq\alpha} \cap L_B^{\supseteq\alpha} \Leftrightarrow x \in A$, $x \in K_A^{\supseteq\alpha}$ and $x \in B$, $x \in L_B^{\supseteq\alpha} \Leftrightarrow x \in A \cap B$, $K_A(x) \supseteq \alpha$ and $L_B(x) \supseteq \alpha \Leftrightarrow x \in A \cap B$ and $(K_A \cap_R L_B)(x) \supseteq \alpha \Leftrightarrow x \in (K_A \cap_R L_B)^{\supseteq\alpha}$.
c) Since $K_A \cap_R L_B \subseteq K_A$ and $K_A \cap_R L_B \subseteq L_B$, then $(K_A \cap_R L_B)^{\cup\alpha} \subseteq K_A^{\cup\alpha}$ and $(K_A \cap_R L_B)^{\cup\alpha} \subseteq L_B^{\cup\alpha}$ by Theorem 3.3 (d) and therefore

$$(K_A \cap_R L_B)^{\cup\alpha} \subseteq K_A^{\cup\alpha} \cap L_B^{\cup\alpha}. \tag{3.1}$$

$x \in K_A^{\cup\alpha} \cap L_B^{\cup\alpha} \Rightarrow x \in A \cap B$, $K_A(x) \cup \alpha = U$ and $L_B(x) \cup \alpha = U \Rightarrow x \in A \cap B$ and $(K_A(x) \cap L_B(x)) \cup \alpha = U \Rightarrow x \in (K_A \cap_R L_B)^{\cup\alpha}$. Therefore,

$$K_A^{\cup\alpha} \cap L_B^{\cup\alpha} \subseteq (K_A \cap_R L_B)^{\cup\alpha}. \tag{3.2}$$

From (3.1) and (3.2), the proof is completed.

d) $x \in H_C^{\cap\alpha} \Rightarrow (K_A \cap_R L_B)(x) \cap \alpha \neq \emptyset \Rightarrow (K_A(x) \cap L_B(x)) \cap \alpha \neq \emptyset \Rightarrow (K_A(x) \cap \alpha) \neq \emptyset$ and $(L_B(x) \cap \alpha) \neq \emptyset \Rightarrow x \in K_A^{\cap\alpha}$ and $x \in L_B^{\cap\alpha} \Rightarrow x \in K_A^{\cap\alpha} \cap L_B^{\cap\alpha}$.

□

The following example shows Theorem 3.10:

Example 3.11. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$, $A = \{e_1, e_2, e_3\}$, $B = \{e_1, e_2, e_4\}$ and let K_A and L_B be following soft sets over $U = \{u_1, u_2, u_3, u_4, u_5\}$

$$K_A = \{(e_1, \{u_1, u_2, u_5\}), (e_2, \{u_1, u_3\}), (e_3, \{u_3, u_4, u_5\})\},$$
$$L_B = \{(e_1, \{u_2, u_4\}), (e_2, \{u_1, u_3, u_5\}), (e_4, \{u_3\})\}.$$

Let $H_C = K_A \cap_R L_B$, where $H(x) = K_A(x) \cap L_B(x)$ for all $x \in C = A \cap B$. Then, $H_C = \{(e_1, \{u_2\}), (e_2, \{u_1, u_3\})\}$,

Let $\alpha = \{u_1, u_3\}$ and $\theta = \{u_2, u_4, u_5\}$. Then, we have the restricted sets as follows:

Since $H_C^{\subseteq\alpha} = \{e_2\}$, $K_A^{\subseteq\alpha} = \{e_2\}$, $L_B^{\subseteq\alpha} = \{e_4\}$, then we have $(K_A \widetilde{\cap} L_B)^{\subseteq\alpha} \supseteq K_A^{\subseteq\alpha} \cap L_B^{\subseteq\alpha}$.

Since $H_C^{\supseteq\alpha} = \{e_2\}$, $K_A^{\supseteq\alpha} = \{e_2\}$, $L_B^{\supseteq\alpha} = \{e_2\}$, $K_A^{\supseteq\beta} = \{e_2, e_3\}$, $L_B^{\supseteq\beta} = \{e_2, e_4\}$, then it is seen that $(K_A \widetilde{\cap} L_B)^{\supseteq\alpha} = K_A^{\supseteq\alpha} \cap L_B^{\supseteq\alpha}$.

Since $H_C^{\cap\theta} = \{e_1\}$, $K_A^{\cap\theta} = \{e_1, e_3\}$, $L_B^{\cap\theta} = \{e_1, e_2\}$, we see that $(K_A \cap_R L_B)^{\cap\theta} = K_A^{\cap\theta} \cap L_B^{\cap\theta}$.

Since $H_C^{\cup\theta} = \{e_2\}$, $K_A^{\cup\theta} = \{e_2\}$, $L_B^{\cup\theta} = \{e_2\}$, we see that $(K_A \cap_R L_B)^{\cup\theta} = K_A^{\cup\theta} \cap L_B^{\cup\theta}$.

Some properties of the restrictive sets according to OR operation "$\vee$" are given as follows:

Theorem 3.12. *Let K_A, L_B be soft sets over U and $\alpha \subseteq U$. Then*

a) $(K_A \vee L_B)^{\subseteq\alpha} = K_A^{\subseteq\alpha} \times L_B^{\subseteq\alpha}$.
b) $(K_A \vee L_B)^{\cap\alpha} \supseteq K_A^{\cap\alpha} \times L_B^{\cap\alpha}$.
c) $(K_A \vee L_B)^{\supseteq\alpha} \supseteq K_A^{\supseteq\alpha} \times L_B^{\supseteq\alpha}$.
d) $(K_A \vee L_B)^{\cup\alpha} \supseteq K_A^{\cup\alpha} \times L_B^{\cup\alpha}$.

