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KENMOTSU MANIFOLDS ADMITTING SCHOUTEN-VAN-KAMPEN CONNECTION

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Abstract. In this paper we study Schouten-Van Kampen connection associated to Kenmotsu manifold with metric g as an almost Ricci soliton. Firstly, we prove that if potential vector field of almost Ricci soliton is point wise collinear with Reeb vector field ξ, then it reduces to a homothetic vector field and if this homothetic vector field is Killing, then almost Ricci soliton reduces to steady Ricci soliton. Next we prove that the potential vector field of an almost Ricci soliton preserves Reeb vector field ξ, and (1,1) tensor ϕ. Finally, we study geometric properties of torse forming and concircular vector fields on a Kenmotsu manifold.

1. Introduction and preliminaries

In 1982, Hamilton[6] made the fundamental observation that Ricci flow is an excellent tool for simplifying the structure of a manifold. It is a process which deforms the metric of manifold M by smoothing out the irregularities. Ricci flow can be defined as the evolution of the Riemannian metric g_0 in time t to $g = g(t)$ by means of the equation

$$\frac{\partial g}{\partial t} = -2S, g(0) = g_0, \tag{1.1}$$

where S is the Ricci tensor of M. A Ricci soliton corresponds to self-similar solution of the Ricci flow and it is defined on a Riemannian manifold (M, g) by a vector field V satisfying the equation

$$\frac{1}{2}(L_V g) + S = \lambda g, \tag{1.2}$$

where L_V is the Lie derivative in the diection of V and λ is a constant. If λ in (1.2) is a smooth function, then the Ricci soliton (g, V, λ) is called an almost Ricci soliton[12]. A Ricci soliton is said to be shrinking, steady or expanding according as λ is negative, zero, positive respectively. Sharma[14] intiated the study of Ricci solitons in contact geometry. Later Tripathi[16], Nagaraja[9], De[3] and others extensively studied the Ricci solitons in contact geometry.

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The Schouten-Van Kampen connection(SVK-connection) is one of the most natural connections adapted to pair the complemetary distributions on differential manifold endowed with an affine connection. In 1978, Solov'ev[13] investigated hyper distribution in Riemannian manifold using SVK-connection on foliated manifolds. Olzako[10] studied the SVK-connection to adapt it to an almost contact metric structure. Recently Ghosh[5], Nagaraja et.al[8], Yildiz et.al[18] studied the curvature properties of Kenmotsu, f-Kenmotsu and Sasakian manifolds admitting SVK-connection. It is interesting to study special vector fields like torse forming vector field on a Kenmotsu manifold. A torse forming vector field was firstly defined and studied by Yano [17]. A nowhere vanishing vector field V on a Riemannian manifold (M, g) is called a torse forming vector field if

$$\nabla_X V = fX + \alpha(X)V, \tag{1.3}$$

where ∇ is the Levi-civita connection of g, f is a smooth function and α is a 1-form on M. Moreover V is called concircular [1] if the 1-form α vanishes identically. The vector V is concurrent[2], [19] if the 1-form α vanishes identically and the function $f = 1$. The vector field V is recurrent if the function $f = 0$. Finally if in (1.3) $f = 0 = \alpha$, the vector field V is called a parallel vector field.

In this paper we study almost Ricci soliton on a Kenmotsu manifold admitting SVK-connection and characterise such a manifold with the help of special vector fields.

We shall present some preliminaries that will be useful for establishing the desired results. A $(2n + 1)$-dimensional smooth manifold M is said to be contact if it admits a global 1-form η such that $\eta \wedge (d\eta)^n \neq 0$ on M. This 1-form is called contact 1-form on M. There exist a unique vector field ξ such that $d\eta(\xi, X) = 0$ and $\eta(\xi) = 1$. Polarizing $d\eta$ on contact sub-bundle D (defined by $\eta = 0$), we obtain Riemannian metric g, and $(1, 1)$ tensor field ϕ such that

$$d\eta(X, Y) = \Phi(X, Y) = g(X, \phi Y), \eta(X) = g(X, \xi), \phi^2(X) = -X + \eta(X)\xi, \tag{1.4}$$

for all X, $Y \in TM$.

