Annals of Mathematics and Computer Science
ISSN: 2789-7206
Vol 10 (2022) 19-26

ON SOME NEW CONTINUED FRACTION EXPANSIONS FOR THE RATIOS OF THE FUNCTION $\rho(a, b)$

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ABSTRACT. We obtain some new continued fraction expansions of the ratios $\rho(aq, bq)/\rho(a, bq)$ and $\rho(aq, bq)/\rho(aq, b)$. We deduce therefrom some special cases of these continued fractions which are analogous to the continued fractions of Ramanujan.

1. INTRODUCTION

Ramanujan has defined the function $\rho(a, b)$ in his lost notebook [22] as

$$\rho(a,b) := \left(1+\frac{1}{b}\right)\sum_{n=0}^{\infty}\frac{(-1)^n q^{n(n+1)/2}a^n b^{-n}}{(-aq)_n}, \tag{1.1}$$

where $a,\ b$ are complex numbers other than 0 and $-q^{-n}$ and

$$(a)_n = \prod_{k=0}^{n-1}(1-aq^k), n = 1, 2, 3, \cdots, |q| < 1. \tag{1.2}$$

Ramanujan, a pioneer in the theory of continued fractions has recorded scores of continued fraction identities in chapter 12 of his second notebook [21] and in his lost notebook [22]. This part of Ramanujan's work has been studied and developed by several authors including Carlitz [12], Gordon [16], Hirschhorn [17], Andrews [4], Al-Salam and Ismail [3], Ramanathan [19, 20], Denis [13, 14, 15], Bhargava and Adiga [8, 9], Bhargava, Adiga and Somashekara [10, 11], Verma, Denis and Srinivasa Rao [28], Singh [23], Bhagirathi [5, 6, 7], Adiga and Somashekara [2], Vasuki [26], Adiga, Denis and Vasuki [1], Vasuki and Madhusudan [27], Somashekara and Fathima [24], Mamta and Somashekara [18], Somashekara, Shalini and Narasimha Murthy [25].

The main purpose of this paper is to establish new continued fraction expansions for the ratios: $\rho(aq, bq)/\rho(a, bq)$ and $\rho(aq, bq)/\rho(aq, b)$, which complement the work of Somashekara, Shalini and Narasimha Murthy [25].

Date: Received: Sep 4, 2022; Accepted: Nov 9, 2022.
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2020 *Mathematics Subject Classification.* Primary 11A55 , 33D15.
Key words and phrases. Basic hypergeometric series, reciprocity theorem, q-continued fraction.

In section 2, we state some key functional relations satisfied by $\rho(a,b)$ which will be used in the proof of our main results. In section 3, we prove our main results and in section 4 we obtain some special cases of our continued fractions which are analogous to the continued fractions of Ramanujan.

2. Preliminary Results

In this section we give some functional relations of $\rho(a,b)$.

Lemma 2.1. *We have*

$$(1+aq)\frac{\rho(a,b)}{(1+\frac{1}{b})} = aq\frac{\rho(aq,b)}{(1+\frac{1}{b})} + \frac{\rho(aq,bq)}{(1+\frac{1}{bq})}, \tag{2.1}$$

$$(1+aq)\frac{\rho(a,bq)}{(1+\frac{1}{bq})} - (1+aq)\frac{\rho(a,b)}{(1+\frac{1}{b})} = \frac{aq}{b}\frac{\rho(aq,b)}{(1+\frac{1}{b})} - \frac{a}{b}\frac{\rho(aq,bq)}{(1+\frac{1}{bq})}, \tag{2.2}$$

$$(1+aq)\frac{\rho(a,bq)}{(1+\frac{1}{bq})} = aq\frac{\rho(aq,bq)}{(1+\frac{1}{bq})} + \frac{\rho(aq,bq^2)}{(1+\frac{1}{bq^2})}, \tag{2.3}$$

$$\rho(a,b) = \frac{\rho(a,bq)}{(1+\frac{1}{bq})} + \frac{a(1+\frac{1}{b})}{(1+aq)(1+\frac{1}{bq})}\rho(aq,bq), \tag{2.4}$$

$$\rho(aq,b) = \frac{\rho(aq,bq)}{(1+\frac{1}{bq})} + \frac{aq(1+\frac{1}{b})}{(1+aq^2)(1+\frac{1}{bq})}\rho(aq^2,bq). \tag{2.5}$$