Proof. Let $K_A \vee L_B = (H, A \times B)$, where $H(x, y) = K_A(x) \cup L_B(y)$ for all $(x, y) \in A \times B$.

a) $(x, y) \in H_{A\times B}^{\subseteq\alpha} \Leftrightarrow K_A(x) \cup L_B(y) \subseteq \alpha \Leftrightarrow K_A(x) \subseteq \alpha$ and $L_B(y) \subseteq \alpha \Leftrightarrow (x, y) \in K_A^{\subseteq\alpha} \times L_B^{\subseteq\alpha}$.
b) $(x, y) \in K_A^{\cap\alpha} \times L_B^{\cap\alpha} \Leftrightarrow x \in K_A^{\cap\alpha}$ and $y \in L_B^{\cap\alpha} \Leftrightarrow K_A(x) \cap \alpha \neq \emptyset$ and $L_B(y) \cap \alpha \neq \emptyset \Rightarrow (K_A(x) \cup L_B(y)) \cap \alpha \neq \emptyset \Rightarrow (x, y) \in (K_A \vee L_B)^{\cap\alpha}$.

c) $(x,y) \in K_A^{\supseteq\alpha} \times L_B^{\supseteq\alpha} \Leftrightarrow x \in K_A^{\supseteq\alpha}$ and $y \in L_B^{\supseteq\alpha} \Leftrightarrow K_A(x) \supseteq \alpha$ and $L_B(y) \supseteq \alpha \Rightarrow K_A(x) \cup L_B(y) \supseteq \alpha \Rightarrow (x,y) \in (K_A \vee L_B)^{\supseteq\alpha}$.

d) $(x,y) \in K_A^{\cup\alpha} \times L_B^{\cup\alpha} \Leftrightarrow x \in K_A^{\cup\alpha}$ and $y \in L_B^{\cup\alpha} \Leftrightarrow K_A(x) \cup \alpha = U$ and $L_B(y) \cup \alpha = U \Rightarrow (K_A(x) \cup L_B(y)) \cup \alpha = U \Rightarrow (x,y) \in (K_A \vee L_B)^{\cup\alpha}$.

□

Some properties of the restrictive sets according to AND operation "$\wedge$" are given as follows:

Theorem 3.13. *Let K_A, L_B be soft sets over U and $\alpha \subseteq U$. Then*

a) $(K_A \wedge L_B)^{\cap\alpha} \subseteq K_A^{\cap\alpha} \times L_B^{\cap\alpha}$.
b) $(K_A \wedge L_B)^{\cup\alpha} = K_A^{\cup\alpha} \times L_B^{\cup\alpha}$.
c) $(K_A \wedge L_B)^{\subseteq\alpha} \supseteq K_A^{\subseteq\alpha} \times L_B^{\subseteq\alpha}$.
d) $(K_A \wedge L_B)^{\supseteq\alpha} = K_A^{\supseteq\alpha} \times L_B^{\supseteq\alpha}$.

Proof. Let $K_A \wedge L_B = (H, A \times B)$, where $H(x,y) = K_A(x) \cap L_B(y)$ for all $(x,y) \in A \times B$.

a) $(x,y) \in H_{A\times B}^{\cap\alpha} \Rightarrow (K_A(x) \cap L_B(y)) \cap \alpha \neq \emptyset \Rightarrow K_A(x) \cap \alpha \neq \emptyset$ and $L_B(y) \cap \alpha \neq \emptyset \Rightarrow x \in K_A^{\cap\alpha}$ and $y \in L_B^{\cap\alpha} \Rightarrow (x,y) \in K_A^{\cap\alpha} \times L_B^{\cap\alpha}$.

b) $(x,y) \in H_{A\times B}^{\cup\alpha} \Leftrightarrow (K_A(x) \cap L_B(y)) \cup \alpha = U \Leftrightarrow (K_A(x) \cup \alpha) \cap (L_B(y) \cup \alpha) = U \Leftrightarrow K_A(x) \cup \alpha = U$ and $L_B(y) \cup \alpha = U \Leftrightarrow x \in K_A^{\cup\alpha}$ and $y \in L_B^{\cup\alpha} \Leftrightarrow (x,y) \in K_A^{\cup\alpha} \times L_B^{\cup\alpha}$.

c) $(x,y) \in K_A^{\subseteq\alpha} \times L_B^{\subseteq\alpha} \Leftrightarrow x \in K_A^{\subseteq\alpha}$ and $y \in L_B^{\subseteq\alpha} \Leftrightarrow K_A(x) \subseteq \alpha$ and $L_B(y) \subseteq \alpha \Rightarrow (K_A(x) \cap L_B(y)) \subseteq \alpha \Rightarrow (x,y) \in (K_A \wedge L_B)^{\subseteq\alpha}$.

d) $(x,y) \in H_{A\times B}^{\supseteq\alpha} \Leftrightarrow K_A(x) \cap L_B(y) \supseteq \alpha \Leftrightarrow K_A(x) \supseteq \alpha$ and $L_B(y) \supseteq \alpha \Leftrightarrow (x,y) \in K_A^{\supseteq\alpha} \times L_B^{\supseteq\alpha}$.

□

The following example shows Theorem 3.12 and 3.13:

Example 3.14. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$, $A = \{e_1, e_2, e_3\}$, $B = \{e_1, e_2, e_4\}$ and let K_A and L_B be following soft sets over $U = \{u_1, u_2, u_3, u_4, u_5\}$

$$K_A = \{(e_1, \{u_1, u_2, u_5\}), (e_2, \{u_1, u_3\}), (e_3, \{u_3, u_4, u_5\})\},$$
$$L_B = \{(e_1, \{u_2, u_4\}), (e_2, \{u_1, u_3, u_5\}), (e_4, \{u_3\})\}.$$

Then,

$K_A \vee L_B =$
$\{((e_1,e_1), \{u_1,u_2,u_4,u_5\}), ((e_1,e_2), \{u_1,u_2,u_3,u_5\}), ((e_1,e_4), \{u_1,u_2,u_3,u_5\}),$
$((e_2,e_1), \{u_1,u_2,u_3,u_4\}), ((e_2,e_2), \{u_1,u_3,u_5\}), ((e_2,e_4), \{u_1,u_3\}),$
$((e_3,e_1), \{u_2,u_3,u_4,u_5\}), ((e_3,e_2), \{u_1,u_3,u_4,u_5\}), ((e_3,e_4), \{u_3,u_4,u_5\})\}.$

$K_A \wedge L_B = \{((e_1,e_1), \{u_2\}), ((e_1,e_2), \{u_1,u_5\}), ((e_1,e_4), \emptyset),$
$((e_2,e_1), \emptyset), ((e_2,e_2), \{u_1,u_3\}), ((e_2,e_4), \{u_3\}),$
$((e_3,e_1), \{u_4\}), ((e_3,e_2), \{u_3,u_5\}), ((e_3,e_4), \{u_3\})\}.$