From these equations one can deduce that

$$\phi \circ \xi = 0, \eta \circ \phi = 0, g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y). \tag{1.5}$$

The structure (ϕ, ξ, η, g) is known as an almost contact metric structure and metric g is called the associated metric. A Riemanninan manifold M together with this structure (ϕ, ξ, η, g) is said to be an almost contact metric manifold, and we denote it by (M, ϕ, ξ, η, g).

An almost contact metric manifold is said to be a Kenmotsu manifold if

$$(\nabla_X \phi)(Y) = -g(X, \phi Y) - \eta(Y)\phi X. \tag{1.6}$$

In a Kenmotsu manifold the following relations hold[3]:

$$\nabla_X \xi = X - \eta(X)\xi \tag{1.7}$$

$$(\nabla_X \eta)(Y) = g(\nabla_X \xi, Y) \tag{1.8}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X \tag{1.9}$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi \tag{1.10}$$

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X) \tag{1.11}$$

$$S(X.\xi) = -2n\eta(X) \tag{1.12}$$

$$S(\phi X, \phi Y) = S(X,Y) + 2n\eta(X)\eta(Y), \tag{1.13}$$

for all X,Y,Z on M, where R denotes the curvature tensor of type $(1,3)$ and S denotes the Ricci tensor.

Throught this paper we assosiate '-' with the quantities with respect to SVK-connection. The SVK-connection related to Levi-civita connection ∇ [10] by

$$\overline{\nabla}_X Y = \nabla_X Y - \eta(Y)\nabla_X \xi + (\nabla_X \eta)(Y)\xi. \tag{1.14}$$

Using (1.7) and (1.8) , the above equation yields

$$\overline{\nabla}_X Y = \nabla_X Y + g(X,Y)\xi - \eta(Y)X. \tag{1.15}$$

By taking $Y = \xi$ in (1.14) and using (1.7), we get

$$\overline{\nabla}_X \xi = 0. \tag{1.16}$$

We now calculate the Riemannian curvature tensor $\overline{R}$ using (1.14) as follows;

$$\overline{R}(X,Y)Z = R(X,Y)Z + g(Y,Z)X - g(X,Z)Y. \tag{1.17}$$

Putting $Z = \xi$ in (1.17), we get

$$\overline{R}(X,Y)\xi = 0. \tag{1.18}$$

Contraction of (1.17) gives

$$\overline{S}(Y,Z) = S(Y,Z) + 2ng(Y,Z), \tag{1.19}$$

and

$$\overline{r} = r + 2n. \tag{1.20}$$

Proposition 1.1. [15] *The Ricci operator Q on a (2n+1)-dimensional Kenmotsu manifold satisfies*

$$(\nabla_X Q)\xi = -QX - 2nX \tag{1.21}$$

$$(\nabla_\xi Q)X = -2QX - 4nX \tag{1.22}$$

for arbitary vector field X.

2. Almost Ricci soliton admitting Schouten-Van-Kampen-connection on Kenmotsu manifold

Theorem 2.1. *Let (M,g) be a Kenmotsu manifold admitting SVK-connection. Let (g,V,λ) and (g,V,μ) be almost Ricci solitons on M with respect to Levi-Civita-connection and SVK-connection respectively. If $\lambda = 2n - \mu$, then*

(i) *potential vector field V is Killing and M is Eienstien.*
(ii) *V transforms ξ, η and ϕ according as $L_V\xi = -4n\xi, L_V\eta = 0, (L_V\phi) = -4n\phi$.*
(iii) *V is a strictly infinitesimal contact transformation.*

Proof. Let (g, V, μ) be an almost Ricci soliton with respect to SVK-connection. Then

$$(\overline{L}_V g)(X,Y) + 2\overline{S}(X,Y) - 2\mu g(X,Y) = 0, \tag{2.1}$$

where $\overline{L}_V$ is Lie derivative along V with respect to SVK-connection, $\overline{S}$ is Ricci tensor with respect to SVK-connection, μ is an arbitary scalar function.
We have

$$(\overline{L}_V g)(X,Y) = g(\overline{\nabla}_X V, Y) + g(\overline{\nabla}_Y V, X). \tag{2.2}$$

Using (1.15) in (2.2), we get

$$\begin{aligned}(\overline{L}_V g)(X,Y) =& g(\nabla_X V, Y) + g(X, \nabla_Y V) + g(X,V)\eta(Y)\\ &+ g(Y,V)\eta(X) - g(X,Y)\eta(V).\end{aligned} \tag{2.3}$$