Proof. For proofs of (2.1) - (2.3), see [25]. Substituting (2.1) in (2.2) we obtain (2.4) and changing a to aq in (2.4) we get (2.5). □

3. Main results

In this section we deduce the continued fraction expansions for the ratios $\rho(aq,bq)/\rho(a,bq)$ and $\rho(aq,bq)/\rho(aq,b)$.

Theorem 3.1. *We have*

$$\frac{\rho(aq,bq)}{\rho(a,bq)} = \frac{1+aq}{A_1+}\;\frac{B_1}{1+}\;\frac{C_1}{A_2+}\;\frac{B_2}{1+}\;\frac{C_2}{A_3+\cdots}\;\frac{C_{n-1}}{A_n+}\;\frac{B_n}{1+\cdots}, \tag{3.1}$$

where

$$A_n = aq^n,\ \ B_n = (1+\frac{1}{bq^n})$$

and

$$C_n = aq^n(1+\frac{1}{bq^n}),\ \ n = 1,2,3,\cdots.$$

Proof. Changing a to aq^n and b to bq^n in (2.3), we obtain

$$\rho(aq^n,bq^{n+1}) = \frac{aq^{n+1}}{(1+aq^{n+1})}\rho(aq^{n+1},bq^{n+1}) + \frac{(1+\frac{1}{bq^{n+1}})}{(1+aq^{n+1})(1+\frac{1}{bq^{n+2}})}\rho(aq^{n+1},bq^{n+2}). \tag{3.2}$$

Changing a to aq^n and b to bq^n in (2.4), we obtain

$$\rho(aq^n,bq^n)=\frac{\rho(aq^n,bq^{n+1})}{(1+\frac{1}{bq^{n+1}})}+\frac{aq^n(1+\frac{1}{bq^n})}{(1+aq^{n+1})(1+\frac{1}{bq^{n+1}})}\rho(aq^{n+1},bq^{n+1}). \tag{3.3}$$

(3.2) and (3.3) can be written as

$$S_n\equiv\frac{\rho(aq^n,bq^{n+1})}{\rho(aq^{n+1},bq^{n+1})}=\frac{aq^{n+1}}{(1+aq^{n+1})}+\frac{(1+\frac{1}{bq^{n+1}})}{(1+aq^{n+1})(1+\frac{1}{bq^{n+2}})}\frac{1}{T_{n+1}}, \tag{3.4}$$

$$T_n\equiv\frac{\rho(aq^n,bq^n)}{\rho(aq^n,bq^{n+1})}=\frac{1}{(1+\frac{1}{bq^{n+1}})}+\frac{aq^n(1+\frac{1}{bq^n})}{(1+aq^{n+1})(1+\frac{1}{bq^{n+1}})}\frac{1}{S_n}. \tag{3.5}$$

Iterating (3.4) and (3.5) with $n=0,1,2,\cdots$ and then taking reciprocals we obtain (3.1) after some simplifications. □

Theorem 3.2. *We have*

$$\frac{\rho(aq,bq)}{\rho(aq,b)}=\frac{B_0}{1+}\;\frac{D_0}{A_1+}\;\frac{B_1}{1+}\;\frac{D_1}{A_2+}\;\frac{B_2}{1+}\;\frac{D_2}{A_3}\;\frac{B_3}{1+\cdots}\;\frac{D_{n-1}}{A_n+}\;\frac{B_n}{1+\cdots}, \tag{3.6}$$

where

$$A_n=aq^n,\;\;B_n=(1+\frac{1}{bq^n})$$

and

$$D_n=aq^{n+1}(1+\frac{1}{bq^n}),\;\;n=0,1,2,\cdots.$$

Proof. Changing a to aq^n and b to bq^n in (2.5), we obtain

$$\rho(aq^{n+1},bq^n)=\frac{\rho(aq^{n+1},bq^{n+1})}{(1+\frac{1}{bq^{n+1}})}+\frac{aq^{n+1}(1+\frac{1}{bq^n})}{(1+aq^{n+2})(1+\frac{1}{bq^{n+1}})}\rho(aq^{n+2},bq^{n+1}). \tag{3.7}$$