Let $\alpha = \{u_1, u_3\}$, $\beta = \{u_3\}$, $\theta = \{u_4, u_5\}$ and $\sigma = \{u_2, u_3, u_4\}$. Then, we have the restricted sets as follows:

$(K_A \vee L_B)^{\subseteq\alpha} = \{(e_2, e_4)\}$, $(K_A \wedge L_B)^{\subseteq\alpha} = \{(e_1, e_4), (e_2, e_1), (e_2, e_2), (e_2, e_4), (e_3, e_4)\}$, $K_A^{\subseteq\alpha} = \{e_2\}$, $L_B^{\subseteq\alpha} = \{e_4\}$ and $K_A^{\subseteq\alpha} \times L_B^{\subseteq\alpha} = \{(e_2, e_4)\}$.

$(K_A \vee L_B)^{\supseteq\alpha} = \{(e_1, e_2), (e_1, e_4), (e_2, e_1), (e_2, e_2), (e_2, e_4), (e_3, e_2)\}$, $(K_A \wedge L_B)^{\supseteq\alpha} = \{(e_2, e_2)\}$, $K_A^{\supseteq\alpha} = \{e_2\}$, $L_B^{\supseteq\alpha} = \{e_2\}$ and $K_A^{\supseteq\alpha} \times L_B^{\supseteq\alpha} = \{(e_2, e_2)\}$.

$(K_A \vee L_B)^{\cap\beta} = \{(e_1, e_2), (e_1, e_4), (e_2, e_1), (e_2, e_2), (e_2, e_4), (e_3, e_1), (e_3, e_2), (e_3, e_4)\}$, $K_A^{\cap\beta} = \{e_2, e_3\}$, $L_B^{\cap\beta} = \{e_2, e_4\}$ and $K_A^{\cap\beta} \times L_B^{\cap\beta} = \{(e_2, e_2), (e_2, e_4), (e_3, e_2), (e_3, e_4)\}$.

$(K_A \wedge L_B)^{\cap\theta} = \{(e_1, e_2), (e_3, e_1), (e_3, e_2)\}$,
$K_A^{\cap\theta} = \{e_1, e_3\}$, $L_B^{\cap\theta} = \{e_1, e_2\}$ and $K_A^{\cap\theta} \times L_B^{\cap\theta} = \{(e_1, e_1), (e_1, e_2), (e_3, e_1), (e_3, e_2)\}$.

$(K_A \vee L_B)^{\cup\sigma} = \{(e_1, e_1), (e_1, e_2), (e_1, e_4), (e_2, e_2), (e_3, e_2)\}$,
$(K_A \wedge L_B)^{\cup\sigma} = \{(e_1, e_2)\}$, $K_A^{\cup\sigma} = \{e_1\}$, $L_B^{\cup\sigma} = \{e_2\}$ and $K_A^{\cup\sigma} \times L_B^{\cup\sigma} = \{(e_1, e_2)\}$.

Some properties of the restrictive sets according to the operation restricted difference "$\sim_R$" are given as follows:

Theorem 3.15. *Let K_A, L_B be soft sets over U and $\alpha \subset U$. Then,*

a) $(K_A \sim_R L_B)^{\subseteq\alpha} \supseteq K_A^{\subseteq\alpha} \setminus L_B^{\subseteq\alpha}$.
b) $(K_A \sim_R L_B)^{\supseteq\alpha} \subseteq K_A^{\supseteq\alpha} \setminus L_B^{\supseteq\alpha}$.
c) $(K_A \sim_R L_B)^{\cup\alpha} \subseteq K_A^{\cup\alpha} \setminus L_B^{\cup\alpha}$.

Proof. $K_A \sim_R L_B = H_C$, where $C = A \cap B$ and $H(x) = K_A(x) \setminus L_B(x)$ for all $x \in C$.

a) $x \in K_A^{\subseteq\alpha} \setminus L_B^{\subseteq\alpha} \Rightarrow x \in K_A^{\subseteq\alpha}$ and $x \notin L_B^{\subseteq\alpha} \Rightarrow K_A(x) \subseteq \alpha$ and $L_B(x) \nsubseteq \alpha \Rightarrow (K_A(x) \setminus L_B(x)) \subseteq K_A(x) \subseteq \alpha \Rightarrow x \in (K_A \sim_R L_B)^{\subseteq\alpha}$.
b) $x \in H_C^{\supseteq\alpha} \Rightarrow \alpha \subseteq (K_A(x) \setminus L_B(x)) \subseteq K_A(x)$ and since $[(K_A(x) \setminus L_B(x)) \cap L_B(x) = \emptyset] \Rightarrow K_A(x) \supseteq \alpha$ and $L_B(x) \nsupseteq \alpha \Rightarrow x \in K_A^{\supseteq\alpha} \setminus L_B^{\supseteq\alpha}$.
c) $x \in H_C^{\cup\alpha} \Rightarrow (K_A(x) \setminus L_B(x)) \cup \alpha = U \Rightarrow (K_A(x) \cap (U \setminus L_B(x))) \cup \alpha = U \Rightarrow (K_A(x) \cup \alpha) \cap ((U \setminus L_B(x)) \cup \alpha) = U \Rightarrow K_A(x) \cup \alpha = U$ and $(U \setminus L_B(x)) \cup \alpha = U \Rightarrow K_A(x) \cup \alpha = U$ and $L_B(x) \cup \alpha \neq U \Rightarrow x \in K_A^{\cup\alpha}$ and $x \notin L_B^{\cup\alpha} \Rightarrow x \in K_A^{\cup\alpha} \setminus L_B^{\cup\alpha}$.