From equation (2.3), we get

$$\begin{aligned}(\overline{L}_V g)(X,Y) =& (L_V g)(X,Y) + g(X,V)\eta(Y) + g(Y,V)\eta(X)\\ &- 2\eta(V)g(X,Y).\end{aligned} \tag{2.4}$$

Putting (2.4) in (2.1) and using (1.2) and (1.19), the equation (2.1) reduces to

$$\begin{aligned}&(L_V g)(X,Y) + 2S(X,Y) + (4n-2\mu)g(X,Y) + g(X,V)\eta(Y)\\ &+ g(Y,V)\eta(X) - 2g(X,Y)\eta(V) = 0,\end{aligned} \tag{2.5}$$

$$If \quad (L_V g)(X,Y) + 2S(X,Y) + (4n-2\mu)g(X,Y) = 0, \tag{2.6}$$

then from (2.5), we have

$$g(X,V)\eta(Y) + g(Y,V)\eta(X) - 2\eta(V)g(X,Y) = 0. \tag{2.7}$$

From equation (2.6),it follows that soliton constant is $\lambda = 2n - \mu$.
Putting $Y = \xi$ in (2.7), it follows that we get $V = \eta(V)\xi$, which is point wise collinear with ξ. If we set $\eta(V) = f$, then $V = f\xi$. Taking covariant derivative of this along X, we get

$$\nabla_X V = (Xf)\xi + fX - f\eta(X)\xi. \tag{2.8}$$

Using (2.8) in

$$(L_V g)(X,Y) = g(\nabla_X V, Y) + g(\nabla_Y V, X), \tag{2.9}$$

we obtain

$$(L_V g)(X,Y) = (Xf)\eta(Y) + (Yf)\eta(X) + 2fg(X,Y) - 2f\eta(X)\eta(Y). \tag{2.10}$$

Using equation (2.10) in (2.6), we get

$$(Xf)\eta(Y)+(Yf)\eta(X)+2fg(X,Y)-2f\eta(X)\eta(Y)+(4n-2\lambda)g(X,Y)+2S(X,Y) = 0. \tag{2.11}$$

Setting $Y = \xi$ in (2.11) gives

$$(Xf) = (2\lambda - (\xi f))\eta(X). \tag{2.12}$$

Replacing X by ξ in (2.12), we obtain $(\xi f) = \lambda$, which when substitued in (2.12) gives

$$(Xf) = \lambda\eta(X) \tag{2.13}$$

for all X. The above equation can be written as

$$df = \lambda\eta. \tag{2.14}$$

Applying d to (2.14) and using ($d^2 = 0$), we obatin

$$d\lambda \wedge \eta + \lambda d\eta = 0 \tag{2.15}$$

$$or \quad \frac{1}{2}\{d\lambda(X)\eta(Y) - d\lambda(Y)\eta(X)\} + \lambda d\eta(X,Y) = 0. \tag{2.16}$$

Taking $Y = \xi$ in (2.16) we obatin $d\lambda = 0$. Again use of this in (2.16) yields $\lambda d\eta = 0$. Since $d\eta$ is non-vanishing we have $\lambda = 0$. Using this in (2.13), we get $(Xf) = 0$ and hence $f = \eta(V)$. And equation (2.11) reduces to

$$2S(X,Y) = -(2f + 4n)g(X,Y) + 2f\eta(X)\eta(Y). \tag{2.17}$$

Using (2.17) in (2.6) and choosing X and Y orthogonal to ξ, we get

$$(L_V g)(X,Y) = 2fg(X,Y). \tag{2.18}$$

Since f is a constant, the vector field V is a homothetic vector field.
Using (2.18) in (2.6), we get

$$S(X,Y) = -(2n + f)g(X,Y). \tag{2.19}$$

On comparing equation (2.17) and (2.19), we obtain $f = 0$. Using this in (2.18), we obtain

$$(L_V g)(X,Y) = 0. \tag{2.20}$$

i.e., the vector field V is Killing and

$$S(X,Y) = -2ng(X,Y). \tag{2.21}$$

i.e., the manifold is Einstien. This completes the proof of (i).
Using equation(2.21) in (1.2), we get