Changing a to aq^n and b to bq^n in (2.1), we obtain

$$\rho(aq^n,bq^n)=\frac{aq^n}{(1+aq^{n+1})}\rho(aq^{n+1},bq^n)+\frac{(1+\frac{1}{bq^n})}{(1+aq^{n+1})(1+\frac{1}{bq^{n+1}})}\rho(aq^{n+1},bq^{n+1}). \tag{3.8}$$

(3.7) and (3.8) can be written as

$$U_n\equiv\frac{\rho(aq^{n+1},bq^n)}{\rho(aq^{n+1},bq^{n+1})}=\frac{1}{(1+\frac{1}{bq^{n+1}})}+\frac{aq^{n+1}(1+\frac{1}{bq^n})}{(1+aq^{n+2})(1+\frac{1}{bq^{n+1}})}\frac{1}{V_{n+1}}, \tag{3.9}$$

$$V_n\equiv\frac{\rho(aq^n,bq^n)}{\rho(aq^{n+1},bq^n)}=\frac{aq^n}{(1+aq^{n+1})}+\frac{(1+\frac{1}{bq^n})}{(1+aq^{n+1})(1+\frac{1}{bq^{n+1}})}\frac{1}{U_n}. \tag{3.10}$$

Iterating (3.9) and (3.10) with $n=0,1,2,\cdots$ and then taking reciprocals we obtain (3.6) after some simplifications. □

4. Special Cases

In this section we extract the following special cases of (3.1) and (3.6).

$$\frac{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q^2)_n}}{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(-q)_n}} = \frac{1+q}{q+}\ \frac{(1+1/q)}{1+}\ \frac{q(1+1/q)}{q^2+}\ \frac{(1+1/q^2)}{1+\cdots}, \tag{4.1}$$

$$\frac{\displaystyle\sum_{n=0}^{\infty}\frac{q^{\frac{n(n+1)}{2}}}{(-q^2)_n}}{\displaystyle\sum_{n=0}^{\infty}\frac{q^{\frac{n(n-1)}{2}}}{(-q)_n}} = \frac{1+q}{q+}\ \frac{(1-1/q)}{1+}\ \frac{q(1-1/q)}{q^2+}\ \frac{(1-1/q^2)}{1+\cdots}, \tag{4.2}$$

$$\frac{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+3)}{2}}}{(-q^3)_n}}{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q^2)_n}} = \frac{1+q^2}{q^2+}\ \frac{(1+1/q)}{1+}\ \frac{q^2(1+1/q)}{q^3+}\ \frac{(1+1/q^2)}{1+\cdots}, \tag{4.3}$$

$$\frac{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(-q^2)_n}}{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n-3)}{2}}}{(-q)_n}} = \frac{1+q}{q+}\ \frac{(1+1/q^2)}{1+}\ \frac{q(1+1/q^2)}{q^2+}\ \frac{(1+1/q^3)}{1+\cdots}, \tag{4.4}$$

$$\frac{\displaystyle\sum_{n=0}^{\infty}\frac{q^{\frac{n(n+3)}{2}}}{(-q^3)_n}}{\displaystyle\sum_{n=0}^{\infty}\frac{q^{\frac{n(n+1)}{2}}}{(-q^2)_n}} = \frac{1+q^2}{q^2+}\ \frac{(1-1/q)}{1+}\ \frac{q^2(1-1/q)}{q^3+}\ \frac{(1-1/q^2)}{1+\cdots}, \tag{4.5}$$