□

Example 3.16. Let $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2\}$, $B = \{e_1, e_3\}$ and let K_A and L_B be following soft sets over $U = \{u_1, u_2, u_3, u_4\}$

$$K_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\},$$
$$L_B = \{(e_1, \{u_2, u_4\}), (e_3\{u_1, u_3\})\}.$$

If $K_A \sim_R L_B = H_C$, where $C = A \cap B$ and $H(x) = K_A(x) \setminus L_B(x)$ for all $x \in C$, then

$$H_C = \{(e_1, \{u_1\})\}.$$

Let $\alpha = \{u_1, u_4\}$, $\beta = \{u_1, u_2\}$. Then, we have the restricted sets as follows:

$H_C^{\subseteq\alpha} = \{e_1\}$, $K_A^{\subseteq\alpha} = \emptyset$, $L_B^{\subseteq\alpha} = \emptyset$, then $(K_A \sim_R L_B)^{\subseteq\alpha} \supseteq K_A^{\subseteq\alpha} \setminus L_B^{\subseteq\alpha}$,

$H_C^{\supseteq\beta} = \emptyset$, $K_A^{\supseteq\beta} = \{e_1\}$, $L_B^{\supseteq\beta} = \emptyset$, then $(K_A \sim_R L_B)^{\supseteq\beta} \subseteq K_A^{\supseteq\beta} \setminus L_B^{\supseteq\beta}$ and

$H_C^{\cup\alpha} = \emptyset$, $K_A^{\cup\alpha} = \{e_2\}$, $L_B^{\cup\alpha} = \emptyset$, then $(K_A \sim_R L_B)^{\cup\alpha} \subseteq K_A^{\cup\alpha} \setminus L_B^{\cup\alpha}$.

Proposition 3.17. [26] *Let K_A be a soft set over U and $\alpha, \beta \subseteq U$. If $\alpha \subseteq \beta$, then $K_A^{\supseteq\beta} \subseteq K_A^{\supseteq\alpha}$.*

Corollary 3.18. *Let K_A be a soft set over U and $\alpha, \beta \subseteq U$. Then, $K_A^{\supseteq(\alpha\cap\beta)} \supseteq K_A^{\supseteq\alpha} \cup K_A^{\supseteq\beta} \supseteq K_A^{\supseteq\alpha} \cap K_A^{\supseteq\beta}$.*

Proof. $K_A^{\supseteq(\alpha\cap\beta)} \supseteq K_A^{\supseteq\alpha}$ and $K_A^{\supseteq(\alpha\cap\beta)} \supseteq K_A^{\supseteq\beta}$ by Theorem 3.17, then we have $K_A^{\supseteq(\alpha\cap\beta)} \supseteq K_A^{\supseteq\alpha} \cup K_A^{\supseteq\beta} \supseteq K_A^{\supseteq\alpha} \cap K_A^{\supseteq\beta}$. □

Proposition 3.19. [26] *Let K_A be a soft set over U and $\alpha, \beta \subseteq U$. If $\alpha \subseteq \beta$, then $K_A^{\subseteq\alpha} \subseteq K_A^{\subseteq\beta}$.*

Corollary 3.20. *Let K_A be a soft set over U and $\alpha, \beta \subseteq U$. Then, $K_A^{\subseteq(\alpha\cap\beta)} \subseteq K_A^{\subseteq\alpha} \cap K_A^{\subseteq\beta} K_A^{\subseteq\alpha} \cup K_A^{\subseteq\beta}$.*

Theorem 3.21. *Let K_A be a soft set over U and $\alpha, \beta \subseteq U$. If $\beta \subseteq \alpha \subseteq U$, then*

$$K_A^{\subseteq(\alpha\setminus\beta)} \subseteq K_A^{\subseteq\alpha} \setminus K_A^{\subseteq\beta}.$$

Proof. If $x \in K_A^{\subseteq(\alpha\setminus\beta)} \Rightarrow K_A(x) \subseteq (\alpha \setminus \beta) \Rightarrow K_A(x) \subseteq \alpha$ and $K_A(x) \nsubseteq \beta \Rightarrow x \in K_A^{\subseteq\alpha}$ and $x \notin K_A^{\subseteq\beta} \Rightarrow x \in K_A^{\subseteq\alpha} \setminus K_A^{\subseteq\beta}$. □

Theorem 3.22. *Let K_A be a soft set over U and $\alpha, \beta \subseteq U$. Then,*

$$K_A^{\supseteq(\alpha\setminus\beta)} \supseteq K_A^{\supseteq\alpha} \setminus K_A^{\supseteq\beta}.$$

Proof. If $x \in K_A^{\supseteq\alpha} \setminus K_A^{\supseteq\beta} \Rightarrow K_A(x) \supseteq \alpha$ and $K_A(x) \nsupseteq \beta \Rightarrow K_A(x) \supseteq (\alpha \setminus \beta) \Rightarrow x \in K_A^{\supseteq(\alpha\setminus\beta)}$. □

Some properties of soft sets that are restricted by α-intersection and α-union are given by the following Theorems:

Theorem 3.23. *Let K_A be a soft set over U and $\alpha, \beta \subseteq U$. Then*

a) $K_A^{\cup(\alpha\cap\beta)} \subseteq K_A^{\cup\alpha} \cap K_A^{\cup\beta}$.
b) $K_A^{\cup(\alpha\cup\beta)} \supseteq K_A^{\cup\alpha} \cup K_A^{\cup\beta}$.

Proof. a) If $x \in K_A^{\cup(\alpha\cap\beta)} \Rightarrow K_A(x) \cup (\alpha \cap \beta) = U \Rightarrow K_A(x) \cup \alpha = U$ and $L_B(x) \cup \beta = U \Rightarrow x \in K_A^{\cup\alpha} \cap K_A^{\cup\beta}$.
b) If $x \in K_A^{\cup\alpha} \cup K_A^{\cup\beta} \Rightarrow K_A(x) \cup \alpha = U$ or $K_A(x) \cup \beta = U \Rightarrow K_A(x) \cup (\alpha \cup \beta) = U \Rightarrow x \in K_A^{\cup(\alpha\cup\beta)}$.

□

Theorem 3.24. *Let K_A be a soft set over U and let $\beta \subseteq \alpha \subseteq U$. Then*

a) $K_A^{\cup(\alpha\setminus\beta)} \subseteq K_A^{\cup\alpha} \setminus K_A^{\cup\beta}$.
b) $K_A^{\cup(\alpha\setminus\beta)} = K_A^{\cup\alpha} \cap K_A^{\cup\beta^t}$, *where* $\beta^t = U \setminus \beta$.