$$(L_V g)(X,Y) = 4ng(X,Y). \tag{2.22}$$

Setting $Y = \xi$ in (2.22), we have

$$(L_V\eta)(X) = 4n\eta(X) + g(X, L_V\xi). \tag{2.23}$$

From which we get

$$(L_V g)(\xi,\xi) = 4n. \tag{2.24}$$

By direct computaion, we get

$$\frac{1}{2}(L_V g)(\xi,\xi) = -\eta(L_V\xi). \tag{2.25}$$

$$and \quad (L_V\eta)(\xi) = -\eta(L_V\xi). \tag{2.26}$$

On comapring last two equations, we obtain

$$\frac{1}{2}(L_V g)(\xi,\xi) = (L_V\eta)(\xi). \tag{2.27}$$

Using (2.24) in (2.26), we get

$$(L_V\eta)(\xi) = 2n. \tag{2.28}$$

Lie differentiating $\eta(X) = g(X,\xi)$ along V and using (2.18), we get

$$(L_V\eta)(X) = 2f\eta(X). \tag{2.29}$$

From (2.29) in (2.23) and using $f = 0$, we have

$$g(X, L_V\xi) = -4n\eta(X), \tag{2.30}$$

For all X and therefore $L_V\xi = -4n\xi$. Using this in (2.23), we get

$$L_V\eta = 0 \tag{2.31}$$

Equation (2.31) shows that V is a strict infinitesimal contact transformation. Taking Lie derivative of the first term of (1.4) along V, and noticing the fact that L_V commutes with exteroir derivative operator d, we get

$$(d(L_V\eta))(X, Y) = (L_V g)(X, \phi Y) + g(X, (L_V\phi)(Y)). \tag{2.32}$$

Applying d on (2.31), we get

$$(d(L_V\eta))(X, Y) = 0. \tag{2.33}$$

Replace Y by ϕY in equation (2.22) to get

$$(L_V g)(X, Y) = 4ng(X, \phi Y). \tag{2.34}$$

Using equation (2.33) and (2.34) in (2.32), we obtain

$$g(X, (L_V\phi)(Y)) = -4ng(X, \phi Y), \tag{2.35}$$

for all X and Y. Therefore the above equation may be written as

$$(L_V\phi) = -4n\phi.$$

This completes the proof of (ii). □

3. **Special vector field on Kemnotsu manifold admitting SVK-connection**

Theorem 3.1. *Let (M, g) be a Kenmotsu manifold admitting SVK-connection, and let the metric g be an almost Ricci soliton whose potential vector field V as torse forming vector field is point wise collinear with ξ, then V reduces to a concircular vector field.*

Proof. On comparing equation (2.8) and (1.3), we get

$$(Xf)\xi = \alpha(X)V + f\eta(X)\xi. \tag{3.1}$$

Take innerproduct with ξ, in equation (3.1), we get

$$(Xf) = \alpha(X)\eta(V) + f\eta(X). \tag{3.2}$$

Again take innerproduct with Y in equation (3.1), we obtain

$$(Xf)\eta(Y) = \alpha(X)g(V, Y) + f\eta(X)\eta(Y). \tag{3.3}$$

Using (3.2) in (3.3), we get

$$\alpha(X)g(\phi V, \phi Y) = 0. \tag{3.4}$$

Since $g(\phi V, \phi Y) \neq 0$, it follows that $\alpha(X) = 0$. Using $\alpha(X) = 0$ in (1.4) we get

$$\nabla_X V = fX. \tag{3.5}$$

□

Theorem 3.2. *Let (M, g) be a (2n+1)-dimensional Kenmotsu manifold admitting SVK-connection. If the metric g is an almost Ricci soliton with potential vector field V is a concircular vector field, then gradient vector field Df is also a concircular vector field.*

Proof. Riemannian curvature tensor of almost Ricci soliton (M, g, V, λ) is given by [4]

$$\begin{aligned} R(X,Y)V =&(X\lambda)Y - (Y\lambda)X + (\nabla_Y Q)X - (\nabla_X Q)Y + \\ &(\nabla_X \phi)Y - (\nabla_Y \phi)X. \end{aligned} \tag{3.6}$$