$$\frac{\displaystyle\sum_{n=0}^{\infty}\frac{q^{\frac{n(n-1)}{2}}}{(-q^2)_n}}{\displaystyle\sum_{n=0}^{\infty}\frac{q^{\frac{n(n-3)}{2}}}{(-q)_n}} = \frac{1+q}{q+}\ \frac{(1-1/q^2)}{1+}\ \frac{q(1-1/q^2)}{q^2+}\ \frac{(1-1/q^3)}{1+\cdots}, \tag{4.6}$$

$$\frac{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+3)}{2}}}{(-q^2)_n}}{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q)_n}} = \frac{1+q}{q+}\ \frac{2}{1+}\ \frac{2q}{q^2+}\ \frac{(1+1/q)}{1+}\ \frac{q^2(1+1/q)}{q^3+\cdots}, \tag{4.7}$$

$$\frac{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(-q)_n}}{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n-3)}{2}}}{(-1)_n}} = \frac{2}{1+}\ \frac{(1+1/q)}{1+}\ \frac{1+1/q}{q+}\ \frac{(1+1/q^2)}{1+}\ \frac{q(1+1/q^2)}{q^2+\cdots}, \tag{4.8}$$

$$\frac{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q)_n}}{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(-1)_n}} = \frac{2}{1+}\ \frac{2}{1+}\ \frac{2}{q+}\ \frac{(1+1/q)}{1+}\ \frac{q(1+1/q)}{q^2+\cdots}, \tag{4.9}$$

$$\frac{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q^3)_n}}{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(-q^2)_n}} = \frac{1+q^2}{q^2+}\ \frac{(1+1/q^2)}{1+}\ \frac{q^2(1+1/q^2)}{q^3+}\ \frac{(1+1/q^3)}{1+\cdots}, \tag{4.10}$$

$$\frac{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(-q^3)_n}}{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n-3)}{2}}}{(-q^2)_n}} = \frac{1+q^2}{q^2+}\ \frac{(1+1/q^3)}{1+}\ \frac{q^2(1+1/q^3)}{q^3+}\ \frac{(1+1/q^4)}{1+\cdots}, \tag{4.11}$$

$$\frac{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n+3)}{2}}}{(-q^4)_n}}{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q^3)_n}} = \frac{1+q^3}{q^3+}\ \frac{(1+1/q^2)}{1+}\ \frac{q^3(1+1/q^2)}{q^4+}\ \frac{(1+1/q^3)}{1+\cdots}, \tag{4.12}$$

$$\frac{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q^2)_n}}{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n+3)}{2}}}{(-q^2)_n}} = \frac{2}{1+}\ \frac{2q}{q+}\ \frac{(1+1/q)}{1+}\ \frac{q^2(1+1/q)}{q^2+\cdots}, \tag{4.13}$$

$$\frac{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n+3)}{2}}}{(-q^3)_n}}{\sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n+5)}{2}}}{(-q^3)_n}} = \frac{2}{1+}\ \frac{2q^2}{q^2+}\ \frac{(1+1/q)}{1+}\ \frac{q^3(1+1/q)}{q^3+\cdots}, \tag{4.14}$$

$$\frac{\sum_{n=0}^{\infty} \frac{q^{\frac{n(n+1)}{2}}}{(q^2)_n}}{\sum_{n=0}^{\infty} \frac{q^{\frac{n(n+3)}{2}}}{(q^2)_n}} = \frac{2}{1+}\ \frac{-2q}{-q+}\ \frac{(1+1/q)}{1+}\ \frac{-q^2(1+1/q)}{-q^2+\cdots}, \tag{4.15}$$

$$\frac{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(-q^2)_n}}{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q^2)_n}} = \frac{1+1/q}{1+}\ \frac{q(1+1/q)}{q+}\ \frac{(1+1/q^2)}{1+}\ \frac{q^2(1+1/q^2)}{q^2+\cdots}, \tag{4.16}$$

$$\frac{\sum_{n=0}^{\infty}\frac{q^{\frac{n(n-1)}{2}}}{(q^2)_n}}{\sum_{n=0}^{\infty}\frac{q^{\frac{n(n+1)}{2}}}{(q^2)_n}} = \frac{1+1/q}{1+}\ \frac{-q(1+1/q)}{-q+}\ \frac{(1+1/q^2)}{1+}\ \frac{-q^2(1+1/q^2)}{-q^2+\cdots}, \tag{4.17}$$