Proof. a) If $x \in K_A^{\cup(\alpha\setminus\beta)} \Rightarrow K_A(x) \cup (\alpha \setminus \beta) = U \Rightarrow K_A(x) \cup \alpha = U$ and $K_A(x) \cup \beta \neq U \Rightarrow x \in K_A^{\cup\alpha}$ and $x \notin K_A^{\cup\beta} \Rightarrow x \in K_A^{\cup\alpha} \setminus K_A^{\cup\beta}$.

b) If $x \in K_A^{\cup(\alpha\setminus\beta)} \Leftrightarrow K_A(x)\cup(\alpha\setminus\beta) = U \Leftrightarrow K_A(x)\cup(\alpha\cap\beta^t) = U \Leftrightarrow (K_A(x)\cup\alpha)\cap(K_A(x)\cup\beta^t) = U \Leftrightarrow K_A(x)\cup\alpha = U$ and $K_A(x)\cup\beta^t = U \Leftrightarrow x \in K_A^{\cup\alpha}$ and $x \in K_A^{\cup\beta^t} \Leftrightarrow x \in K_A^{\cup\alpha} \cap K_A^{\cup\beta^t}$.

□

Proposition 3.25. [6] *Let K_A be a soft set over U and let $\emptyset \neq \alpha, \beta \subseteq U$. Then*

a) $K_A^{\cap\alpha} \cap K_A^{\cap\beta} \subseteq K_A^{\cap(\alpha\cup\beta)}$. *Here the equality does not hold in general, even if* $\alpha \cap \beta = \emptyset$.

b) $K_A^{\cap\alpha} \cup K_A^{\cap\beta} = K_A^{\cap(\alpha\cup\beta)}$.

c) $K_A^{\cap(\alpha\cap\beta)} \subseteq K_A^{\cap\alpha} \cap K_A^{\cap\beta}$.

Proposition 3.26. [6] *Let K_A be a soft set over U and let $\emptyset \neq \alpha \subseteq U$. Then*

a) $K_A^{\cap\alpha} \subseteq suppK_A$.

b) *If $\alpha \subseteq K(x)$ for all $x \in A$, then $K_A^{\cap\alpha} = suppK_A = A$.*

c) *If $K(x) \neq \emptyset$ and $K(x) \subseteq \alpha$ for all $x \in A$, $K_A^{\cap\alpha} = suppK_A = A$.*

d) *If $K_A = \mathcal{U}_A$, then $K_A^{\cap\alpha} = suppK_A = A$.*

e) *If $K_A = \Phi_A$ or $suppK_A = \emptyset$, then $K_A^{\cap\alpha} = \emptyset$.*

Proposition 3.27. [6] *Let K_A be a soft set over U and let $\emptyset \neq \alpha \subseteq U$. Then*

a) *If $\alpha \subseteq \beta$, then $K_A^{\cap\alpha} \subseteq K_A^{\cap\beta}$.*

b) $(F^c, A)^{\cap\alpha} = \{x \in A | \alpha \setminus K(x) \neq \emptyset\}$.

c) $K_A^{\cap\alpha^t} = \{x \in A | K(x) \setminus \alpha \neq \emptyset\}$.

d) $(F^c, A)^{\cap\alpha^t} = \{x \in A | U \setminus (K(x) \cup \alpha) \neq \emptyset\}$.

Proposition 3.28. [6] *Let K_A be a soft set over U and let $\emptyset \neq \alpha \subsetneq U$. If $K(x) \cup \alpha \neq U$ for all $x \in A$, then*

a) $suppK_A \subseteq (F^c, A)^{\cap\alpha^t}$.

b) $K_A^{\cap\alpha} \subseteq (F^c, A)^{\cap\alpha^t}$.

Theorem 3.29. *Let K_A be a soft set over U and $\alpha, \beta \subseteq U$. If $\beta \subseteq \alpha \subseteq U$, then*

$$K_A^{\cap(\alpha\setminus\beta)} \supseteq K_A^{\cap\alpha} \setminus K_A^{\cap\beta}.$$

Proof. $x \in K_A^{\cap\alpha} \setminus K_A^{\cap\beta} \Leftrightarrow x \in K_A^{\cap\alpha}$ and $x \notin K_A^{\cap\beta} \Leftrightarrow K_A(x) \cap \alpha \neq \emptyset$ and $K_A(x) \cap \beta = \emptyset \Rightarrow K_A(x) \cap (\alpha \setminus \beta) \neq \emptyset \Leftrightarrow x \in K_A^{\cap(\alpha\setminus\beta)}$. □

4. α-Intersections Applied to Group Theory Via Soft Set

Definition 4.1. [9] Let G be a group and K_G be a soft set over U. Then K_G is called a soft int-group of G over U if

i) $K(xy) \supseteq K(x) \cap K(y)$ for all $x, y \in G$,

ii) $K(x^{-1}) = K(x)$ for all $x \in G$.

Theorem 4.2. [9] *A soft set K_G over U is a soft int-group over U if and only if $K(xy^{-1}) \supseteq K(x) \cap K(y)$ for all $x, y \in G$.*

Theorem 4.3. [9] *Let K_G be a soft int-group over U. Then, $K(e_G) \supseteq K(x)$ for all $x \in G$. Here, e_G is the identity element of the group G.*

Now, we are ready to introduce the concept of soft intersection subgroup of a group G generated by $\alpha \subseteq U$.

Definition 4.4. Let G be a group, $\emptyset \neq \alpha \subseteq U$ and K_G be a soft set over U. If the soft set $(K, K_G^{\cap\alpha})$ is a soft int-group over U, then it is called a *soft intersection subgroup of* G *generated by the set* α and denoted by $< \alpha >_\cap \widetilde{\leq} G$.