We use (3.5) in $R(X,Y)V = \nabla_X\nabla_Y V - \nabla_Y\nabla_X V - \nabla_{[X,Y]}V$, to obtain

$$R(X,Y)V = (Xf)Y - (Yf)X. \tag{3.7}$$

Using (3.7) in (1.17), we get

$$\overline{R}(X,Y)V = (Xf)Y - (Yf)X + g(Y,V)X - g(X,V)Y. \tag{3.8}$$

Contracting (3.8) with respect to orthonormal basis $(e_i)_1^{2n+1}$, we obtain

$$\overline{S}(Y,V) = -2n(Yf) + 2ng(Y,V). \tag{3.9}$$

On comparing (3.9) and (1.19), we get

$$S(Y,V) = -2n(Yf). \tag{3.10}$$

Putting $Y = \xi$ in above equation, we obtain

$$S(\xi,V) = -2n(\xi f). \tag{3.11}$$

Comparing (3.11) and (1.12), we get

$$\eta(V) = (\xi f). \tag{3.12}$$

Taking innerproduct with ξ and choosing $X = \xi$ in (3.8), we obtain

$$g(\overline{R}(\xi,Y)V,\xi) = (\xi f)\eta(Y) - (Yf) + g(Y,V) - \eta(V)\eta(Y). \tag{3.13}$$

Again putting $X = \xi$ and taking inner product with ξ in equation (1.17), we get

$$g(\overline{R}(\xi,Y)V,\xi) = 0. \tag{3.14}$$

On comparing (3.13) and (3.14) and using (3.12), we get

$$(Yf) = g(Y,V). \tag{3.15}$$

$$or \quad Df = V. \tag{3.16}$$

Taking covariant derivative with respect to Z in the foregoing equation, from (3.5) it follows that Df is a concircular vector field. □

Remark 3.3. Applying Poincare lemma i.e.,($d^2 = 0$) to last obtained equation, we get fZ=0, where Z is arbitary vector field which is non-zero, therefore $f = 0$. Putting $f = 0$ and $\alpha(X) = 0$ in (1.3), then V becomes a parallel vector field.

Theorem 3.4. *Let (M, g) be a Kenmotsu manifold admitting SVK-connection. If g is an almost Ricci soliton with potential vector field V as a concircular vector field, then almost Ricci soliton reduces to Ricci soliton with soliton constant $\lambda = f + c$.*

Proof. Using (1.6) in equation (3.6) and putting $X = \xi$, we get

$$R(\xi, Y)V = (\xi\lambda)Y - (Y\lambda)\xi + (\nabla_Y Q)\xi - (\nabla_\xi Q)Y + \phi Y. \tag{3.17}$$

Using equations (2.18) and (2.19) in (3.17), we get

$$R(\xi, Y)V = (\xi\lambda)Y - (Y\lambda)\xi + QY + 2nY + \phi Y. \tag{3.18}$$

Take inner product with ξ in equation (3.18) and using (1.12), we get

$$g(R(\xi, Y)V, \xi) = (\xi\lambda)\eta(Y) - (Y\lambda). \tag{3.19}$$

Using equation (3.19) in (1.17), we get

$$g(\overline{R}(\xi, Y)V, \xi) = (\xi\lambda)\eta(Y) - (Y\lambda) + g(Y, V) - \eta(Y)\eta(V). \tag{3.20}$$

On comparing (3.20) and (3.13) , we obtain

$$(Y\lambda) - (Yf) = (\xi\lambda)\eta(Y) - (\xi\lambda)\eta(Y). \tag{3.21}$$

$$g(X, D(\lambda - f)) = (\xi(\lambda - f))\eta(X), \tag{3.22}$$

for all X. Therefore we have

$$d(f + \lambda) = (\xi(\lambda - f))\eta. \tag{3.23}$$

Applying d and taking wedge product with $d\eta$, we obtain

$$(\xi(\lambda - f))\eta \wedge d\eta = 0. \tag{3.24}$$

Since $\eta \wedge (d\eta)^n$ is volume element of M, $\eta \wedge d\eta \neq 0$, we have

$$(\xi(\lambda - f)) = 0. \tag{3.25}$$

Using (3.25) in (3.22), we obtain

$$g(X, D(\lambda - f)) = 0, \tag{3.26}$$

for all X. Hence we have

$$D(\lambda - f) = 0. \tag{3.27}$$

On integrating, we obtain $\lambda - f = c$ a constant. Hence nature of the soliton depends on both λ and f. □

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