$$\frac{\sum_{n=0}^{\infty}\frac{q^{\frac{n(n-1)}{2}}}{(-q^2)_n}}{\sum_{n=0}^{\infty}\frac{q^{\frac{n(n+1)}{2}}}{(-q^2)_n}} = \frac{1-1/q}{1+}\ \frac{q(1-1/q)}{q+}\ \frac{(1-1/q^2)}{1+}\ \frac{q^2(1-1/q^2)}{q^2+\cdots}, \tag{4.18}$$

$$\frac{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(q^2)_n}}{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(q^2)_n}} = \frac{1-1/q}{1+}\ \frac{-q(1-1/q)}{-q+}\ \frac{(1-1/q^2)}{1+}\ \frac{-q^2(1-1/q^2)}{-q^2+\cdots}, \tag{4.19}$$

$$\frac{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q^3)_n}}{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+3)}{2}}}{(-q^3)_n}} = \frac{1+1/q}{1+}\ \frac{q^2(1+1/q)}{q^2+}\ \frac{(1+1/q^2)}{1+}\ \frac{q^3(1+1/q^2)}{q^3+\cdots}, \tag{4.20}$$

$$\frac{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(-q^3)_n}}{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q^3)_n}} = \frac{1+1/q^2}{1+}\ \frac{q^2(1+1/q^2)}{q^2+}\ \frac{(1+1/q^3)}{1+}\ \frac{q^3(1+1/q^3)}{q^3+\cdots}, \tag{4.21}$$

$$\frac{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n-1)}{2}}}{(-q)_n}}{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q)_n}} = \frac{2}{1+}\ \frac{1}{1+}\ \frac{(1+1/q)}{1+}\ \frac{q(1+1/q)}{q+}\ \frac{(1+1/q^2)}{1+\cdots}, \tag{4.22}$$

$$\frac{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+3)}{2}}}{(-q^2)_n}}{\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+5)}{2}}}{(-q^2)_n}} = \frac{1+q}{1+}\ \frac{q(1+1/q)}{q+}\ \frac{2}{1+}\ \frac{2q^2}{q^2+\cdots}, \tag{4.23}$$

$$\frac{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(-q)_n}}{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n+3)}{2}}}{(-q)_n}} = \frac{1+q}{1+}\ \frac{(1+q)}{1+}\ \frac{2}{1+}\ \frac{2q}{q+}\ \frac{1+1/q}{1+\cdots}. \tag{4.24}$$

Proof. Setting $a = 1, b = 1$ in (3.1) and (3.6) and using the definition (1.1) we obtain (4.1) and (4.13) after some simplifications.

Substituting $a = 1, b = -1$; $a = q, b = -1$ and $a = q^2, b = q$ in (3.1) and using (1.1) we obtain (4.2), (4.5) and (4.12) respectively.

Substituting $a = q, b = 1$; $a = 1, b = q$ and $a = 1, b = -q$ in in (3.1) and using (1.1) we obtain (4.3), (4.4) and (4.6) after some simplification respectively. Similarly we obtain (4.7) - (4.11) from (3.1) by substituting $a = 1, b = 1/q$; $a = 1/q, b = 1$; $a = 1/q = b$; $a = q = b$ and $a = q, b = q^2$ respectively.

Substituting $a = q, b = 1$; $a = -1, b = 1$; $a = 1, b = q$ and $a = -1, b = q$ in (3.6) and using the definition (1.1) we obtain (4.14) - (4.17) respectively. Similarly (4.18) - (4.24) are obtained by substituting $a = 1, b = -q$; $a = -1, b = -q$; $a = q = b$; $a = q, b = q^2$; $a = 1/q, b = 1$; $a = 1, b = 1/q$ and $a = 1/q = b$ respectively. □

Acknowledgement. The author would like to thank Prof.D.D.Somashekara, Department of Mathematics, University of Mysore for his guidance and sustaining encouragement.

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