Example 4.5. Let $G = (\mathbb{Z}_6, +)$ be the group, the soft set K_G over $U = S_3$, where $K : G \longrightarrow P(U)$ is a set-valued function defined by
$K(0) = \{(1), (12), (132), (123)\}$, $K(1) = \{(23)\}$, $K(2) = \{(12)\}$,
$K(3) = \{(1), (132), (123)\}$, $K(4) = \{(12)\}$, $K(5) = \{(23)\}$.
Let $\alpha = \{(132), (123)\} \subseteq U$. Then

$$K_G^{\cap\alpha} = \{0, 3\}$$

and
$< \alpha >_\cap = (K, K_G^{\cap\alpha}) = \{(0, \{(1), (12), (132), (123)\}), (3, \{(132), (123)\})\} \widetilde{\leq} G.$

Let $\beta = \{(12)\} \subseteq U$. Then

$$K_G^{\cap\beta} = \{0, 2, 4\}$$

and
$< \beta >_\cap = (K, K_G^{\cap\beta}) = \{(0, \{(1), (12), (132), (123)\}), (2, \{(12)\}), (4, \{(12)\})\} \widetilde{\leq} G.$
Therefore, the soft set $\{(0, K(0)), (2, K(2)), (4, K(4))\}$ is a soft intersection subgroup of G generated by the element $(12) \in S_3$.

Theorem 4.6. *Let K_G be a soft set over U. Then, $(K, \{e_G\}) \widetilde{\leq} G$. But the soft intersection subgroup $(K, \{e_G\})$ is not set generated.*

Proof. Since $K : \{e_G\} \longrightarrow P(U)$ is a function such that

$$K(xy^{-1}) = K(e_G) \supseteq K(e_G) \cap K(e_G) = K(x) \cap K(y)$$

for all $x, y \in \{e_G\}$, then $(K, \{e_G\}) \widetilde{\leq} G$ by Theorem 4.2.
For the rest of the proof, we have $(K, \{0\}) \widetilde{\leq} \mathbb{Z}_6$ in Example 4.5. But there is no $\alpha \subseteq S_3$ such that $(K, \{0\}) = < \alpha >_\cap$, since $K(e_G) \supseteq K(x)$ for all $x \in G$. □

Definition 4.7. The soft subgroup $(K, \{e_G\})$ of G is called the trivial soft subgroup of G.

Proposition 4.8. *If $< \alpha >_\cap \widetilde{\leq} G$, then the generator α doesn't have to be unique. Furthermore, if $< \{a\} >_\cap = < \{b\} >_\cap$ for $a, b \in U$, then $< \{a, b\} >_\cap = < \{a\} >_\cap$.*

Proof. Let $\alpha = \{a\}$, $\beta = \{b\}$ for $a, b \in U$ such that $< \alpha >_\cap = < \beta >_\cap$. Consider the sets

$$A = \{x \in G | K(x) \cap \{a\} \neq \emptyset\} = \{x \in G | K(x) \cap \{b\} \neq \emptyset\}$$

and

$$B = \{x \in G | K(x) \cap \{a, b\} \neq \emptyset\}.$$

Obviously $A \subseteq B$.
Let $x \in B$. Then,

$$\begin{aligned} K(x)\cap\{a,b\}\neq\emptyset &\Rightarrow K(x)\cap\{a\}\neq\emptyset \text{ or } K(x)\cap\{b\}\neq\emptyset \\ &\Rightarrow x\in A \text{ or } x\in A \\ &\Rightarrow x\in A \end{aligned}$$

Hence $B\subseteq A$. Therefore, $K_G^{\cap\{a\}}=K_G^{\cap\{b\}}=K_G^{\cap\{a,b\}}$, which implies that $<\{a,b\}>_\cap=<\{a\}>_\cap$. For the rest of the proof, we have Example 4.9. □

Example 4.9. In Example 4.5, if we take $\alpha=\{(132)\}$ and $\beta=\{(123)\}$, then it is seen that
$<\alpha>_\cap=<\beta>_\cap=<(132),(123)>_\cap$.

The following proposition gives a relation between set generated soft subgroups of a group and subgroups of that group:

Proposition 4.10. *Let K_G be a soft int-group over U and $\emptyset\neq\alpha\subseteq U$. If $<\alpha>_\cap \widetilde{\leq} G$, then $K_G^{\cap\alpha}$ is a subgroup of G.*

Proof. Since $<\alpha>_\cap=(K,K_G^{\cap\alpha})\widetilde{\leq}G$, then $\emptyset\neq K_G^{\cap\alpha}\subseteq G$ and $K(xy^{-1})=K(x)\cap K(y)$ for all $x,y\in K_G^{\cap\alpha}$. Since $F:K_G^{\cap\alpha}\longrightarrow P(U)$ is function, then $xy^{-1}\in K_G^{\cap\alpha}$ for all $x,y\in K_G^{\cap\alpha}$. Therefore $K_G^{\cap\alpha}\leq G$. □

Proposition 4.11. *Let G be a group and K_G be a soft int-group over U. If $<\alpha>_\cap \widetilde{\leq} G$, then $F(e_G)\cap\alpha\neq\emptyset$. But the converse is not true, in general.*

Proof. Let $<\alpha>_\cap \widetilde{\leq} G$. Then, by Proposition 4.10, the set $K_G^{\cap\alpha}=\{x\in G|K(x)\cap\alpha\neq\emptyset\}$ is a subgroup of G, Since $K(x)\cap\alpha\neq\emptyset$ and $F(e_G)\supseteq K(x)$ for all $x\in K_G^{\cap\alpha}$, then $K(e_G)\cap\alpha\neq\emptyset$. For the rest of the proof, we have Example 4.12 : □

Example 4.12. In Example 4.5 , if we take $\lambda=\{(1),(12),(123)\}$, then it is seen that $K(0)\cap\lambda\neq\emptyset$. But $K_G^{\cap\lambda}=\{0,2,3,4\}$ and then $(K,K_G^{\cap\lambda})$ is not a soft int-subgroup of G.

Proposition 4.13. *Let $<\alpha>_\cap \widetilde{\leq} G$ and $<\beta>_\cap \widetilde{\leq} G$. If $\alpha\subseteq\beta$, then $<\alpha>_\cap \widetilde{\subseteq} <\beta>_\cap$.*

Proof. Let $x\in K_G^{\cap\alpha}$. Then, $K(x)\cap\alpha\neq\emptyset$. Since $\alpha\subseteq\beta$, $K(x)\cap\beta\neq\emptyset$. Hence $K_G^{\cap\alpha}\subseteq K_G^{\cap\beta}$. Thus, $(K,K_G^{\cap\alpha})\widetilde{\subseteq}(K,K_G^{\cap\beta})$ implying that $<\alpha>_\cap \widetilde{\subseteq} <\beta>_\cap$. □

Theorem 4.14. [9] *Let G be a group and K_G, T_G be soft sets over U. If $K_G\widetilde{\leq}G$ and $T_G\widetilde{\leq}G$, then $K_G\cap_R T_G\widetilde{\leq}G$.*

Corollary 4.15. *If $<\alpha>_\cap \widetilde{\leq} G$ and $<\beta>_\cap \widetilde{\leq} G$, then $<\alpha>_\cap \cap_R <\beta>_\cap$ is also a soft int-subgroup of G. But it does not have to be set-generated.*

Proof. If $<\alpha>_\cap \widetilde{\leq} G$ and $<\beta>_\cap \widetilde{\leq} G$, then $<\alpha>_\cap \cap_R <\beta>_\cap \widetilde{\leq} G$ by Theorem 4.14. We have the following example for the rest of the proof. □

Example 4.16. Let $G=(\mathbb{Z}_{12},+)$ be group, K_G be the soft set over $U=\{u_1,u_2,u_3,...,u_{15}\}$. Here, U represent 15 different cancer diseases, where $F:G\longrightarrow P(U)$ is a set-valued function defined by
$K(0)=\{u_1,u_3,u_5,u_6,u_7,u_9,u_{11}\}$, $K(1)=\{u_2,u_4\}$, $K(2)=\{u_3,u_6,u_7,u_{11}\}$, $K(3)=\{u_1,u_5,u_9\}$,

$K(4) = \{u_3, u_6, u_7, u_{11}\}$, $K(5) = \{u_1, u_8\}$, $K(6) = \{u_1, u_3, u_5, u_6, u_7, u_9, u_{11}\}$, $K(7) = \{u_2, u_{10}\}$,
$K(8) = \{u_3, u_6, u_7, u_{11}\}$, $K(9) = \{u_1, u_5, u_9\}$, $K(10) = \{u_3, u_6, u_7, u_{11}\}$, $K(11) = \{u_2, u_8\}$. Let $\alpha = \{u_{11}\}$ and $\beta = \{u_5\}$. Then, $K_G^{\cap\alpha} = \{0, 2, 4, 6, 8, 10\}$ and $K_G^{\cap\beta} = \{0, 3, 6, 9\}$ are subgroups of G and the soft sets

$$\begin{aligned}(K, K_G^{\cap\alpha}) =&\{(0, \{u_1, u_3, u_5, u_6, u_7, u_9, u_{11}\}), (2, \{u_3, u_6, u_7, u_{11}\}), (4, \{u_3, u_6, u_7, u_{11}\}),\\ &(6, \{u_1, u_3, u_5, u_6, u_7, u_9, u_{11}\}), (8, \{u_3, u_6, u_7, u_{11}\}), (10, \{u_3, u_6, u_7, u_{11}\})\},\end{aligned}$$

$$\begin{aligned}(K, K_G^{\cap\beta}) =&\{(0, \{u_1, u_3, u_5, u_6, u_7, u_9, u_{11}\}), (3, \{u_1, u_5, u_9\}), (6, \{u_1, u_3, u_5, u_6, u_7, u_9, u_{11}\}),\\ &(9, \{u_1, u_5, u_9\})\}\end{aligned}$$

are soft int-groups. Hence $<\alpha>_\cap \widetilde{\leq} G$ and $<\beta>_\cap \widetilde{\leq} G$. Now,

$$<\alpha>_\cap \cap_R <\beta>_\cap = \{(0, \{u_1, u_3, u_5, u_6, u_7, u_9, u_{11}\}), (6, \{u_1, u_3, u_5, u_6, u_7, u_9, u_{11}\})\} \widetilde{\leq} G.$$

But there is no subset θ of U such that $<\alpha>_\cap \cap_R <\beta>_\cap = <\theta>_\cap$.

We have obtained that the intersections of set-generated subgroups do not necessarily have to be a set-generated subgroup. The following theorem gives us the answer of the question "What will the generators be, if the intersection is also a set-generated subgroup?"

Theorem 4.17. *Let G be a group, $<\alpha>_\cap \widetilde{\leq} G$ and $<\beta>_\cap \widetilde{\leq} G$. Then,* ***either***

$$<\alpha>_\cap \cap_R <\beta>_\cap \text{ is a trivial subgroup}$$

or ***if*** *there exists $\sigma \subseteq U$ such that*

$$<\alpha>_\cap \cap_R <\beta>_\cap = <\sigma>_\cap,$$

then $\emptyset \neq \sigma \subseteq \alpha \cup \beta$.

Proof. If $<\alpha>_\cap \widetilde{\cap} <\beta>_\cap$ is a trivial subgroup, we are done. Assume that there exists a $\sigma \subseteq U$ such that trivial subgroup is not equal to $<\alpha>_\cap \cap_R <\beta>_\cap = <\sigma>_\cap$. Then, $<\sigma>_\cap \widetilde{\leq} G$ by Theorem 4.14. Since $<\alpha>_\cap \cap_R <\beta>_\cap = <\sigma>_\cap$, then we have

$$V \cap \alpha \neq \emptyset \text{ and } V \cap \beta \neq \emptyset \Leftrightarrow V \cap \sigma \neq \emptyset \tag{4.1}$$

for $(x, V) \in <\sigma>_\cap$. The requirement (4.1) holds for:

i) $\alpha \subseteq \beta \Rightarrow \sigma = \beta$,
ii) $\beta \subseteq \alpha \Rightarrow \sigma = \alpha$,
iii) $\alpha \cap \beta \neq \emptyset \Rightarrow \sigma = \alpha \cup \beta$
iv) $\alpha \neq \beta$ and $\alpha \cap \beta \neq \emptyset \Rightarrow \sigma = \alpha \cup \beta$.

Although the requirement (4.1) also holds for $\sigma \supseteq \alpha \cup \beta$, it is enough to show there exists $\sigma \subseteq U$ such that $\emptyset \neq \sigma \subseteq \alpha \cup \beta$ to complete the proof. □

Definition 4.18. [9] Let K_G and T_H be soft int-groups over U. Then, the product of soft int-groups K_G and T_H is defined as $K_G \times T_H = (Q, G \times H)$, where $Q(x, y) = K(x) \times T(y)$ for all $(x, y) \in G \times H$.

Theorem 4.19. [9] *If K_G and T_H are soft int-groups over U, then so is $K_G \times T_H$ over $U \times U$.*

In the following theorem, we show that the product of two set-generated soft intersection subgroups is also a set-generated soft intersection subgroup.

Theorem 4.20. *Let G_1, G_2 be groups and K_{G_1}, T_{G_2} be two soft sets over U_1 and U_2, respectively. If there exist $\alpha \subseteq U_1$ and $\beta \subseteq U_2$ such that $< \alpha >_\cap \widetilde{\leq} G_1$ and $< \beta >_\cap \widetilde{\leq} G_2$, then*

$$< \alpha >_\cap \times < \beta >_\cap = < \alpha \times \beta >_\cap .$$

Proof. Let $< \alpha >_\cap \widetilde{\leq} G_1$ and $< \beta >_\cap \widetilde{\leq} G_2$. Then $K_{G_1}^{\cap\alpha} \leq G_1$ and $T_{G_2}^{\cap\beta} \leq G_2$ by Proposition 4.10 and

$$(K, K_{G_1}^{\cap\alpha}) \times (T, T_{G_2}^{\cap\beta}) \widetilde{\leq} G_1 \times G_2$$

by Theorem 4.19. Now, let $(x, K(x)) \in < \alpha >_\cap$ and $(y, T(y)) \in < \beta >_\cap$. Then $K(x) \cap \alpha \neq \emptyset$ and $T(y) \cap \beta \neq \emptyset$. Then we have

$$\begin{aligned} K(x) \cap \alpha \neq \emptyset \text{ and } T(y) \cap \beta \neq \emptyset &\Leftrightarrow (K(x) \times T(y)) \cap (\alpha \times \beta) \neq \emptyset \\ &\Leftrightarrow (x, K(x)) \in < \alpha >_\cap \text{ and } (y, T(y)) \in < \beta >_\cap \\ &\Leftrightarrow ((x,y), K(x) \times T(y)) \in (H, (H, G_1 \times G_2)^{\cap(\alpha\times\beta)}) \end{aligned}$$

where $(H, G_1 \times G_2)$ is a soft set over $U_1 \times U_2$ and where $H : G_1 \times G_2 \longrightarrow P(U_1 \times U_2)$ is a set-valued function defined by $H(x,y) = K(x) \times T(y)$. Hence the proof is completed. □

Example 4.21. Let $G_1 = (\mathbb{Z}_4, +)$ be the group, K_{G_1} be the soft set over $U = \{u_0, u_1, u_2, u_3\}$, where U represents the mobile phone brands and where $K : G_1 \longrightarrow P(U)$ is a set-valued function defined by
$K(\bar{0}) = \{u_1, u_2, u_3\}$, $K(\bar{1}) = \{u_0\}$, $K(\bar{2}) = \{u_1, u_3\}$, $K(\bar{3}) = \{u_2\}$. For $\alpha = \{u_3\}$, $K_{G_1}^{\cap\alpha} = \{\bar{0}, \bar{2}\}$ and

$$< \alpha >_\cap = \{(\bar{0}, \{u_1, u_2, u_3\}), (\bar{2}, \{u_1, u_3\})\} \widetilde{\leq} G_1.$$

Let $G_2 = (\mathbb{Z}_6, +)$ be the group, T_{G_2} be the soft set over $V = \{v_0, v_1, v_2, v_3, v_4, v_5\}$, where V represents the mobile phone features and where $T : G_2 \longrightarrow P(V)$ is a set-valued function defined by
$T(\bar{0}) = \{v_0, v_1, v_2, v_5\}$, $T(\bar{1}) = \{v_3, v_4\}$, $T(\bar{2}) = \{v_4\}$, $T(\bar{3}) = \{v_0, v_2\}$, $T(\bar{4}) = \{v_3\}$, $T(\bar{5}) = \{v_4\}$. For $\beta = \{v_2\}$, $T_{G_2}^{\cap\beta} = \{\bar{0}, \bar{3}\}$ and

$$< \beta >_\cap = \{(\bar{0}, \{v_0, v_1, v_2, v_5\}), (\bar{3}, \{v_0, v_2\})\} \widetilde{\leq} G_2.$$

Then,
$< \alpha >_\cap \times < \beta >_\cap$ is a soft set given by:
$\{((\bar{0}, \bar{0}), \{(u_1, v_0), (u_1, v_1), (u_1, v_2), (u_1, v_5), (u_2, v_0), (u_2, v_1), (u_2, v_2), (u_2, v_5),$
$(u_3, v_0)(u_3, v_1)(u_3, v_2)(u_3, v_5)\}), ((\bar{2}, \bar{0}), \{(u_1, v_0), (u_1, v_1), (u_1, v_2), (u_1, v_5),$
$(u_3, v_0), (u_3, v_1), (u_3, v_2), (u_3, v_5)\}),$
$((\bar{0}, \bar{3}), \{(u_1, v_0), (u_1, v_2), (u_2, v_0), (u_2, v_2), (u_3, v_0), (u_3, v_2)\}),$
$((\bar{2}, \bar{3}), \{(u_1, v_0), (u_1, v_2), (u_3, v_0), (u_3, v_2)\})\}$.

Now, the soft set $(H, G_1 \times G_2)$ over $U \times V$, where $H : G_1 \times G_2 \to P(U \times V)$ is a set-valued function defined by $H(x,y) = K(x) \times T(y)$ for all $(x,y) \in G_1 \times G_2$. It is easily seen that $< \alpha \times \beta >_\cap = < \alpha >_\cap \times < \beta >_\cap$.

5. Conclusion

Throughout this paper, we deal with the algebraic soft substructures of groups given in the papers [9] and [26]. We have introduced a novel concept called *set-generated soft subgroups of a group* using non-empty subsets of initial universe set. By theoretical aspect, we have applied some of the operations defined on soft sets to set-generated soft subgroups. Furthermore, we have given some relationships between the generators of soft subgroups and studied their related properties with several examples. Especially, this paper includes applications of α-intersection, which is one of the restrictive sets introduced in Section 3. Similar studies can be done on α-lower, α-upper and α-union with groups and other algebraic structures such as fields, modules, vector spaces and algebras.

Conflict of interest

The authors declare that they have no conflict of interest.

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